Journal of Interdisciplinary Mathematics
ISSN: 0972-0502 (Print), ISSN: 2169-012X (Online)
Vol. 26 (2023), No. 3, pp. 563–571
DOI : 10.47974/JIM-1682

Role of interspecific competition among predators and group size : A dynamical approach

Shivam [†]
Department of Mathematics
Graphic Era University
Dehradun
Uttrakhand
India

Teekam Singh [*]
Department of Computer Science and Engineering
Graphic Era University
Dehradun
Uttrakhand
India

Arvind Kumar [§]
Department of Mathematics
Dyal Singh College
University of Delhi
Delhi
India

Ankit Kumar [‡]
Saroj Kumar Pandey
GLA University
Mathura
Uttar Pradesh
India

[†] *E-mail:* `shivam@ma.geu.ac.in`
[*] *E-mail:* `tsingh@ma.iitr.ac.in (Corresponding Author)`
[§] *E-mail:* `arvindmathsdu@gmail.com`
[‡] *E-mail:* `kumar.ankit@gla.ac.in`

Ramu Dubey [#]
Department of Mathematics
J.C.B.U. of Science and Technology, YMCA
Faridabad
Uttar Pradesh
India

Kamred Udham Singh [$]
School of Computing
Graphic Era Hill University
Dehradun
Uttrakhand
India

Abstract

Living in a group benefit all living beings for their survival and fulfilling needs. This paper proposes and analyzes a prey-predator system with the interspecific death and functional response Hassell-Varley. First, we discuss the existing stationary points of the system and their permanence. Then, by taking the Hassell-Varley coefficient (g) as the bifurcation parameter, we examined the Hopf-bifurcation analysis of the coexistence stationary point. The role of interspecific death rates to different group forms of predator is also examined. Finally, quantitative analysis has been fetched out to validate the qualitative outcomes and findings. The investigations exhibits that the group formation among predators and interspecific death rates are sensitive factors to the system's stability.

Subject Classification: *70K75, 34C23, 37G10.*

Keywords: *Prey-predator system, Interspecific competition, Hopf-bifurcation, Hassell-Varley functional response.*

1. Introduction

The most common behavior that is observed among organisms is group behavior. The formation of groups is defined by the organism's nature and needs. Some organisms used this behavior to protect themselves from others, whereas some used it to capture more food with less effort. Therefore, the study of this behavior in predator-prey relationships gained more interests. The survivability of prey or predators can be improved

[#] *E-mail:* **rdubeyjiya@gmail.com**
[$] *E-mail:* **kamredudhamsingh@gmail.com**

by studying these interactions since they can reveal system imbalances brought on by human disruption.

The choice of functional response, which specifies the predator's intake rate as a function of prey, is one of the environmental variables that might alter population dynamics when analyzing the predator-prey relationship. As a result, researchers have recognized and studied several functional responses, including the Crowley-Martin, Beddington-DeAngelis type, Holling types I through IV, as well as advanced versions of these types [1]-[2]. In this study, the limit of the cluster formed by predators is represented by the Hassell-Varley (H-V) interaction term, and it is subordinate on the density of predators. The Hassell-Varley parameter $0 < g \leq 1$ determines the size of the predators group. A proportion-subordinate predator-prey system is produced for $g = 1,$ where no group forms. These limitations are characteristic of earthbound predators, and they are generated when $g = \frac{1}{2}.$ Very compact groups arise when $g = \frac{1}{3},$ as shown in aquatic predators [3].

Hassel-Varley functional response has been used in numerous studies over the years, including Li and Tian's [4] study of the moveable conduct investigation of an assessment-controlled predator-prey system with an exponent terror impact. A discrete H-V response predator-prey model with an assessment power was studied by Xie et al. [5] who also discovered the prerequisites for the entity and uniqueness of recurrent modalities. Pathak et al. [6] investigated the Hassell-Varley response food chain model, looked at the impacts of delay in time on the model, and concluded that the model might be employed for pest control.

From literature review, it is observed that the prey-predator system is studied accompanied by liner death rate, no study has been carried out for quadratic death rate which is due to interspecific death between population for their survival. Therefore, here in the current study, we construct a prey-predator system accompanied by H-V interaction term and quadratic death rate in predator species. The current manuscript is structured as follows: In sec. 2, we composed a prey-predator system accompanied by Hassel-Varley functional response and quadratic death rate among predators. In sec. 3, we discussed the existing equilibrium points and their stability, and the Hopf-bifurcation theory has been carried out in sec. 4. In sec. 5, we verify the derived results and draw conclusions in sec. 6.

