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Ideal grill compactness space

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Abstract

In this study, we have employed the concepts of ideal and grill collections with a new definition of compactness. We call it ideal grill compact space, where we have gotten the important characteristics and relationships of those spaces.

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1. Introduction

Kuratwaski [9] and Vaidyanathswamy [15] were the first to define the concept of ideal topological space. In [8] Jankovic and Hamlett proposed a new topology from a topology space (Ӽ, T) induced by an ideal on Ӽ and described as follows:

Let $\mathcal{J}$ be an ideal on a topological space (Ӽ, T). The operator $(\ )^{*}: p(Ӽ) \rightarrow p(Ӽ)$ is said to be a local function of д with respect to T and $\mathcal{J}$ [15] given by

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$д^*(\mathcal{I},\mathrm{T}) = \{$x: д ∩ Ű ∉ $\mathcal{I}$, for every open Ű $of\ x$ }. Then $\beta(\mathrm{T},\mathcal{I}) = \{$V − д: V ∈ T, д ∈ $\mathcal{I}\}$ is a base for the topology $\mathrm{T}^*(\mathcal{I})$on Ӽ which is strictly finer than T in general. In [10] Newcomb introduced the concept of compactness modulo in 1967. There are contemporary researchers who work in the field of ideal spaces for years on fuzzy, soft, proximity, and center sets and we mention [1, 2, 3, 5, 11, 16]. In 1947, Choquet [4] started a great idea of the grill, which later proved to be a useful instrument for investigations of various topological spaces, such as nets and filters. Following Choquet [4], a grill $\mathcal{G}$ on a space Ӽ is a collection of subsets of a non-empty set Ӽ such that:

(i) $\emptyset \notin \mathcal{G}$.

(ii) д ∈ $\mathcal{G}$ and д ⊆ 𝔐 then 𝔐 ∈ $\mathcal{G}$.

(iii) д, 𝔐 ⊆ Ӽ and д ∪ 𝔐 ∈ $\mathcal{G}$ then д ∈ $\mathcal{G}$ or 𝔐 ∈ $\mathcal{G}$.

A topological space (Ӽ, T) with a grill $\mathcal{G}$ on Ӽ is represented by (Ӽ, T, $\mathcal{G}$). Roy and Mukherjee [12] established a new topology on the grill for any topological space (X, T) and they called it the generated topology which is finer than T with the basis β that for any 𝔎 ∈ β, there exists Ű ∈ T and д ∉ $\mathcal{G}$ such that 𝔎 = Ű − д and denoted by $\mathrm{T}_{\mathcal{G}}$. Roy and Mukherjee [13] defined a kind of compactness concerning the grill. In this research, we introduce a new definition of ideal grill spaces, namely $\mathrm{I}_{\mathcal{G}}$-compact space. Also, we give useful characterizations of $\mathrm{I}_{\mathcal{G}}$-compact space.

2. Preliminaries

Definition 2.1[10]: A subset Ҕ of ideal space (Ӽ, T, $\mathcal{I}$) is called $\mathcal{I}$ -compact if for every open cover $\{\mathfrak{H}_{\tilde{\alpha}}: \tilde{\alpha} \in \Omega\}$ of Ҕ, there exists a finite subsets Ω_0 of Ω such that Ҕ $- \bigcup_{\tilde{\alpha}\in\Omega_0} \mathfrak{H}_{\tilde{\alpha}} \in \mathcal{I}$.The space(Ӽ,T, $\mathcal{I}$) is called $\mathcal{I}$ -compact if Ӽ is $\mathcal{I}$ -compact a subset. So (Ӽ, T) is compact if and only if (Ӽ, T, $\{\emptyset\}$) is $\{\emptyset\}$ -compact.

Definition 2.2[13]: Let (Ӽ,T, $\mathcal{G}$) be grill space. A space Ӽ is called $\mathcal{G}$ −compact if every open cover $\{$Ừ$_{\tilde{\alpha}}: \tilde{\alpha} \in \Lambda\}$ of Ӽ, if there exists a finite subset Λ_0 of Λ such that Ӽ $- \bigcup_{\tilde{\alpha}\in\Lambda_0}$ Ừ$_{\tilde{\alpha}} \notin \mathcal{G}$.

