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Basic properties of soft Np-open sets

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Abstract

This paper aims to present and study the soft topological properties of soft Np - closure, soft Np - interior, soft Np - exterior, and soft Np - boundary of a soft set using the notion of soft Np- open and soft Np- closed sets. Also, we discuss the soft product of soft Np-open sets and give examples.

Subject Classification: *05A05, 05B10.*

Keywords: *Soft Np-open set, Soft Np-closure, Soft Np-nterior, Soft Np-exterior, Soft Np-boundary.*

1. Introduction

Molodtsov [1] introduced the notion of soft sets theory. He utilized soft sets theory to a variety of fields, such as operations research, smoothness of functions, probability theory, game theory, and measurement theory. The idea of soft open sets was used by Shabir and Naz [2] to describe soft topological spaces. They also studied several concepts such as soft closed sets, soft closure, soft separation axioms, and soft subspace. Hussain and Ahmad [3] studied the soft interior, soft exterior, soft closure, soft boundary of soft sets and obtained many results related to these concepts. Ilango and Ravindran [4] and Mahmood and Ail [5] introduced and studied soft $\mathrm{p}-$open set and soft $\mathrm{Np}-$open set. In this article we define soft $\mathrm{Np}-$closure, soft $\mathrm{Np}-$interior, soft $\mathrm{Np}-$exterior and soft $\mathrm{Np}-$the boundary of soft sets and study many of their properties. Also we discuss the soft product of soft $\mathrm{Np}-$open sets.

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2. Preliminaries.

Definition. 2.1:[1]: An ordered pair (E, J) is soft set over V if $E: J \to P(V)$ is a function where J is a set of parameters. This soft set can be written as:

$$(E, J) = \{(j, E(j)) : j \in J, E(j) \in P(V)\}.$$

Definition 2.2:[6]: (E, J) is called.

(i) Null denoted by $\tilde{\emptyset}$ in case $E(j) = \emptyset, \forall\, j \in J$.

(ii) Absolute denoted by $\tilde{V}$ in case $E(j) = V, \forall\, j \in J$.

Definition 2.3:[6],[3]: If (E_1, J_1) and (E_2, J_2) are soft sets. Then:

(i) A soft union of (E_1, J_1) and (E_2, J_2) is (E, J), where $J = J_1 \cup J_2$ *and* $\forall\, j \in J$.

$$E(j) = \begin{cases} E_1(j) & \text{if } j \in J_1 - J_2. \\ E_2(j) & \text{if } j \in J_2 - J_1. \\ E_1(j) \cup E_2(j) & \text{if } j \in J_1 \cap J_2. \end{cases}$$

We write $(E, J) = (E_1, J_1) \tilde{\cup} (E_2, J_2)$

(ii) A soft intersection of (E_1, J_1) and (E_2, J_2) is the soft set (E, J), where $J = J_1 \cap J_2$ and $\forall\, j \in J, E(j) = E_1(j) \cap E_2(j)$.
We write $(E, j) = (E_1, J_1) \tilde{\cap} (E_2, J_2)$.

Definition 2.4:[7]: If (E_1, J_1) and (E_2, J_2) are soft sets over V, then (E_1, J_1) is soft subset of (E_2, J_2) in case $J_1 \subseteq J_2$ and $E_1(j) \subseteq E_2(j)$, $\forall\, j \in J_1$, and we write $(E_1, J_1) \tilde{\subseteq} (E_2, J_2)$.

Definition 2.5:[2]: A family $\tilde{\rho}$ of soft sets is called soft topology on V if:

(i) $\tilde{V}, \tilde{\emptyset} \tilde{\in} \tilde{\rho}$.

(ii) If $(E_1, J), (E_2, J) \tilde{\in} \tilde{\rho}$, then $(E_1, J) \tilde{\cap} (E_2, J) \tilde{\in} \tilde{\rho}$.

(iii) If $(E_\alpha, J) \tilde{\in} \tilde{\rho}, \forall\, \alpha \in \Psi$, then $\tilde{\cup}\{(E_\alpha, J) : \alpha \in \Psi\} \tilde{\in} \tilde{\rho}$.

The triple $(V, \tilde{\rho}, J)$ is called soft topological space (or STS).

