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Mathematical models of free kick and penalty in soccer

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Abstract

This paper presents a mathematical model for free kick and penalty in Soccer. The modeling is done by assuming the ball moves inside the air fluid and takes into account the influence of the direction, speed, rotation, and the elevation angle of the ball with the ground, and gravitational acceleration of aerodynamic forces such as drag force and Magnus force. The results of this modeling can explain physically how a player can do a very spectacular free kick, the ball soars high over the posse and suddenly dips towards the goal. Moreover, if the player gets the optimal value of the components, goals can be scored from these shots.

Subject Classification: *00A05 Mathematics in general, 00A27 Lists of open problems.*

Keywords: *Mathematical model, Free kick, Penalty, Ball motion, Soccer.*

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1. Introduction

Peretti (2019), explains that the results of analysis using mathematics are important, one of which is mathematical modeling. Mathematical modeling is a process in which real life situations are expressed using mathematics, meaning that real life problems are translated into mathematics, solved using symbols, and the solutions are tested again in real life. Mathematical modeling refers to the process of making mathematical representations of real world scenarios, so that through mathematical modeling it can make predictions and provide insights such as generating various possible solutions to help decision making of the problems encountered (Leela and Commissioning, 2009). To examine situations through mathematics, it is necessary to construct mathematical models of problem-driven natural and social phenomena. Mathematical modeling can be thought of as an iterative process consisting of the following steps: (i) thinking about a problem by identifying a specific problem; (ii) make assumptions and identify variables to be considered; (iii) doing math means writing down the relevant mathematical equations and simplifying them; (iv) solving equations; (v) comparing the results with the data, if the results agree then the mathematical model has been completed, but if not then modification of the equation is required such as making it more complex and incorporating new processes; (vi) the stage continues with model refinement to improve model performance if the results do not agree; (vii) then the mathematical model has been completed, then reports the results of a clear report about the model and its implementation so that the model can be understood by others (Carrejo and Marshall, 2007; Gablonsky and Lang, 2005).

Football is the most popular sport (Luo & Xie, 2018), with high paying players. Therefore, every shot is precious. So the players need to have good kicking skills to control the rotation and trajectory of the ball. Curved trajectories in football have received attention and become a research topic for researchers (Bray and Kerwin, 2003; Huang et al., 2018). Like Leela and Comissiong (2009) in their research on the success of a penalty kick to score a goal, it requires fast shots and range of kick angles which has an impact on the outcome of the curved trajectory of the ball and the ball is difficult for the goalkeeper to reach. Furthermore Carré et al. (2002), who demonstrated a ball trajectory simulation based on the foot impact location. Furthermore, Huang et al (2018) in their research explained flight style, analyzed a mathematical model of a curved trajectory using aerodynamic theory, and simulated a kick trajectory in order to verify the proposed mathematical model.

In elite football, approximately one-third of goals are scored either directly or indirectly from a set play, irrespective of the tournament. Hence, preparation and planning of set plays from both offensive and defensive points of view are important for winning games. Carling et al (2007), explained that offensive set plays to get goals are carried out through free kicks and penalties. In line with this, the research results of Ensum et al (2000) have shown that compared to a short pass which then sends the ball towards the goal, a direct kick from a free kick in the center area is more effective as a goal scoring offensive set play.

Furthermore Bray and Kerwin (2003) on a mathematical model of ball flight have shown that there is little chance for the goalkeeper to prevent a goal from a direct shot originating from a free kick. Several studies have explained kicks in soccer, but the mathematical models in kicking soccer are still minimal, especially on free kicks. Therefore, a free kick mathematical model is needed in football, so this research was conducted in order to develop, determine, and analyze the completion of a free kick mathematical model.

2. Free Kick

2.1 *Model Building*

When the ball is kicked, it means that there is a direction and speed given to the ball, which will make the ball spin. Furthermore, it will affect the curve of the ball's flight. During the flight of the ball, there is a Magnus effect which describes the curved trajectory of the ball or known as a banana kick, this kick causes the ball to go into the goal. This ever happened in 1997 by a Brazilian player Roberto Carlos who took a free kick from 35 meters. Just then, Carlos kicked the ball in the bottom right corner then kicked the ball high and to the right but also spinning around its axis. His shot sent the ball flying wide of the players then bounced to the left and bounced into the goalpost. In this case, Newton explained that objects will move in the same direction and speed until a force is applied to them. When the kick is taken, there is a direction and speed given the ball. The flight of the ball starts with a direct route where air flows on both sides of the ball slowing the ball down. The air flowing on the ball can move in the direction of the ball rotation or against the ball rotation. If both occur, the meaning in this case both occur is that there is air that is opposite and in the direction of the ball's rotation. Air moving in the opposite direction to the ball's rotation causes the pressure on the ball to increase, while on the other hand (air in the direction of the ball's rotation) causes lower pressure. The difference between the pressures makes the ball curve towards the lower pressure zone. This is called the Magnus effect (Badiru, 2010; Goff and Carré, 2009; Huang et al., 2018).