Theorem 2.3 [6]: *Every $\mathcal{I}$ -compact subset of Hausdorff's ideal space is* T^*-*closed.*

Remark 2.4 [10]: If $(\mathfrak{X}, \mathfrak{T}, \mathcal{J})$ is an ideal space and $\mathfrak{K}$ is a subset of $\mathfrak{X}$, then $(\mathfrak{K}, \mathfrak{T}_{\mathfrak{K}}, \mathcal{J}_{\mathfrak{K}})$ where $\mathfrak{T}_{\mathfrak{K}}$ is the relative topology on $\mathfrak{K}$ and $\mathcal{J}_{\mathfrak{K}} = \left\{\mathrm{I}_{\mathfrak{K}} \cap j : j \in \mathcal{J}\right\}$ *is an ideal* topological subspace.

Theorem 2.5[14]: *If $G_i \in \mathcal{G}$ for all $i \in I$ then* $\bigcup_{i \in I} G_i \in \mathcal{G}$.

Remark 2.6: [7,10]:

i. If $\mathfrak{F}: (\mathfrak{X}, \mathrm{T}, \mathcal{J}) \rightarrow (\mathrm{Y}, \acute{\ddot{\upsilon}})$ is one to one function and $\mathcal{J}$ is ideal on $\mathfrak{X}$, then $\mathfrak{F}(\mathcal{J}) = \left\{\mathfrak{F}(д): д \in \mathcal{J}\right\}$ *is an ideal on* Y.

ii. If $\mathfrak{F}: (\mathfrak{X}, \mathrm{T}) \rightarrow (\mathrm{Y}, \acute{\ddot{\upsilon}}, \mathfrak{f})$ is one to function and $\mathfrak{f}$ is ideal on Y then $\mathfrak{F}^{-1}(\mathfrak{f}) = \{\mathfrak{F}^{-1}(\mathrm{B}) : \mathrm{B} \in \mathfrak{f}\}$ *is an ideal on* $\mathfrak{X}$.

iii. If $\mathfrak{f}: (\mathfrak{X}, \mathrm{T}, \mathcal{G}) \rightarrow (\mathrm{Y}, \acute{\ddot{\upsilon}})$ is one to one function and $\mathcal{G}$ is a grill on $\mathfrak{X}$, then $\mathfrak{f}(\mathcal{G}) = \left\{\mathfrak{f}(д): д \in \mathcal{G}\right\}$ *is an grill on* Y.

v. If $\mathfrak{f}: (\mathfrak{X}, \mathrm{T}) \rightarrow (\mathrm{Y}, \acute{\ddot{\upsilon}}, \mathcal{G}')$ is one to function $\mathcal{G}'$ is a grill on Y, then $\mathfrak{f}^{-1}(\mathcal{G}') = \{\mathfrak{f}^{-1}(\mathrm{B}) : \mathrm{B} \in \mathcal{G}'\}$ *is an grill on* $\mathfrak{X}$.

Theorem 2.7[7]: *If $\mathfrak{f}: (\mathfrak{X}, \mathrm{T}) \rightarrow (\mathrm{Y}, \ddot{\upsilon})$ is a function such that $\mathcal{J}$ and $\mathcal{G}_1$ are ideal respective grill on $\mathfrak{X}$. Then:*

i. $\mathfrak{f} = \{ \mathrm{A} \subseteq \mathrm{Y} : \mathfrak{f}^{-1}(\mathrm{A}) \in \mathcal{J}\}$ *is an ideal on* Y.

ii. $\mathcal{G}_2 = \{ \mathcal{B} \subseteq \mathrm{Y} : \mathfrak{f}^{-1}(\mathcal{B}) \in \mathcal{G}_1\}$ *is a grill on* Y.

