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Describing Bateman-Burgers' equation in one and two dimensions using Homotopy perturbation method

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Abstract

Homotopy perturbation method spread to express a good solution of one of most important equation in various areas in science which is the Bateman-Burgers' Equation. Homotopy perturbation method (HPM) investigate a particular success in solving linear and nonlinear Betman differential equations in both one and two dimension cases.

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1. Introduction

Burgers' equation or Bateman-Burgers equation (also known as viscous Burgers' equation) is considered as one of most important partial differential equations spreading over several areas of science and applied mathematics such as gas dynamics, fluid mechanics, and nonlinear acoustics [1].

The familiar Burgers' equation was initially determined by Bateman (1915)[2, 3]. In 1948 Burgers [4] expressed this equation to obtain some characteristic description of turbulent fluid in a channel caused by the interaction of the opposite effects of diffusion and convection, that is why it is commonly known as "Burgers' equation.

The general form of Bateman-Burgers equation in one space dimension is:

$$\frac{du}{dt} - \Upsilon \frac{d^2u}{dx^2} + u\frac{du}{dx} = 0 \tag{1}$$

where: (Υ) diffusion coefficient or kinematic viscosity [5]. The kinematic viscosity is the (more common) dynamic viscosity divided by the density. The dynamic viscosity has dimension $ML^{-1}T^{-1}$ (Nsm^{-2} in SI-units), while the kinematic viscosity has dimension L^2T^{-1}.

This equation controls three terms; time evolution, Nonlinearity and diffusion which considered as the easiest nonlinear shape formula for diffusive waves in fluid dynamics [6]. The term $u\frac{du}{dx}$ is considered as an impedance term that will interrupt waves and the term $\Upsilon\frac{d^2u}{dx^2}$ is a diffusion term similar to the one which appears in the heat equation.

Obtaining $u(x, t)$ as a field and Υ as diffusion coefficient (or Kinematic viscosity), then familiar formula of Burgers equation (which is also recognized as viscous Burgers equation) in one dimensional space is:

$$\frac{du}{dt} + u\frac{du}{dx} = \Upsilon\frac{d^2u}{dx^2} \tag{2}$$

When the $(\Upsilon = 1)$ Burgers equation take the form:

$$\frac{du}{dt} = \frac{d^2u}{dx^2} - u\frac{du}{dx} \tag{3}$$

When the diffusion term is absent $(\Upsilon = 0)$, burgers equation is considered as inviscid Burgers' equation

$$\frac{du}{dt} + u\frac{du}{dx} = 0 \tag{4}$$

The two-dimensional Burger's equation takes the form [7]

$$u_t + uu_x + vu_y = \frac{1}{Re}(u_{xx} + u_{xx}) \tag{5}$$

$$v_t + uu_x + vv_y = \frac{1}{Re}(v_{xx} + v_{yy}) \tag{6}$$

That are the same as incompressible Navier-tokes equation with the pressure gradient terms removed.

Subject to the initial conditions

$$u(x, y, 0) = \phi_1(x, y) \tag{7}$$

$$v(x, y, 0) = \phi_2(x, y) \tag{8}$$

Where $(x, y) \in \Omega$, and boundary conditions

$$u(x, y, t) = \zeta(x, y, t) \tag{9}$$

$$v(x, y, t) = \xi(x, y, t) \tag{10}$$

Where $(x, y) \in \partial\Omega, t > 0$,

$\Omega = \{(x, y) : a \leq x \leq b,\ a \leq y \leq b\}$ is the computational domain; $u(x, y, t)$ and $v(x, y, t)$ are the velocity components to be determined ; ϕ_1, ϕ_2, ζ, and ξ are known functions; u_t is the unsteady term; uu_x is the non-linear convection term; Re is the Reynolds number, and $\frac{1}{R}(u_{xx} + u_{yy})$ is the diffusion term[8].

2. Methodology

The (HPM) proposed first by Ji-Huan in (1999), for solving partial differential and integral equations, linear or nonlinear. This method proves it's efficiency; it's developed continuously such as q-Homotopy Analysis Aboodh Transform Method (q-HAATM) to work out and spread for more and more fractional differential equation[9]. This method HPM which does not require a small parameter in equation has advantage in that provides an approximate solution to a wide range of nonlinear differential equation [10]. Otherwise this method proves more precise description and can spread over higher order nonlinearity term than other methods as quadrature method as an example [6].