Mathematical Model

The prey-predator system having H-V functional response is defined as

$$\frac{du}{dt} = u(1-u) - \frac{auv}{(u+v)^g}$$
$$\frac{dv}{dt} = \frac{suv}{(u+v)^g} - dv^2 \tag{1}$$
$$v(0) > 0, u(0) > 0.$$

where $u(t)$ and $v(t)$ denotes the density of prey and predators' population at time t. a denotes search efficiency of v for u, s is the metamorphosis rate of u to v & d is the mortality rate of v. Here we consider quadratic death rate to define natural and interspecific death rate [7],[8] and [9].

3. Stability Analysis

We first determine the model's existing equilibrium points and, subsequently, its stability. For computing equilibrium points of the model (1), we put the right-hand side of model (1) equal to zero, we have

1. $E_a = (1,0)$, axial equilibrium point.
2. $E^* = (u^*, v^*)$, the coexistence equilibrium point.

Since, model (1) is too much complicated to get an analytical expression of u^* and v^*, hence we studied them by numerical simulations.

After having an equilibrium point for (1), next we discuss its stability. At any point (u,v) the variational matrix for model (1) is defined as

$$V = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}, \tag{2}$$

where

$$c_{11} = -av(u+v)^{-(1+g)}(u - gu + v) - 2u + 1$$
$$c_{12} = -(u + v - gv)(u+v)^{-(1+g)}au$$
$$c_{21} = (u+v)^{-(1+g)}(u - gu + v)sv$$
$$c_{22} = (u+v)^{-(1+g)}(u + v - gv)su - 2dv$$

Now, at $E_a = (1,0)$ the matrix (2) is defined as

$$V_a = \begin{bmatrix} -1 & -a \\ 0 & s \end{bmatrix}$$

By Routh-Hurwitz (R-H) stability criteria, the model (1) is said to be stable if the eigenvalues of variational matrix (2) are having negative real parts.

At $E_a = (1,0)$ the eigenvalues are -1 and s. Since s cannot be negative, so the model (1) at axial equilibrium point is saddle.

At $E^* = (u^*, v^*)$ matrix (2) is

$$V^* = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}$$

$$= \begin{bmatrix} 1-2u^* - av^*(u^*+v^*)^{-1-g}(u^* - gu^* + v^*) & -au^*(u^*+v^*)^{-1-g}(u^*+v^*-gv^*) \\ sv^*(u^*+v^*)^{-1-g}(u^*-gu^*+v^*) & -2dv^* + su^*(u^*+v^*)^{-1-g}(u^*+v^*-gv^*) \end{bmatrix}$$

To determine the stability at E^*, we examined the V^* matrix characteristic equation i.e.,

$$\lambda^2 - Trace[V^*]\lambda + Det[V^*] = 0 \tag{3}$$

For the stability of model (1), the eigenvalue of (3), must have negative real parts. According to R-H theory, the system (1) is asymptotically stable locally at $E^* = (u^*, v^*)$, if and only if

1. $Trace[V^*] < 0$,
2. $Det[V^*] > 0$.

4. Hopf-Bifurcation Analysis

After the stability at E^*, now we discuss the Hopf-bifurcation analysis of model (1). The Hopf-bifurcation will arise when the model loses its stability due to the pair of purely imaginary eigen values.

At $E^* = (u^*, v^*)$, the model (1) will experience Hopf-bifurcation at $g = \overline{g}$, if $Trace[V^*] = 0$, and $\frac{d}{dg}\mathrm{Re}[\overline{g}] \neq 0$.

To examine the Hopf-bifurcation at $g = \overline{g}$, the $Trace[V^*] = 0$, then the characteristic equation (3) becomes

$$\lambda^2 + Det[V^*] = 0 \tag{4}$$

Now, Eq. (4) must has a pair of purely imaginary eigenvalues $\lambda_{1,2} = \pm i\rho$, where $\rho = \sqrt{Det[V^*]}$ at $g = \overline{g}$. To show the transversality condition, let

$\lambda_{1,2} = A(g) \pm iB(g)$, where $Z(g) = \frac{Trace[V^*]}{2}$ and $B(g) = \sqrt{Det[V^*] - \frac{Trace^2[V^*]}{4}}$ for some value of g. Now,

$$\frac{d}{dg}\mathrm{Re}[g]\Big|_{g=\bar{g}} = \frac{1}{2}Trace[V^*]\Big|_{g=\bar{g}} \neq 0.$$

Hence, the transversality condition is satisfied.