3. $\mathcal{J}_{\mathcal{G}}$-Compact Space

Definition 3.1: A subset $\mathfrak{M}$ of ideal grill topological space $(\mathrm{X}, \mathrm{T}, \mathcal{J}, \mathcal{G})$ is called ideal grill compact (briefly $\mathcal{J}_{\mathcal{G}}$-compact) if for every open cover $\{\breve{\mathrm{G}}_{\tilde{\alpha}} : \tilde{\alpha} \in \Lambda\}$ of $\mathfrak{M}$, there exists a finite subset $\{\breve{\mathrm{G}}_{\tilde{\alpha}_{\hat{\imath}}} : \hat{\imath} = 1,2, \dots\dots, n\}$ such that $\mathfrak{M} - \bigcup_{i=1}^{n} \breve{\mathrm{G}}_{\tilde{\alpha}_i} \in \mathcal{J}$ and $\bigcup_{i=1}^{n} \breve{\mathrm{G}}_{\tilde{\alpha}_i} \in \mathcal{G}$. The space$(\mathfrak{X}, \mathrm{T})$ is called be $\mathcal{J}_{\mathcal{G}}$-compact if $\mathfrak{X}$ is $\mathcal{J}_{\mathcal{G}}$-compact.

Remark 3.2:

(i) A space$(X, \mathrm{T}, \mathcal{J}, \mathcal{G})$ is called an ideal grill topological space such that $\mathcal{J}$ *and* $\mathcal{G}$ are ideal resp. grill on $\mathfrak{X}$ denoted by $\mathcal{J}_{\mathcal{G}}$–topological space.

(ii) It is clear that $(\mathfrak{X}, \mathrm{T})$is compact iff $\{\emptyset\}_{p(\mathfrak{X})-\{\emptyset\}} -$ compact space such that $\mathcal{J} = \{\emptyset\}$ and $\mathcal{G} = p(\mathfrak{X}) - \{\emptyset\}$.

Example 3.3: Every indiscrete space is $\mathcal{I}_{\mathcal{G}}$-compact space for any ideal $\mathcal{I}$ (grill $\mathcal{G}$) on Ӽ.

Remark 3.4:

(i) Every compact space is $\mathcal{I}_{\mathcal{G}}$-compact for any ideal (grill) on Ӽ.

(ii) Every $\mathrm{I}_{\mathcal{G}}$-compact space is $\mathcal{I}$-compact and $\mathcal{G}$-compact.

(iii) The hereditary property of $\mathcal{I}_{\mathcal{G}}$-compact space is achieved if the subset is closed concerning the space, which is also one of the elements of the grill.

The opposite of Remark (3.4, i, ii) is not generally true, as shown in the following examples.

Example3.5: Suppose that (Ӽ, T)be co-countable topological space on uncountable set Ӽ such that $p \in$ Ӽ$, \mathcal{I}_p = \{ \tilde{\tilde{A}} \subseteq X : \tilde{\tilde{A}} \in$ P(Ӽ)$, p \notin \tilde{\tilde{A}} \}$is an ideal on Ӽ and $\mathcal{G}_p = \{ \tilde{\tilde{A}} \subseteq$ Ӽ$: \tilde{\tilde{A}} \in$ P(Ӽ)$, p \in \tilde{\tilde{A}} \}$is a gril on Ӽ .Then (Ӽ, T) is $(\mathcal{I}_p)_{\mathcal{G}_p}$-compact space, but not compact space.

Example 3.6: Each finite subset of a space Ӽ is $\mathcal{I}_f$ −compact space but not $(\mathcal{I}_f)_{\mathcal{G}\infty}$ −compact space.

Proposition 3.7: Every $\mathcal{G}$-compact subset of a T_2 −grill space is $\mathrm{T}_{\mathcal{G}}$ −closed.