Consider the nonlinear partial differential equation of the form

$$\frac{du}{dt} = \frac{d^2u}{dx^2} + f(x) \tag{11}$$

With initial condition

$$u(x,0) = u_0(x) \tag{12}$$

The Homotopy Perturbation Method (HPM) consists the following steps [11]:

1) Applying the Homotopy Perturbation Method on Eq. (11), we get

$$H(v,p) = (1-p)\left(\frac{du}{dt} - \frac{du_0}{dt}\right) + p\left(\frac{du}{dt} - \frac{d^2u}{dx^2} - f(x)\right) = 0$$

$$H(v,p) = \frac{du}{dt} - \frac{du_0}{dt} - p\frac{du}{dt} + p\frac{du_0}{dt} + p\frac{du}{dt} - p\frac{d^2u}{dx^2} - pf(x) = 0$$

$$H(v,p) = \frac{du}{dt} - \frac{du_0}{dt} + p\left(\frac{du_0}{dt} - \frac{d^2u}{dx^2} - f(x)\right) = 0$$

Then

$$\frac{du}{dt} - \frac{du_0}{dt} = p\left(\frac{d^2u}{dx^2} + f(x) - \frac{du_0}{dt}\right) \tag{13}$$

2) Suppose that the solution of the Eq.(11) take the form

$$u = u_0 + pu_1 + p^2u_2 + p^3u_3 + \dots \tag{14}$$

3) Substituting Eq. (14) into Eq.(13) to get

$$\begin{aligned}
\frac{du_0}{dt} + p\frac{du_1}{dt} + p^2\frac{du_2}{dt} + p^3\frac{du_3}{dt} - \frac{du_0}{dt} \\
&= p\left[\frac{d^2u_0}{dx^2} + p\frac{d^2u_1}{dx^2} + p^2\frac{d^2u_2}{dx^2} + p^3\frac{d^2u_3}{dx^2} + f(x) - \frac{du_0}{dt}\right] \\
&= p\frac{d^2u_0}{dx^2} + p^2\frac{d^2u_1}{dx^2} + p^3\frac{d^2u_2}{dx^2} + p^4\frac{d^2u_3}{dx^2} + pf(x) - p\frac{du_0}{dt}
\end{aligned} \tag{15}$$

4) Comparing the coefficients of equal powers of p to get

$$p^0 : \frac{du_0}{dt} - \frac{du_0}{dt} = 0 \tag{16.a}$$

$$\to u_0 = u(x,0) = u_0(x)$$

$$p^1 : \frac{du_1}{dt} = \frac{d^2u_0}{dx^2} + f(x) - \frac{du_0}{dt} \tag{16.b}$$

$$p^2 : \frac{du_2}{dt} = \frac{d^2u_1}{dx^2} \tag{16.c}$$

$$p^3 : \frac{du_3}{dt} = \frac{d^2u_2}{dx^2} \tag{16.d}$$

5) Taking integral from (0) to (t) on Eq.(16($a-d$)), to get

$$u_1 = \int_0^t \left[\frac{d^2u_0}{dx^2} + f(x) - \frac{du_0}{dt} \right] dt \tag{17a}$$

$$u_2 = \int_0^t \left[\frac{d^2u_1}{dx^2} \right] dt \tag{17b}$$

$$u_3 = \int_0^t \left[\frac{d^2u_2}{dx^2} \right] dt \tag{17c}$$

6) Then the solution will be

$$u(x,t) = u_0 + u_1 + u_2 + u_3 + \ldots \tag{18}$$

3. An Approximate Solution For Non-Linear Bateman Equation By Homotopy Perturbation Method In (1-D).

Consider the Bateman equation of the form:

$$\frac{du}{dt} = \frac{d^2u}{dx^2} - u\frac{du}{dx} \tag{3}$$

With initial condition is:

$$u(x,0) = 2 - \frac{2}{x} \tag{19}$$

Applying the Homotopy Perturbation Method we get:

$$H(v,p) = (1-p)\left(\frac{du}{dt} - \frac{du_0}{dt}\right) + p\left(\frac{du}{dt} + u\frac{du}{dx} - \frac{d^2u}{dx^2}\right) = 0$$

$$\frac{du}{dt} - \frac{du_0}{dt} + p\left(\frac{du_0}{dt} + u\frac{du}{dx} - \frac{d^2u}{dx^2}\right) = 0$$

$$\frac{du}{dt} - \frac{du_0}{dt} = -p\left(\frac{du_0}{dt} + u\frac{du}{dx} - \frac{d^2u}{dx^2}\right) \tag{20}$$