Definition 2.6:[8]: (E,J) is soft point if $\exists\, j \in J$ and $v \in V$ with $E(j) = \{v\}$ and $E(s) = \emptyset$ for each $s \in J \backslash \{j\}$ and we write by $\tilde{v} = (j, \{v\})$.

Proposition 2.7:[9],[7]: Let $(V, \tilde{\rho}, J)$ be soft topological space with $(E, J) \tilde{\subseteq} \tilde{V}$. Then.

(i) $\tilde{v} \tilde{\in} cl((E, J)) \Leftrightarrow (O, J) \tilde{\cap} (E, J) \neq \tilde{\emptyset}$ for all soft open set (O, J) containing $\tilde{v}$.

(ii) $\tilde{v} \tilde{\in} int\big((E, J)\big) \Leftrightarrow \exists\, (O, J) \tilde{\in} \tilde{\rho}$ such that, $\tilde{v} \tilde{\in} (O, J) \tilde{\subseteq} (E, J)$

Definition 2.8: Let $(V, \tilde{\rho}, J)$ be soft space, then $(E, J) \tilde{\subseteq} \tilde{V}$ is called

(i) The soft $p-$ open set **[4]** if $(E, J) \tilde{\subseteq} int(cl(E, J))$.

(ii) The soft $\mathrm{Np}-$ open (soft $\mathrm{Npre}-\mathrm{open}$) set **[5]** if for each $\tilde{v} \widetilde{\in} (E, J)$, there is soft $p-$ open (T, J) with $\tilde{v} \widetilde{\in} (T, J)$ and $(T, J) - (E, J)$ is finite.

The collection of every soft $\mathrm{Np}-\mathrm{open}$ (soft $\mathrm{Np}-$ closed) sets in $(V, \tilde{\rho}, J)$ is denoted by $\tilde{\rho}_{\mathrm{Np}}$ $(\tilde{\rho}^{c}_{\mathrm{Np}})$ respectively.

3. Basic Properties of Soft $\mathrm{Np}-\mathrm{Open}$ Sets

Definition 3.1: If (E, J) is soft subset of $(V, \tilde{\rho}, J)$, then:

(i) $\mathrm{Npcl}((E, J)) = \widetilde{\bigcap}\{(M, J) \widetilde{\in} \tilde{\rho}^{c}_{\mathrm{Np}} : (E, J) \widetilde{\subseteq} (M, J)\}$ is soft $\mathrm{Np}-$ closure of (E, J).

(ii) $\mathrm{Npint}((E, J)) = \widetilde{\bigcup}\{(T, J) \widetilde{\in} \tilde{\rho}_{\mathrm{Np}} : (T, J) \widetilde{\subseteq} (E, J)\}$ is soft $\mathrm{Np}-$ interior of (E, J).

Theorem 3.2: *If (M, J) and (G, J) are soft subsets of $(V, \tilde{\rho}, J)$, then:*

(i) $(M, J) \widetilde{\subseteq} \mathrm{Npcl}((M. J)) \widetilde{\subseteq} \mathrm{cl}((M, J))$.

(ii) $\mathrm{Npcl}((M, J))$ is the smallest soft $\mathrm{Np}-$ closed set containing (M, J).

(iii) $\mathrm{Npcl}((M, J)) = (M, J) \Leftrightarrow (M. J) \widetilde{\in} \tilde{\rho}^{c}_{\mathrm{Np}}$.

(iv) $\mathrm{Npcl}(\mathrm{Npcl}((M, J)) = \mathrm{Npcl}((M, J))$.

(v) $\widetilde{V} - \mathrm{Npcl}((M, J)) = \mathrm{Npint}(\widetilde{V} - (M, J))$.

(vi) If $(M, J) \widetilde{\subseteq} (G, J)$, then $\mathrm{Npcl}((M, J)) \widetilde{\subseteq} \mathrm{Npcl}((G, J))$.

(vii) $\tilde{v} \widetilde{\in} \mathrm{Npcl}((M, J))$ iff $(U, J)\widetilde{\bigcap}(M, J) \neq \widetilde{\emptyset}$ for every soft $\mathrm{Np}-$ open set (U, J) which contains $\tilde{v}$.

(viii) $\mathrm{Npcl}((M, J))\widetilde{\bigcup}\mathrm{Npcl}((G, J)) \widetilde{\subseteq} \mathrm{Npcl}((M, J)\widetilde{\bigcup}(G, J))$.