Proof: Suppose that $\mathfrak{L}$ is $\mathcal{G}$-compact subset of a T_2 −grill space (Ӽ, T, $\mathcal{G}$) and $x \notin \mathfrak{L}$, then $x \in$ Ӽ $- \mathfrak{L}$.Thus for each $y \in \mathfrak{L}$, then there exist open sets $\mathfrak{D}_y$, $\mathfrak{H}_y$ such that $x \in \mathfrak{D}_y$ and $y \in \mathfrak{H}_y$ and $\mathfrak{D}_y \cap \mathfrak{H}_y = \emptyset$.So $\{\mathfrak{H}_y : y \in \mathfrak{L}\}$ is an open cover of $\mathfrak{L}$, then there exists a finite subsets $\left\{\mathfrak{H}_{y_{i}} : i = 1,2, \ldots, k\right\}$ such that $\mathfrak{L} - \bigcup_{i=1}^{k} \mathfrak{H}_{y_i} \notin \mathcal{G}$ .Now $x \notin cl(\mathfrak{H}_y)$for each $y \in \mathfrak{L}$, then $x \notin$cl$(\bigcup_{i=1}^{k} \mathfrak{H}_{y_i})$ .Suppose that $\mathfrak{N} =$ Ӽ $-$ cl$(\bigcup_{i=1}^{k} \mathfrak{H}_{y_i})$, since $\mathfrak{L} - cl(\bigcup_{i=1}^{k} \mathfrak{H}_{y_i}) \subseteq \mathfrak{L} - \bigcup_{i=1}^{k} \mathfrak{H}_{y_i}$, then $\mathfrak{L} - cl(\bigcup_{i=1}^{k} \mathfrak{H}_{y_i}) \notin \mathcal{G}$ .So $\mathfrak{N} \cap \mathfrak{L} \notin \mathcal{G}$, thus $x \notin \phi_{\mathcal{G}}(\mathfrak{L})$. Hence $\mathfrak{L}$ is $\mathrm{T}_{\mathcal{G}}$ −closed set.

There is a relationship between $\mathcal{I}_{\mathcal{G}}$-compact and T^* −closed ($\mathrm{T}_{\mathcal{G}}$ −closed) through the following proposition.

Proposition 3.8: Every $\mathcal{I}_{\mathcal{G}}$-a compact subset of a T_2 − space is T^* −closed and $\mathrm{T}_{\mathcal{G}}$ −closed.

Proof: Suppose that $\mathfrak{H}$ be $\mathcal{I}_{\mathcal{G}}$-a compact subset of a T_2- space X, then by Remark (3.4,(ii)), $\mathfrak{H}$ is $\mathcal{I}$-compact and $\mathcal{G}$ –compact sets. Thus by Theorem (2.3) and Proposition(3.7), $\mathfrak{H}$ is T^{*} $-$closed and $T_{\mathcal{G}}$ $-$closed.

Proposition 3.9: Let $(X, T, \mathcal{I}, \mathcal{G})$be $\mathcal{I}_{\mathcal{G}}$ $-$Topologyical space.Then $(X, T)$is $\mathcal{I}_{\mathcal{G}}$-compact if and only if for any family $\{\tilde{V}_{\dot{\rho}} : \dot{\rho} \in \Omega\}$ of closed subsets of X with $\bigcap_{\dot{\rho}\in\Omega} \tilde{V}_{\dot{\rho}} = \emptyset$, then there exists a finite subfamily $\left\{\tilde{V}_{\dot{\rho}_i} : i = 1,2, \dots, n\right\}$ such that $\bigcap_{i=1}^{n} \tilde{V}_{\dot{\rho}_i} \in \mathcal{I}$ and $\bigcap_{i=1}^{n} \tilde{V}_{\dot{\rho}_i} \notin \mathcal{G}$.

Proof:$\rightarrow$ Let$\{\tilde{V}_{\dot{\rho}} : \dot{\rho} \in \Omega\}$ be family of closed subsets of X such that $\bigcap_{\dot{\rho}\in\Omega} \tilde{V}_{\dot{\rho}} = \emptyset$, then $\{X - \tilde{V}_{\dot{\rho}} : \dot{\rho} \in \Omega\}$is open cover of X. Since X is $\mathcal{I}_{\mathcal{G}}$-compact space, then there exists a finite subset $\{X - \tilde{V}_{\dot{\rho}_i} : i = 1,2, \dots, n\}$ such that $X - \bigcup_{i=1}^{n}(X - \tilde{V}_{\dot{\rho}_i}) \in \mathcal{I}$ and $\bigcup_{i=1}^{n}(X - \tilde{V}_{\dot{\rho}_i}) \in \mathcal{G}$ that is

$$\bigcap_{i=1}^{n} \tilde{V}_{\dot{\rho}_i} \in \mathcal{I} \text{ and } \bigcap_{i=1}^{n} \tilde{V}_{\dot{\rho}_i} \notin \mathcal{G} .$$