Suppose that the solution take the form:

$$u = u_0 + pu_1 + p^2u_2 + p^3u_3 + \ldots \tag{21}$$

Substituting Eq. (21) into Eq. (20) we get

$$\frac{du_0}{dt}+p\frac{du_1}{dt}+p^2\frac{du_2}{dt}+p^3\frac{du_3}{dt}-\frac{du_0}{dt} = -[pu_0+p^2u_1+p^3u_2+p^4u_3]\left[\frac{du_0}{dx}+p\frac{du_1}{dx}+p^2\frac{du_2}{dx}+p^3\frac{du_3}{dx}\right] + p\left[\frac{d^2u_0}{dx^2}+p\frac{d^2u_1}{dx^2}+p^2\frac{d^2u_2}{dx^2}+p^3\frac{d^2u_3}{dx^2}\right]-p\frac{du_0}{dt} \quad (22)$$

Then

$$\begin{aligned}\frac{du_0}{dt}+p\frac{du_1}{dt}+p^2\frac{du_2}{dt}+p^3\frac{du_3}{dt}-\frac{du_0}{dt} &= -\Big[pu_0\frac{du_0}{dx}+p^2u_0\frac{du_1}{dx}+p^3u_0\frac{du_2}{dx}+p^4u_0\frac{du_3}{dx}\\ &+p^2u_1\frac{du_0}{dx}+p^3u_1\frac{du_1}{dx}+p^4u_1\frac{du_2}{dx}+p^5u_1\frac{du_3}{dx}\\ &+p^3u_2\frac{du_0}{dx}+p^4u_2\frac{du_1}{dx}+p^5u_2\frac{du_2}{dx}+p^6u_2\frac{du_3}{dx}\\ &+p^4u_3\frac{du_0}{dx}+p^5u_3\frac{du_1}{dx}+p^6u_3\frac{du_2}{dx}+p^7u_3\frac{du_3}{dx}\Big]\\ &+p\frac{d^2u_0}{dx^2}+p^2\frac{d^2u_1}{dx^2}+p^3\frac{d^2u_2}{dx^2}+p^4\frac{d^2u_3}{dx^2}-p\frac{du_0}{dt}\end{aligned} \quad (23)$$

Comparing the coefficients of equal powers of p we get

$$p:\frac{du_0}{dt}-\frac{du_0}{dt}=0 \quad (24.a)$$

$$\to u_0=u(x,0)=2-\frac{2}{x}$$

$$p^1:\frac{du_1}{dt}=-u_0\frac{du_0}{dx}+\frac{d^2u_0}{dx^2}-\frac{du_0}{dt} \quad (24.b)$$

$$p^2:\frac{du_2}{dt}=-\left(u_0\frac{du_1}{dx}+u_1\frac{du_0}{dx}\right)+\frac{d^2u_1}{dx^2} \quad (24.c)$$

$$p^3:\frac{du_3}{dt}=-\left(u_0\frac{du_2}{dx}+u_1\frac{du_1}{dx}-u_2\frac{du_0}{dx}\right)+\frac{d^2u_2}{dx^2} \quad (24.d)$$