The Magnus force is formulated as

$$F_M = S_p \omega \times v, \tag{1}$$

where ω is an angular velocity vector, v is a linear ball speed, and S_p is a spinning parameter. The value of the spinning parameter depends on the dimensions of the object and the fluid condition in which the object is moving. The cross product shows that the direction of the Magnus force is perpendicular to the direction and the axis of rotation of the object. Further, Equation (1) could be rewritten as

$$F_M = \frac{1}{2} \rho A C_M \tau (\omega \times v), \tag{2}$$

where ρ is air density, A is cross section area, C_M is coefficient of Magnus force, and τ is radius of the object.

There are several forces working and a ball when floating in the air, namely the Magnus force that is perpendicular to the speed of the ball and the direction of the axis of rotation, the air friction force (drag force) which is the opposite direction of the ball, and the gravity force towards the center of the earth. If the free body diagram is described in two dimensions and the ball only rotates in the front direction, the movement of ball motion can be illustrated in Figure 1, where F_D, F_M, F_W, v and ω represent the drag force, Magnus force, gravity, linear velocity, and angular rotation speed of the ball, respectively.

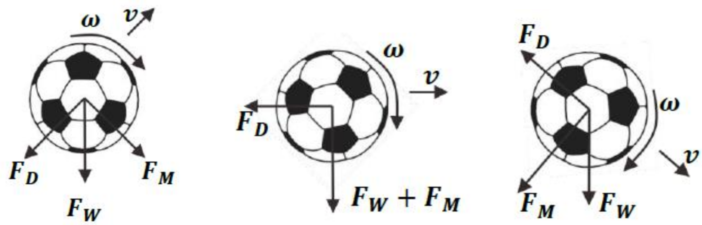

Figure 1
The movement of ball motion

Hence, the ball motion can be derived based on Newton's second law as follows.

$$m\frac{d^2r}{dt^2} = m\frac{dv}{dt} = F_W + F_D + F_M, \tag{3}$$

where $F_W = mg, F_D = -\frac{1}{2}\rho AC_D v^2 \hat{v}$, and $F_M = \frac{1}{2}\rho AC_M r(\omega \ \times v)$. Here, C_D and C_M represent the coefficient of drag force and Magnus force, respectively. Since $\hat{v} = \frac{v}{|v|}$, where $v = \sqrt{v_x^2 + v_y^2 + v_z^2}$, we obtain

$$F_D = -\frac{1}{2}\rho AC_D v(v_x + v_y + v_z), \text{ and}$$

$$F_M = \frac{1}{2}\rho AC_M r(\omega_x + \omega_y + \omega_z) \ \times (v_x + v_y + v_z).$$

If the equations are described in the axis x, y, and z, we have

$$F_{Dx} = -\frac{1}{2}\rho AC_D vv_x, \ F_{Mx} = \frac{1}{2}\rho AC_M r \ (\omega_y v_z - \omega_z v_y),$$

$$F_{Dy} = -\frac{1}{2}\rho AC_D vv_y, \ F_{My} = \frac{1}{2}\rho AC_M r(\omega_z v_x - \omega_x v_z),$$

$$F_{Dz} = -\frac{1}{2}\rho AC_D vv_z, \ F_{Mz} = \frac{1}{2}\rho AC_M r(\omega_x v_y - \omega_y v_x).$$

Thus, the ball motion can also be described in the axis x, y, and z as follow.

$$m\frac{dv_x}{dt} = -\frac{1}{2}\rho A\left(C_D vv_x + C_M r(\omega_z v_y - \omega_y v_z)\right), \tag{4}$$

$$m\frac{dv_y}{dt} = -\frac{1}{2}\rho A\left(C_D vv_y + C_M r(\omega_x v_z - \omega_z v_x)\right), \tag{5}$$

$$m\frac{dv_z}{dt} = -\frac{1}{2}\rho A\left(C_D vv_z + C_D r(\omega_y v_x - \omega_x v_y)\right) - mg. \tag{6}$$

Equations (4), (5), and (6) are the differential equations from the motion of the ball as a result of a free kick in soccer.