(ix) $\mathrm{Npcl}((M, J)\widetilde{\bigcap}(G, J)) \widetilde{\subseteq} \mathrm{Npcl}((M, J))\widetilde{\bigcap}\mathrm{Npcl}((G, J))$.

Proof: (i), (ii), (vi) and (ix) are clear.

(iii) If $(M, J) = \mathrm{Npcl}((M, J))$, then from definitions (3.1) and (ii), we get $\mathrm{Npcl}((M, J))$ is soft $\mathrm{Np}-$ closed. Hence (M, J) is soft $\mathrm{Np}-$ closed. Conversely, by (i), we obtain, $(M, J) \widetilde{\subseteq} \mathrm{Npcl}((M, J))$. Since $(M, J) \widetilde{\subseteq} (M, J)$ and (M, J) is soft $\mathrm{Np}-$ closed, then $\mathrm{Npcl}\big((M, J)\big) \widetilde{\subseteq} (M, J)$ by ii . That is $\mathrm{Npcl}((M, J)) = (M, J)$.

(iv) Since $\mathrm{Npcl}((M, J)) \widetilde{\in} \tilde{\rho}^{c}_{\mathrm{Np}}$, then by (iii) $\mathrm{Npcl}(\mathrm{Npcl}((M, J))) = \mathrm{Npcl}((M, J))$.

(v) Since $\mathrm{Npcl}((M, J)) = \widetilde{\bigcap}\{(T, J) : (M, J) \widetilde{\subseteq} (T, J), (T, J) \widetilde{\in} \tilde{\rho}^{c}_{\mathrm{Np}}\}$. Then $[\mathrm{Npcl}((M, J))]^{c} = \widetilde{\bigcup}\{(T, J)^{c} : (T, J)^{c} \widetilde{\subseteq} (M, J)^{c}, (T, J)^{c} \widetilde{\in} \tilde{\rho}_{\mathrm{Np}}\} = \mathrm{Npint}((M, J)^{c})$.

(vii) Suppose that $\tilde{v} \widetilde{\in} \mathrm{Npcl}((M, J))$. If there exists $(U, J) \widetilde{\in} \tilde{\rho}_{\mathrm{Np}}$ with $\tilde{v} \widetilde{\in} (U, J)$ and $\widetilde{\emptyset} = (U, J)\widetilde{\bigcap}(M, J)$, then $(M, J) \widetilde{\subseteq} (U, J)^{c}$ and $(U, J)^{c}$ is soft $\mathrm{Np}-$ closed in $\widetilde{V}$, so by (vi) and (iii), we obtain $\mathrm{Npcl}((M, J)) \widetilde{\subseteq} (U, J)^{c}$, and then $\tilde{v} \widetilde{\in} \mathrm{Npcl}((M, J)) \widetilde{\subseteq} (U, J)^{c}$, but this is a contradiction. Thus $(U, J) \widetilde{\bigcap} (M, J) \neq \widetilde{\emptyset}$ for each soft $\mathrm{Np}-$ open set (U, J) containing $\tilde{v}$.

Conversely, if $\tilde{v} \tilde{\notin} \mathrm{Npcl}((M,J))$, then $\tilde{v} \tilde{\in} (\mathrm{Npcl}((M,J))^c$ and $(\mathrm{Npcl}((M,J)))^c$ is soft $\mathrm{Np}-$ open such that $(M,J)\tilde{\cap}[(\mathrm{Npcl}((M,J)))^c] \neq \tilde{\emptyset}$. But this is a contradiction. Hence $\tilde{v} \tilde{\in} \mathrm{Npcl}((M,J))$.

(viii) Since $(M,J) \tilde{\subseteq} (M,J)\tilde{\cup}(G.J)$ & $(G,J) \tilde{\subseteq} (M,J)\tilde{\cup}(G,J)$ then by (vi) $\mathrm{Npcl}((M,J)) \tilde{\subseteq} \mathrm{Npcl}((M,J)\tilde{\cup}(G,J))$ and $\mathrm{Npcl}((G,J)) \tilde{\subseteq} \mathrm{Npcl}((M,J)\tilde{\cup}(G,J))$. Hence $\mathrm{Npcl}((M,J))\tilde{\cup}\mathrm{Npcl}((G,J)) \tilde{\subseteq} \mathrm{Npcl}((M,J)\tilde{\cup}(G,J))$.