Taking integral from (0) to (t) on Eq.(24($a-d$)), we get

$$\begin{aligned}u_1&=\int_0^t\left[-\left(2-\frac{2}{x}\right)\frac{d}{dx}\left(2-\frac{2}{x}\right)+\frac{d^2}{dx^2}\left(2-\frac{2}{x}\right)-\frac{d}{dt}\left(2-\frac{2}{x}\right)\right]dt\\ &=\int_0^t\left[-\left(2-\frac{2}{x}\right)\left(\frac{2}{x^2}\right)-\frac{4}{x^3}\right]dt\end{aligned} \quad (25)$$

$$\rightarrow u_1 = -\frac{4}{x^2}t$$

$$\begin{aligned} u_2 &= -\int_0^t \left[-\left(\left(2-\frac{2}{x}\right)\frac{d}{dx}\left(\frac{-4}{x^2}t\right) + \left(\frac{-4}{x^2}t\right)\frac{d}{dx}\left(2-\frac{2}{x}\right) \right) + \frac{d^2}{dx^2}\left(\frac{-4}{x^2}t\right) \right] dt \\ &= \int_0^t \left[-\frac{16}{x^3}t + \frac{16}{x^4}t + \frac{8}{x^4}t - \frac{24}{x^4}t \right] dt \\ &= -\frac{16}{x^3}\frac{t^2}{2} \end{aligned} \tag{26}$$

$$\rightarrow u_2 = -\frac{8}{x^3}t^2$$

$$\begin{aligned} u_3 &= \int_0^t \Bigg[-\left(\left(2-\frac{2}{x}\right)\frac{d}{dx}\left(\frac{-8}{x^3}t^2\right) + \left(\frac{-4}{x^2}t\right)\frac{d}{dx}\left(\frac{-4}{x^2}t\right) - \left(\frac{-8}{x^3}t^2\right)\frac{d}{dx}\left(2-\frac{2}{x}\right) \right) \\ &\quad + \frac{d^2}{dx^2}\left(\frac{-8}{x^3}t^2\right) \Bigg] dt \\ &= \int_0^t \left[-\left(\left(2-\frac{2}{x}\right)\left(\frac{-24}{x^4}t^2\right) + \left(\frac{-4}{x^2}t\right)\left(\frac{8}{x^3}t\right) + \left(\frac{-8}{x^3}t^2\right)\left(\frac{2}{x^2}\right) \right) + \left(\frac{-96}{x^5}t^2\right) \right] dt \\ &= \int_0^t \left[\frac{-48}{x^4}t^2 + \frac{48}{x^5}t^2 + \frac{32}{x^5}t^2 + \frac{16}{x^5}t^2 - \frac{96}{x^5}t^2 \right] dt \\ &= \frac{-48}{x^4}\frac{t^3}{3} \end{aligned} \tag{27}$$

$$\rightarrow u_3 = \frac{-16}{x^4}t^3$$

The solution will be

$$u(x,t) = u_0 + u_1 + u_2 + u_3 + \ldots \tag{28}$$

$$u(x,t) = 2 - \frac{2}{x} - \frac{4}{x^2}t - \frac{8}{x^3}t^2 - \frac{16}{x^4}t^3 - \ldots \tag{28}$$

The approximate solution is

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t)$$

$$u(x,t) = 2 - \frac{2}{x}\left(1 + \frac{2}{x}t + \frac{4}{x^2}t^2 + \frac{8}{x^3}t^3 \ldots\right)$$

$$u(x,t) = 2 - \frac{2}{x}\left(1 - \frac{2}{x}t\right)^{-1} = 2 - \frac{2}{x\left(1 - \frac{2}{x}t\right)}$$

$$u(x,t) = 2 - \frac{2}{(x-2t)} \tag{29}$$

4. An Approximate Solution For Non-Linear Bateman Equation By Homotopy Perturbation Method In (2-D)

Consider the two dimensional nonlinear Burger's differential equation of the form

$$\frac{du}{dt} + u\frac{du}{dx} + v\frac{du}{dy} = \frac{1}{Re}\left(\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2}\right) \tag{5}$$

$$\frac{dv}{dt} + u\frac{dv}{dx} + v\frac{dv}{dy} = \frac{1}{Re}\left(\frac{d^2v}{dx^2} + \frac{d^2v}{dy^2}\right) \tag{6}$$