Conversely: Let $\{F_{\alpha} : \alpha \in \Omega\}$be open cover of X ,then $\{X - F_{\alpha} : \alpha \in \Omega\}$ is family of closed subsets of X with $\bigcap_{\alpha\in\Omega}(X - F_{\alpha}) = \emptyset$, thus by hypothesis, then there exist finite subsets $\{X - F_{\alpha_i} : i=1,2,\dots,m\}$such that $\bigcap_{i=1}^{m}(X - F_{\alpha_i}) \in \mathcal{I}$ and $\bigcap_{i=1}^{m}(X - F_{\alpha_i}) \notin \mathcal{G}$.Thus $X - \bigcup_{i=1}^{m} F_{\alpha_i} \in \mathcal{I}$ and $\bigcup_{i=1}^{m} F_{\alpha_i} \in \mathcal{G}$.Hence X is $\mathcal{I}_{\mathcal{G}}$-compact space.

Proposition 3.10: If $(X, T, \mathcal{I}, \mathcal{G})is\mathcal{I}_{\mathcal{G}}$-compact space such that $\mathcal{I} \subseteq \tilde{I}$ and $\mathcal{G} \subseteq \mathcal{G}'$, then it is $\tilde{I}_{\mathcal{G}'}$-compact space, where $\tilde{I}$ and $\mathcal{G}'$ are ideal resp. grill on X.

Proof: The proof is evident.

The main Theorem

Theorem 3.11: *If* $\mathfrak{K}$ *and* $\mathfrak{E}$ *are* $\mathcal{I}_{\mathcal{G}}$*-compact sets in ideal grill space* $(X, T, \mathcal{I}, \mathcal{G})$*, then* $\mathfrak{K} \cup \mathfrak{E}$ *is* $\mathcal{I}_{\mathcal{G}}$*-compact.*

Proof: Let $\{G_{\tilde{\alpha}} : \tilde{\alpha} \in \Omega\}$is an open cover of $\mathfrak{K} \cup \mathfrak{E}$.Since $\mathfrak{K}$ *and* $\mathfrak{E}$are $\mathcal{I}_{\mathcal{G}}$-compact sets, then there exists a finite subset $\{G_{\tilde{\alpha}_i} : i = 1,2, \dots, n\}$ and$\{G_{\tilde{\alpha}_l} : l = 1,2, \dots, m\}$ such that $\mathfrak{K} - \bigcup_{i=1}^{n} G_{\tilde{\alpha}_i} \in \mathcal{I}$ and $\bigcup_{i=1}^{n} G_{\tilde{\alpha}_i} \in \mathcal{G}$,so $\mathfrak{E} - \bigcup_{l=1}^{m} G_{\tilde{\alpha}_l} \in \mathcal{I}$ and $\bigcup_{l=1}^{m} G_{\tilde{\alpha}_l} \in \mathcal{G}$.

Suppose that $\mathfrak{K} - \bigcup_{i=1}^{n} G_{\tilde{\alpha}_i} = \mathcal{I}_1$ and $\mathfrak{E} - \bigcup_{l=1}^{m} G_{\tilde{\alpha}_l} = \mathcal{I}_2$, then $\mathfrak{K} = \bigcup_{i=1}^{n} G_{\tilde{\alpha}_i} \cup \mathcal{I}_1$ and $\mathfrak{E} = \bigcup_{l=1}^{m} G_{\tilde{\alpha}_l} \cup \mathcal{I}_2$, thus $\mathfrak{K} \cup \mathfrak{E} = (\bigcup_{i=1}^{n} G_{\tilde{\alpha}_i} \cup \mathcal{I}_1) \cup (\bigcup_{l=1}^{m} G_{\tilde{\alpha}_l} \cup \mathcal{I}_2) = (\bigcup_{i=1}^{n} G_{\tilde{\alpha}_i} \cup \bigcup_{l=1}^{m} G_{\tilde{\alpha}_l}) \cup (\mathcal{I}_1 \cup \mathcal{I}_2)$. But $\mathfrak{K} \cup \mathfrak{E} - (\bigcup_{i=1}^{n} G_{\tilde{\alpha}_i} \cup \bigcup_{l=1}^{m} G_{\tilde{\alpha}_l}) = (\mathcal{I}_1 \cup \mathcal{I}_2) \in \mathcal{I}$ and $\bigcup_{i=1}^{n} G_{\tilde{\alpha}_i} \cup \bigcup_{l=1}^{m} G_{\tilde{\alpha}_l} \in \mathcal{G}$. Hence $\mathfrak{K} \cup \mathfrak{E}$ is $\mathcal{I}_{\mathcal{G}}$-compact.