5. Results and Discussion

In this segment, we use quantitative analysis to validate the obtained qualitative analytical outcomes. For that, we fixed numeric parameters value as: a=2, s=1 and consider g and d as bifurcation parameters. For fixed $d = 0.5$ and $g = \frac{2}{3}$, we have (1, 0) and (0.025, 0.156) as the equilibrium points. At $E^* = (0.025, 0.156)$, the calculated eigenvalues are $-0.029 \pm 0.143i$. As obtained characteristic value have negative real-part, hence the model (1) is asymptotically stable at E^*.

Next, we fixed the value of $d = 0.5$ and check the influence of Hassell-Varley coefficient (g) to the model. In Figure 1, as the value of g is increased from 1/3 to 2/3, the population density of both population is decreased. At g=1, both populations are going to extinct. In Figure 2, we show the bifurcation diagram of the population with respect to bifurcation parameter g. From Figures 1 and 2, it is clear that in predator-prey relationship, the group size of predators also affects the model's stability and makes the whole system endangered.

In Figure (3), we show the population dynamics for fixed $g = \frac{1}{3}$ and varying d. As the value of d increases from 0.5 to 1, the prey population increase whereas predator population decreases. In Figure (4), we have population dynamics for fixed $g = 1/2$ with varying d.Here also increase

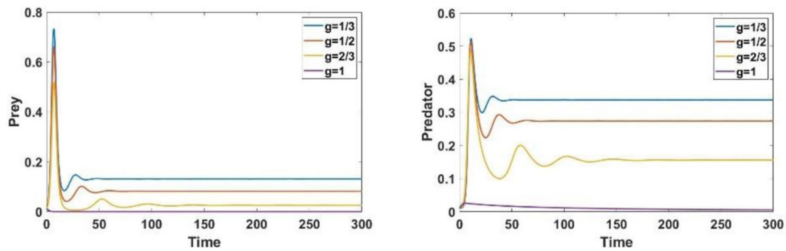

Figure 1

Time series evolution at t = 300 with fixed parameters value and varying g

in the value of d decreases the predator population and increases the prey population. But for $g = 2/3$ we have an interesting result in Figure (5), with an increase in d, both population increases simultaneously.

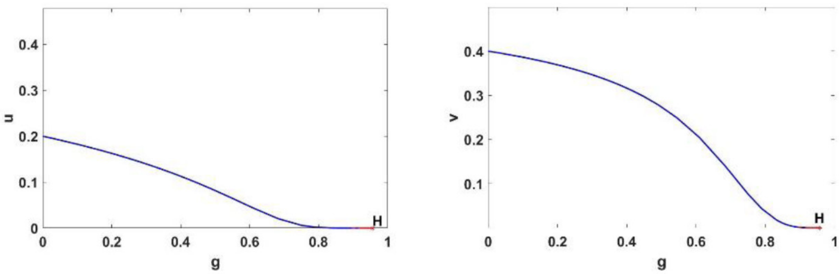

Figure 2

The bifurcation diagram with respect to bifurcation parameter g for fixed values of parameter where H is Hopf-bifurcation point.

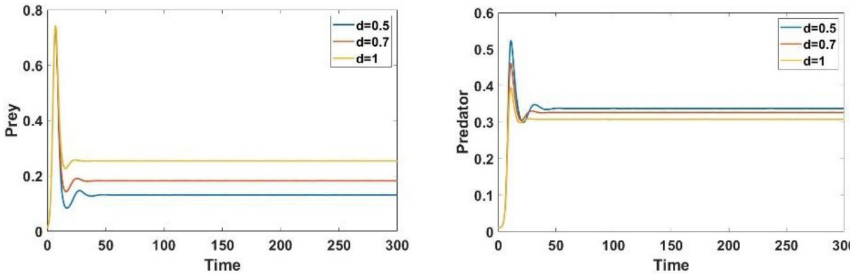

Figure 3

Population distribution evolution for fixed $g = 1/3$ with varying d.