Theorem 3.3: *If (M,J) and (G,J) are soft sets in $(V,\tilde{\rho},J)$. Then:*

(i) $\mathrm{int}((M,J) \tilde{\subseteq} \mathrm{Npint}(M,J)) \tilde{\subseteq} (M,J)$.

(ii) $\mathrm{Npint}((M.J)) = (M,J) \Leftrightarrow (M,J) \tilde{\in} \tilde{\rho}_{\mathrm{Np}}$.

(iii) $\mathrm{Npint}(M,J)) = \mathrm{Npint}(\mathrm{Npint}(M,J)))$.

(iv) $(\mathrm{Npint}((M,J)))^c = \mathrm{Npcl}((M,J)^c)$.

(v) $\tilde{v} \tilde{\in} \mathrm{Npint}((M,J)) \Leftrightarrow \exists\ (T,J) \tilde{\in} \tilde{\rho}_{\mathrm{Np}}$ such that $\tilde{v} \tilde{\in} (T,J) \tilde{\subseteq} (M.J)$.

(vi) If $(M,J) \tilde{\subseteq} (G,J)$, then $\mathrm{Npint}((M,J)) \tilde{\subseteq} \mathrm{Npint}((G,J))$.

(vii) $\mathrm{Npint}((M,J))\tilde{\cup}\mathrm{Npint}((G,J)) \tilde{\subseteq} \mathrm{Npint}((M,J)\tilde{\cup}(G,J))$.

(viii) $\mathrm{Npint}((M,J)\tilde{\cap}(G,J)) \tilde{\subseteq} \mathrm{Npint}((M,J))\tilde{\cap}\mathrm{Npint}((G,J))$.

Proof: Obvious.

Definition 3.4: If $(V,\tilde{\rho},J)$ is STS and $(M,J) \tilde{\subseteq} \tilde{V}$. The soft $\mathrm{Np}-$ exterior of (M,J) is defined as $\mathrm{Npext}((M,J)) = \mathrm{Npint}((M,J)^c)$.

Theorem 3.5: *If $(V,\tilde{\rho},J)$ is STS and $(M,J),(G,J) \tilde{\subseteq} \tilde{V}$. Then:*

(i) $\mathrm{Npext}((M,J))$ is soft $\mathrm{Np}-$ open.

(ii) $\mathrm{Npext}((M,J)) = (\mathrm{Npcl}((M,J)))^c$.

(iii) If $(M,J) \tilde{\subseteq} (G,J) \Rightarrow \mathrm{Npext}((G,J)) \tilde{\subseteq} \mathrm{Npext}((M,J))$.

(iv) $\mathrm{Npext}((M,J))\tilde{\cup}\ \mathrm{Npext}((G,J)) \tilde{\subseteq} \mathrm{Npext}((M,J)\tilde{\cap}(G,J))$.

(v) $\mathrm{Npext}((M,J)\tilde{\cup}(G,J)) \tilde{\subseteq} \mathrm{Npext}((M,J))\tilde{\cap}\mathrm{Npext}((G,J))$.

Proof: (i) & (v) are clear.

(ii) $\mathrm{Npext}((M,J)) = \mathrm{Npint}((M,J)^c) = (\mathrm{Npcl}((M,J)))^c$(By Theorem (3.2),(v)).

(iii) If $(M,J) \tilde{\subseteq} (G,J)$, then $(G,J)^c \tilde{\subseteq} (M,J)^c$.
So $\mathrm{Npint}((G,J)^c) \tilde{\subseteq} \mathrm{Npint}((M,J)^c)$
Thus $\mathrm{Npext}((G,J)) \tilde{\subseteq} \mathrm{Npext}((M,J))$.

(iv) $\mathrm{Npext}((M,J))\tilde{\cup}\ \mathrm{Npext}((G,J)) = \mathrm{Npint}((M.J)^c)\tilde{\cup}\ \mathrm{Npint}((G,J)^c)$
$\tilde{\subseteq}\ \mathrm{Npint}[((M,J)\tilde{\cap}(G.J))^c]$.
$= \mathrm{Npext}((M\,J)\tilde{\cap}(G,J))$.