Proposition 3.12: If $\mathfrak{H}$ is a closed subset of ideal grill space $(X, T, \mathcal{I}, \mathcal{G})$ and $\mathfrak{N}$ is $\mathcal{I}_{\mathcal{G}}$-a compact subset of X such that $\mathfrak{H} \in \mathcal{G}$, then $\mathfrak{H} \cap \mathfrak{N}$ is $\mathcal{I}_{\mathcal{G}}$-compact set in X.

Proof: Let $\{G_{\dot{\eta}} : \dot{\eta} \in \Omega\}$ is open cover of $\mathfrak{H} \cap \mathfrak{N}$, then $(\mathfrak{H} \cap \mathfrak{N}) \cup \mathfrak{H}^c \subseteq \bigcup_{\dot{\eta} \in \Omega} G_{\dot{\eta}} \cup \mathfrak{H}^c$, thus $\mathfrak{N} \cup \mathfrak{H}^c \subseteq \bigcup_{\dot{\eta} \in \Omega} G_{\dot{\eta}} \cup \mathfrak{H}^c$. So $\{G_{\dot{\eta}} : \dot{\eta} \in \Omega\} \cup \mathfrak{H}^c$ is an open cover of $\mathfrak{N}$. Since $\mathfrak{N}$ is $\mathcal{I}_{\mathcal{G}}$-compact set in X, then there exists a finite subsets $\{G_{\dot{\eta}_i} \cup \mathfrak{H}^c : i = 1,2, \ldots, n\}$ such that $\mathfrak{N} - \bigcup_{i=1}^{n}(G_{\dot{\eta}_i} \cup \mathfrak{H}^c) \in \mathcal{I}$ and $\bigcup_{i=1}^{n} G_{\dot{\eta}_i} \cup \mathfrak{H}^c \in \mathcal{G}$. But $\mathfrak{N} - \bigcup_{i=1}^{n}(G_{\dot{\eta}_i} \cup \mathfrak{H}^c) = (\mathfrak{H} \cap \mathfrak{N}) - \bigcup_{i=1}^{n} G_{\dot{\eta}_i}$, then $(\mathfrak{H} \cap \mathfrak{N}) - \bigcup_{i=1}^{n} G_{\dot{\eta}_i} \in \mathcal{I}$ Since $\mathfrak{H} \in \mathcal{G}$ and $\bigcup_{i=1}^{n} G_{\dot{\eta}_i} \cup \mathfrak{H}^c \in \mathcal{G}$, then $\bigcup_{i=1}^{n} G_{\dot{\eta}_i} \in \mathcal{G}$. Therefore $\mathfrak{H} \cap \mathfrak{N}$ is $\mathcal{I}_{\mathcal{G}}$-compact set in X.

Proposition 3.13: If $£ : (X, T, \mathcal{I}, \mathcal{G}) \to (Y, Ъ)$ is continuous, bijective function such that $(X, T, \mathcal{I}, \mathcal{G})$ is $\mathcal{I}_{\mathcal{G}}$-compact, then (Y, Ъ, £(I), £($\mathcal{G}$)) is $(£(\mathcal{I})_{£(\mathcal{G})})$ −compact.