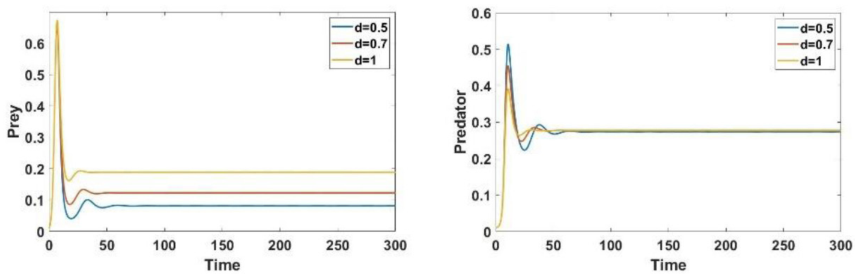

Figure 4

Population distribution evolution for fixed $g = 1/2$ with varying d.

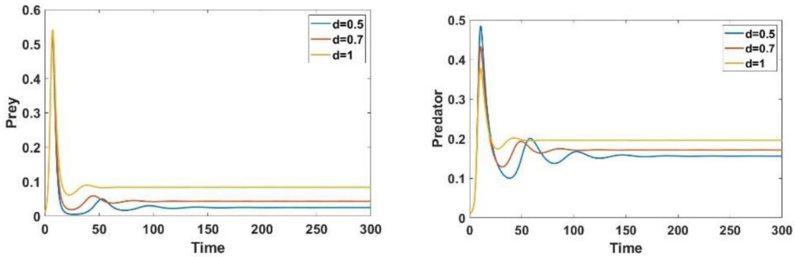

Figure 5

Time series evolution for fixed $g = \frac{2}{3}$ with varying d.

6. Conclusions

In this paper, we discuss about the role of Hassell-Varley coefficient (g) and the interspecific death rate (d) to the predator-prey relationship. The size of population-formed groups is defined by the Hassell-Varley functional response. Organisms in an ecosystem form different group sizes for their need and survival. Thus, we concentrated on predator-prey interactions in which predators establish groups and engage in interspecific rivalry. First, we define the dynamical model and determine its stability using the Jacobian approach. Then we investigate the influence of the controllable parameter d on the model using Hopf-bifurcation. From the results, we conclude that group size is an crucial factor that can affects the dynamics of the predator-prey relationship. Moreover, larger the group size more will be the interspecific competition among individual and makes the system endangered.

References

[1] Fu, Shengmao, and Huisen Zhang. "Effect of hunting cooperation on the dynamic behavior for a diffusive Holling type II predator-prey model." *Communications in Nonlinear Science and Numerical Simulation* 99 : 105807 (2021).

[2] Agarwal, Manju, and Rachana Pathak. "Harvesting and Hopf Bifurcation in a predator-prey model with Holling Type IV Functional Response." *International Journal of Mathematics and Soft Computing* 2, no. 1 : 99 (2017).

[3] Hsu, Sze-Bi, Tzy-Wei Hwang, and Yang Kuang. "Global dynamics of a predator-prey model with Hassell-Varley type functional response." *Discrete Contin. Dyn. Syst. Ser. B* 10, no. 4 : 857-871 (2008).

[4] Li, Huanmeng, and Yuan Tian. "Dynamic behavior analysis of a feedback control predator-prey model with exponential fear effect and Hassell-Varley functional response." *Journal of the Franklin Institute* 360, no. 4 : 3479-3498 (2023).

[5] Xie, Xiaoli, Chunhua Zhang, Xiaoxing Chen, and Jiangyong Chen. "Almost periodic sequence solution of a discrete Hassell–Varley predator-prey system with feedback control." *Applied Mathematics and Computation* 268 : 35-51 (2015).

[6] Pathak, Sweta, Alakes Maiti, and G. P. Samanta. "Rich dynamics of a food chain model with Hassell–Varley type functional responses." *Applied Mathematics and Computation* 208, no. 2 : 303-317 (2009).

[7] Yuan, Sanling, Chaoqun Xu, and Tonghua Zhang. "Spatial dynamics in a predator-prey model with herd behavior." *Chaos: An Interdisciplinary Journal of Nonlinear Science* 23, no. 3 : 033102 (2013).

[8] Singh, Teekam, and Sandip Banerjee. "Spatiotemporal model of a predator–prey system with herd behavior and quadratic mortality." *International Journal of Bifurcation and Chaos* 29, no. 04 : 1950049 (2019).

[9] Shivam, Teekam Singh, and Mukesh Kumar. "Spatiotemporal dynamical analysis of a predator–prey system with fear and group defense in prey." *Journal of Biological Systems* 30, no. 02 : 387-422 (2022).