Definition 3.6: Let $(V,\tilde{\rho},J)$ be soft space with $(M,J) \tilde{\subseteq} \tilde{V}$. The soft $\mathrm{Np}-$ boundary of (M,J) is defined as $\mathrm{Npb}((M,J)) = \mathrm{Npcl}((M,J)) - \mathrm{Npint}((M,J))$.

Theorem 3.7: *If* $(V, \tilde{\rho}, J)$ *is soft space with* $(T, J) \tilde{\subseteq} \tilde{V}$. *Then:*

(i) $\mathrm{Npb}((\mathrm{T},\mathrm{J}))\tilde{\cap}\mathrm{Npint}((\mathrm{T},\mathrm{J})) = \tilde{\emptyset}$.

(ii) $\mathrm{Npb}((\mathrm{T},\mathrm{J})) = \mathrm{Npcl}((\mathrm{T},\mathrm{J})^{c})\tilde{\cap}\mathrm{Npcl}((\mathrm{T},\mathrm{J}))$.

(iii) $\mathrm{Npint}((\mathrm{T},\mathrm{J}))\tilde{\cup}\mathrm{Npb}((\mathrm{T},\mathrm{J})) = \mathrm{Npcl}((\mathrm{T},\mathrm{J}))$.

(iv) $[\mathrm{Npb}((\mathrm{T},\mathrm{J}))]^{c} = \mathrm{Npint}((\mathrm{T},\mathrm{J}))\tilde{\cup}\mathrm{Npext}((\mathrm{T},\mathrm{J}))$.

(v) $\mathrm{Npb}((\mathrm{T},\mathrm{J}))\tilde{\cup}\mathrm{Npext}((\mathrm{T},\mathrm{J}))\tilde{\cup}\mathrm{Npint}((\mathrm{T},\mathrm{J})) = \tilde{\mathrm{V}}$.

Proof: **(i)** and **(v)** are clear.

(ii) $\mathrm{Npb}((\mathrm{T},\mathrm{J})) = \mathrm{Npcl}((\mathrm{T},\mathrm{J})) - \mathrm{Npint}((\mathrm{T},\mathrm{J})) = \mathrm{Npcl}((\mathrm{T},\mathrm{J}))\tilde{\cap}$ $(\mathrm{Npint}((\mathrm{T},\mathrm{J})))^{c} = \mathrm{Npcl}((\mathrm{T},\mathrm{J}))\tilde{\cap}\mathrm{Npcl}((\mathrm{T},\mathrm{J})^{c})$. (By Theorem (3.3),(iv)).

(iii) $\mathrm{Npint}((\mathrm{T},\mathrm{J}))\tilde{\cup}\mathrm{Npb}((\mathrm{T},\mathrm{J})) = \mathrm{Npint}((\mathrm{T},\mathrm{J}))\tilde{\cup}[\mathrm{Npcl}((\mathrm{T},\mathrm{J}))\tilde{\cap}$ $(\mathrm{Npint}((\mathrm{T},\mathrm{J})))^{c}] = [\mathrm{Npint}((\mathrm{T},\mathrm{J}))\tilde{\cup}\mathrm{Npcl}((\mathrm{T},\mathrm{J}))]\tilde{\cap}[\mathrm{Npint}((\mathrm{T},\mathrm{J})\tilde{\cup}$ $(\mathrm{Npint}((\mathrm{T},\mathrm{J})))^{c}] = \mathrm{Npcl}((\mathrm{T},\mathrm{J})\tilde{\cap}\tilde{\mathrm{V}} = \mathrm{Npcl}((\mathrm{T},\mathrm{J}))$.

(iv) $[\mathrm{Npb}((\mathrm{T},\mathrm{J}))]^{c} = [\mathrm{Npcl}((\mathrm{T},\mathrm{J}))\tilde{\cap}(\mathrm{Npint}((\mathrm{T},\mathrm{J})))^{c}]^{c} = [\mathrm{Npcl}((\mathrm{T},\mathrm{J}))]^{c}\tilde{\cup}$ $\mathrm{Npint}((\mathrm{T},\mathrm{J})) = \mathrm{Npint}((\mathrm{T},\mathrm{J})^{c})\tilde{\cup}\mathrm{Npint}((\mathrm{T},\mathrm{J})) =$ $\mathrm{Npint}((\mathrm{T},\mathrm{J}))\tilde{\cup}\mathrm{Npext}((\mathrm{T},\mathrm{J}))$.