2.2 *Model Solving*

The movement of the ball can be derived into a two-dimensional model assuming that it does not have a side spin. Thus, $\omega_z = \omega_x = 0$ which results in no Magnus and drag forces in the direction of the y axis, i.e., $F_{My} = 0$ and $F_{Dy} = 0$. Next, the following accelerations in x and y axis are obtained.

$$a_x = -\frac{\rho A}{2m}\left(C_D v v_x - C_M r \omega_y v_z\right)$$
$$a_z = -\frac{\rho A}{2m}\left(C_D v v_z - C_D r \omega_y v_x\right) - g$$

Thus, if the position and velocity of the ball at time t is known, the speed at x and z axis at time $t + \triangle_t$ can be formulated as follow.

$$v_x(t + \triangle_t) = v_x(t) + \triangle_v = v_x(t) + a_x(t).t,$$
$$v_z(t + \triangle_t) = v_z(t) + \triangle_v = v_z(t) + a_z(t).t.$$
$$v_{x0} = v_0 \cos\theta,\ v_{z0} = v_0 \sin\theta,$$

where v_0, v_{x0}, v_{z0}, and θ represent the initial velocity, initial velocity in x axis, initial velocity in z axis, and the elevation angle when the ball leaves the ground. The position of the ball will changes in speed. Since the change of the ball position in a certain time interval is equal to the average speed of the ball in the same interval, the average speed in the interval time $\triangle_t$ can be presented as

$$\bar{v} = v(t) + \frac{1}{2}a(t)\triangle_t.$$

Hence, the following position of the ball at time $t + \triangle_t$ is obtained.

$$x(t + \triangle_t) = x(t) + v_x(t)\triangle_t + \frac{1}{2}a_x(t)(\triangle_t)^2 \quad (7)$$

$$z(t + \triangle_t) = z(t) + v_z(t)\triangle_t + \frac{1}{2}a_z(t)(\triangle_t)^2. \quad (8)$$

Equations (7) and (8) are the solution of the motion of the ball as a result of a free kick in soccer. Based on the above solution, if the top spin is given to a ball, it is able to maintain its height so that not too far above the crossbar. Thus, the accuracy of the kick will be better compared to a kick without a top spin that seems to soar. Moreover, the ball with top spin only need few second to land on the ground. The Magnus force does not significantly affect the speed of the ball at horizontal direction, but highly affects the speed of the ball in the vertical direction. This is because the Magnus force with a top spin is directly proportional to the sine value of the elevation angle. However, at a greater elevation angle, the influence of the Magnus force at the speed in the horizontal and vertical directions will look higher.

3. Penalty

In this section, formulated the best penalty kick taken with the best angle. This is important to note given the difference in shot yield between the angles of fire on the ground and in the air. Let α and β represent the angles for aerial and ground shots, respectively. We are assuming that shot will travel in a straight line and there is no air resistance.

We start by resolving the initial velocity v_0 into horizontal and vertical components as follow.

$$v_x = v_0 \cos \beta_0$$

$$v_v = v_0 \sin \alpha_0,$$

where α_0 and β_0 are the initial angles. Due to gravity, lack of acceleration for the best penalty kicks while on the pitch. Hence, the equation of ball motion in the horizontal direction is

$$x(t) = vt,$$

where the distance represented by $x(t)$ that can be reached during the time *(t)* with a certain speed *(v)*.

$$x(t) = v_0 t \cos \beta_0.$$

Let l represents the horizontal distance between the goal-line and the penalty spot, and T is the time needed for a ball to reach the distance. Thus we have

$$l = v_0 T \cos \beta_0.$$

Further, the presence of a horizontal and vertical component during a shot in the air, so it is obtained

$$x(t) = v_0 t \cos \alpha_0.$$

Suppose the horizontal distance from the penalty spot to the goal line is expressed as k. Then to reach that horizontal distance, how long does it take for the ball to be expressed as T, then the following equation is obtained

$$k = v_0 T \cos \alpha_0.$$

Similarly, we have the vertical equation of ball motion as follows.

$$y(t) = v_0 t \sin \alpha_0 + \frac{1}{2} g t^2,$$

where g is the acceleration due to gravity. Suppose the vertical distance between the best spot in the air just below the crossbar and the goal line on the pitch is expressed as h. Hence, we obtain the following model.

$$h = v_0 t \sin \alpha_0 + \frac{1}{2} g t^2.$$

Since $T = \frac{k}{v_0 \cos \alpha_0}$, for a given initial angle α_0, we can obtain the following initial velocity formula such that the ball goes in the best direction.

$$v_0 = \frac{k}{\cos \alpha_0} \sqrt{\frac{-g}{2(k \tan \alpha_0 - h)}}.$$

From the above models, we found that both the angle and velocity are the important components needed in the penalty shot. If the player gets the optimal value of the components, goals can be scored from these shots.

4. Conclusions

Based on the formula of the position of the ball, if the ball is kicked by giving a top spin, the ball will experience a Magnus force downward along its path which results in the ball not being too high. In this condition, the speed of the ball can be maintained. The speed in the vertical direction of the ball with a top spin will increase with time. Hence the ball will swooping at a fairly high speed. Further, we found that both the angle and velocity are the important components needed in the penalty shot. If the player gets the optimal value of the components, goals can be scored from these shots.

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