Proof: Let $\{\tilde{V}_{\ddot{\omega}} : \ddot{\omega} \in \Omega\}$ is an open cover of Y, then $\{ £^{-1}(\tilde{V}_{\ddot{\omega}}) : \ddot{\omega} \in \Omega\}$ is open cover of X. Given $(X, T, \mathcal{I}, \mathcal{G})$ is $\mathcal{I}_{\mathcal{G}}$-compact space, then there exists finite subsets $\{£^{-1}(\tilde{V}_{\ddot{\omega}_i}) : i = 1,2,\ldots,n\}$ such that $X - \bigcup_{i=1}^{n} £^{-1}(\tilde{V}_{\ddot{\omega}_i}) \in \mathcal{I}$ and $\bigcup_{i=1}^{n} £^{-1}(\tilde{V}_{\ddot{\omega}_i}) \in \mathcal{G}$. Then $£(X - \bigcup_{i=1}^{n} £^{-1}(\tilde{V}_{\ddot{\omega}_i})) \in £(\mathcal{I})$ and $£(\bigcup_{i=1}^{n} £^{-1}(\tilde{V}_{\ddot{\omega}_i})) \in £(\mathcal{G})$, since $£$ is bijective, thus $Y - \bigcup_{i=1}^{n} \tilde{V}_{\ddot{\omega}_i}) \in £(\mathcal{I})$ and $\bigcup_{i=1}^{n} \tilde{V}_{\ddot{\omega}_i} \in £(\mathcal{G})$. so Y is $(£(\mathcal{I})_{£(\mathcal{G})})$ −compact.

Proposition 3.14: If $(X, T, \mathcal{I}, \mathcal{G}_1)$ is is $\mathcal{I}_{\mathcal{G}_1}$-compact space and $f : (X, T) \to (Y, Ţ)$ is onto and continuous function such that $\mathcal{G}_2 = \{B \subseteq Y : f^{-1}(B) \in \mathcal{G}_1\}$, $\check{j} = \{\tilde{A} \subseteq Y : f^{-1}(\tilde{A}) \in \mathcal{I}\}$, then $(Y, Ţ, \check{j}, \mathcal{G}_2)$ is $\check{j}_{\mathcal{G}_2}$-compact space.

Proof: Let $\{\tilde{H}_{\dot{\rho}} : \dot{\rho} \in \Omega\}$ is an open cover of Y, then $\{f^{-1}(\tilde{H}_{\dot{\rho}}) : \dot{\rho} \in \Omega\}$ is an open cover of X. Thus by Theorem (2.7), $\check{j}$ is ideal on Y and $\mathcal{G}_2$ is a grill on Y. Since X is $\mathcal{I}_{\mathcal{G}_1}$-compact space, then there exists finite subset

$\{f^{-1}(\check{H}_{\rho_\alpha}) : \alpha = 1,2, \ldots, n\}$ such that $X - \bigcup_{\alpha=1}^{n} f^{-1}(\check{H}_{\rho_\alpha}) \in \mathcal{J}$ and $\bigcup_{\alpha=1}^{n} f^{-1}(\check{H}_{\rho_\alpha}) \in \mathcal{G}_1$.

But $X - \bigcup_{\alpha=1}^{n} f^{-1}(\check{H}_{\rho_\alpha}) = f^{-1}(Y - \bigcup_{\alpha=1}^{n} \check{H}_{\rho_\alpha})$ and $f^{-1}(\bigcup_{\alpha=1}^{n} \check{H}_{\rho_\alpha}) \in \mathcal{G}_1$, then $Y - \bigcup_{\alpha=1}^{n} \check{H}_{\rho_\alpha} \in \check{J}$ and $\bigcup_{\alpha=1}^{n} \check{H}_{\rho_\alpha} \in \mathcal{G}_2$. Thus $(Y, T, \check{J}, \mathcal{G}_2)$ is $\check{J}_{\mathcal{G}_2}$-compact space.

Proposition 3.15: If $f : (X, T) \to (Y, T, \mathcal{J}, \mathcal{G})$ be open and bijective function such that $(Y, T, \mathcal{J}, \mathcal{G})$ is $\mathcal{J}_{\mathcal{G}}$-compact space, then $(X, T, f^{-1}(\mathcal{J}), f^{-1}(\mathcal{G}))$ is $f^{-1}(\mathcal{J})_{f^{-1}(\mathcal{G})}$-compact space.

Proof: similar proof to the proposition (3.13).

4. Conclusion

We have presented a new definition for $\mathcal{J}_{\mathcal{G}}$-compact space and study the relationship between $\mathcal{J}_{\mathcal{G}}$-compact space and $\mathcal{J}$ -compact ($\mathcal{G} -$ compact) space. We have also clarified the properties of $\mathcal{J}_{\mathcal{G}}$-compact space concerning union and intersection and effect of the continuous function on $\mathcal{J}_{\mathcal{G}}$-compact space**.**

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