Theorem 3.8: *Let* (G, J_1) *and* (K, J_2) *be non-null soft subsets of* $(V, \tilde{\rho}, J_1)$ *and* $(W, \tilde{\sigma}, J_2)$ *respectively. Then:*

(i) $\mathrm{cl}[(\mathrm{G},\mathrm{J}_1) \times (\mathrm{K},\mathrm{J}_2)] = \mathrm{cl}((\mathrm{G},\mathrm{J}_1)) \times \mathrm{cl}((\mathrm{K},\mathrm{J}_2))$.

(ii) $\mathrm{int}[(\mathrm{G},\mathrm{J}_1) \times (\mathrm{K},\mathrm{J}_2)] = \mathrm{int}((\mathrm{G},\mathrm{J}_1)) \times \mathrm{int}((\mathrm{K},\mathrm{J}_2))$.

Proof:

(i) Let $(\tilde{\mathrm{v}}, \tilde{\mathrm{w}}) \tilde{\in} \mathrm{cl}((\mathrm{G},\mathrm{J}_1) \times (\mathrm{K},\mathrm{J}_2))$, then by Proposition (2.7),(i) $[(\mathrm{G},\mathrm{J}_1)$ $\times (\mathrm{K},\mathrm{J}_2)]\tilde{\cap}[(\mathrm{O},\mathrm{J}_1) \times (\mathrm{U},\mathrm{J}_2)] \neq \tilde{\emptyset}$ for each soft open set $(\mathrm{O},\mathrm{J}_1) \times (\mathrm{U},\mathrm{J}_2)$ in $\tilde{\mathrm{V}} \times \tilde{\mathrm{W}}$ containing $(\tilde{\mathrm{v}}, \tilde{\mathrm{w}})$. Hence $[(\mathrm{G},\mathrm{J}_1)\tilde{\cap}(\mathrm{O},\mathrm{J}_1)] \times [(\mathrm{K},\mathrm{J}_2)\tilde{\cap}(\mathrm{U},\mathrm{J}_2)] \neq \tilde{\emptyset}$ for each soft open set $(\mathrm{O},\mathrm{J}_1) \times (\mathrm{U},\mathrm{J}_2)$ in $\tilde{\mathrm{V}} \times \tilde{\mathrm{W}}$ containing $(\tilde{\mathrm{v}}, \tilde{\mathrm{w}})$. So $(\mathrm{G},\mathrm{J}_1)\tilde{\cap}(\mathrm{O},\mathrm{J}_1) \neq \tilde{\emptyset}$ and $(\mathrm{K},\mathrm{J}_2)\tilde{\cap}(\mathrm{U},\mathrm{J}_2) \neq \tilde{\emptyset}$ for each soft open sets $(\mathrm{O},\mathrm{J}_1)$ in $\tilde{\mathrm{V}}$ and $(\mathrm{U},\mathrm{J}_2)$ in $\tilde{\mathrm{W}}$ containing $\tilde{\mathrm{v}}$ and $\tilde{\mathrm{w}}$ respectively. Hence $\tilde{\mathrm{v}} \tilde{\in} \mathrm{cl}((\mathrm{G},\mathrm{J}_1))$ and $\tilde{\mathrm{w}} \tilde{\in} \mathrm{cl}((\mathrm{K},\mathrm{J}_2))$. Thus $(\tilde{\mathrm{v}}, \tilde{\mathrm{w}}) \tilde{\in} \mathrm{cl}((\mathrm{G},\mathrm{J}_1)) \times \mathrm{cl}((\mathrm{K},\mathrm{J}_2))$. Therefore $\mathrm{cl}[(\mathrm{G},\mathrm{J}_1) \times (\mathrm{K}\,\mathrm{J}_2)] \tilde{\subseteq} \mathrm{cl}((\mathrm{G},\mathrm{J}_1)) \times \mathrm{cl}((\mathrm{K},\mathrm{J}_2))$. Similarly, we prove $\mathrm{cl}((\mathrm{G},\mathrm{J}_1)) \times \mathrm{cl}((\mathrm{K},\mathrm{J}_2)) \tilde{\subseteq} \mathrm{cl}[(\mathrm{G}\,\mathrm{J}_1) \times (\mathrm{K},\mathrm{J}_2)]$

(ii) Follows from Proposition (2.7),(ii).

Proposition 3.9: If $(\mathrm{G},\mathrm{J}_1)$ and $(\mathrm{K},\mathrm{J}_2)$ are non-null soft subsets of $(\mathrm{V},\tilde{\rho},\mathrm{J}_1)$ and $(\mathrm{W},\tilde{\sigma},\mathrm{J}_2)$ respectively. Then $(\mathrm{G},\mathrm{J}_1) \times (\mathrm{K},\mathrm{J}_2)$ is soft $\mathrm{p}-\mathrm{open}$ in $\tilde{\mathrm{V}} \times \tilde{\mathrm{W}}$ iff $(\mathrm{G},\mathrm{J}_1)$ and $(\mathrm{K},\mathrm{J}_2)$ are soft $\mathrm{p}-\mathrm{open}$ in $\tilde{\mathrm{V}}$ and $\tilde{\mathrm{W}}$ respectively.

Proof: Using Theorem (3.8),(i) and (ii).

Theorem 3.10: *Let* (G, J_1) *and* (K, J_2) *be non-null soft subsets of* $(V, \tilde{\rho}, J_1)$ *and* $(W, \tilde{\sigma}, J_2)$ *respectively. Then if* $(G, J_1) \times (K, J_2)$ *is soft* $Np -$ *open in* $\tilde{V} \times \tilde{W}$, *then* $(G, J_1) \tilde{\in} \tilde{\rho}_{Np}$ *and* $(K, J_2) \tilde{\in} \tilde{\sigma}_{Np}$ *respectively.*

Proof: Let $\tilde{v} \tilde{\in} (G, J_1)$. Then $(\tilde{v}, \tilde{w}) \tilde{\in} (G, J_1) \times (K, J_2)$ for some $\tilde{w} \tilde{\in} (K, J_2)$, since $(G, J_1) \times (K, J_2)$ is soft $Np -$ open in $\tilde{V} \times \tilde{W}$, hence $\exists$ soft $p -$ open set $(O, J_1) \times (U, J_2)$ in $\tilde{V} \times \tilde{W}$ with $(\tilde{v}, \tilde{w}) \tilde{\in} (O, J_1) \times (U, J_2)$, and $(O, J_1) \times (U, J_2) - (G, J_1) \times (K, J_2)$ is finite, but $(O J_1) \times (U, J_2) - (G, J_1) \times (K, J_2) = [((O, J_1) - (G, J_1)) \times (U, J_2)] \tilde{\cup} [(O, J_1) \times ((U, J_2) - (K. J_2))]$. Hence $((O, J_1) - (G, J_1)) \times (U, J_2)$ is finite, and so $(O, J_1) - (G, J_1)$ is finite. Since $(O, J_1) \times (U, J_2)$ is soft $p -$ open in $\tilde{V} \times \tilde{W}$ containing $(\tilde{v}, \tilde{w})$, then by Proposition (3.9), (O, J_1) is soft $p -$ open in $\tilde{V}$ with $\tilde{v} \tilde{\in} (O, J_1)$ and $(O, J_1) - (G, J_1)$ is finite. Thus $(G, J_1) \tilde{\in} \tilde{\rho}_{Np}$ in $\tilde{V}$. Similarly, $(K, J_2) \tilde{\in} \tilde{\sigma}_{Np}$ in $\tilde{W}$.

4. Conclusions

(1) Let $(V, \tilde{\rho}, J)$ be STS and $(M, J), (G, J) \tilde{\subseteq} \tilde{V}$. Then:

- **(i)** $Npext((M, J)) \tilde{\cup} Npext((G, J)) \tilde{\subseteq} Npext((M, J) \tilde{\cap} (G, J))$.
- **(ii)** $Npint[(M, J) \tilde{\cap} (G, J)] \tilde{\subseteq} Npint(M, J)) \tilde{\cap} Npint((G, J))$.
- **(iii)** $Npcl((M, J)) \tilde{\cap} Npcl[(M, J)^c] = Npb((M, J))$.

(2) If $(G, J_1) \times (K, J_2)$ is soft $Npre -$ open in $(V \times W, \tilde{\rho} \times \tilde{\sigma}, J_1 \times J_2)$, then (G, J_1) and (K, J_2) are soft $Npre -$ open in $(V, \tilde{\rho}, J_1)$ and $(W, \tilde{\sigma}, J_2)$ respectively.

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