With initial conditions

$$u(x,y,0) = 2x + y, \tag{30}$$

$$v(x,y,0) = x - 2y. \tag{31}$$

Applying Homotopy Perturbation Method and suppose $Re = 1$, then

$$H(v,p) = (1-p)\left(\frac{du}{dt} - \frac{du_0}{dt}\right) + p\left(\frac{du}{dt} + u\frac{du}{dx} + v\frac{du}{dy} - \frac{d^2u}{dx^2} - \frac{d^2u}{dy^2}\right) = 0$$

$$H(v,p) = (1-p)\left(\frac{dv}{dt} - \frac{dv_0}{dt}\right) + p\left(\frac{dv}{dt} + u\frac{dv}{dx} + v\frac{dv}{dy} - \frac{d^2v}{dx^2} - \frac{d^2v}{dy^2}\right) = 0$$

$$\frac{du}{dt} - \frac{du_0}{dt} = -p\left(\frac{du_0}{dt} + u\frac{du}{dx} + v\frac{du}{dy} - \frac{d^2u}{dx^2} - \frac{d^2u}{dy^2}\right) \tag{32}$$

$$\frac{dv}{dt} - \frac{dv_0}{dt} = -p\left(\frac{dv_0}{dt} + u\frac{dv}{dx} + v\frac{dv}{dy} - \frac{d^2v}{dx^2} - \frac{d^2v}{dy^2}\right) \tag{33}$$

Suppose that the solution take the form:

$$u = u_0 + pu_1 + p^2u_2 + p^3u_3 + \ldots \tag{34.a}$$

$$v = v_0 + pv_1 + p^2v_2 + p^3v_3 + \ldots \tag{34.b}$$

Substituting Eq. (34. ($a-b$)) into Eq. (32) and Eq. (33), we get

$$\begin{aligned}&\frac{du_0}{dt}+p\frac{du_1}{dt}+p^2\frac{du_2}{dt}+p^3\frac{du_3}{dt}-\frac{du_0}{dt}\\&\quad=-p\frac{du_0}{dt}\\&\quad-[pu_0+p^2u_1+p^3u_2+p^4u_3]\left[\frac{du_0}{dx}+p\frac{du_1}{dx}+p^2\frac{du_2}{dx}+p^3\frac{du_3}{dx}\right]\\&\quad-[pv_0+p^2v_1+p^3v_2+p^4v_3]\left[\frac{du_0}{dy}+p\frac{du_1}{dy}+p^2\frac{du_2}{dy}+p^3\frac{du_3}{dy}\right]\\&\quad+p\frac{d^2u_0}{dx^2}+p^2\frac{d^2u_1}{dx^2}+p^3\frac{d^2u_2}{dx^2}+p^4\frac{d^2u_3}{dx^2}+p\frac{d^2u_0}{dy^2}+p^2\frac{d^2u_1}{dy^2}\\&\quad+p^3\frac{d^2u_2}{dy^2}+p^4\frac{d^2u_3}{dy^2}\end{aligned}\tag{35}$$

And

$$\begin{aligned}&\frac{dv_0}{dt}+p\frac{dv_1}{dt}+p^2\frac{dv_2}{dt}+p^3\frac{dv_3}{dt}-\frac{du_0}{dt}\\&\quad=-p\frac{dv_0}{dt}\\&\quad-[pv_0+p^2v_1+p^3v_2+p^4v_3]\left[\frac{dv_0}{dy}+p\frac{dv_1}{dy}+p^2\frac{dv_2}{dy}+p^3\frac{dv_3}{dy}\right]\\&\quad-[pu_0+p^2u_1+p^3u_2+p^4u_3]\left[\frac{dv_0}{dx}+p\frac{dv_1}{dx}+p^2\frac{dv_2}{dx}+p^3\frac{dv_3}{dx}\right]\\&\quad+p\frac{d^2v_0}{dx^2}+p^2\frac{d^2v_1}{dx^2}+p^3\frac{d^2v_2}{dx^2}+p^4\frac{d^2v_3}{dx^2}+p\frac{d^2v_0}{dy^2}+p^2\frac{d^2v_1}{dy^2}\\&\quad+p^3\frac{d^2v_2}{dy^2}+p^4\frac{d^2v_3}{dy^2}\end{aligned}\tag{36}$$

Comparing the coefficients of equal powers of p we get

$$p:\frac{du_0}{dt}-\frac{du_0}{dt}=0\tag{37.a}$$

$$\rightarrow u_0=u(x,0)=2x+y$$

$$\frac{dv_0}{dt}-\frac{dv_0}{dt}=0\tag{37.b}$$

$$\rightarrow v_0=v(x,0)=x-2y$$

$$p^1:\frac{du_1}{dt}=-\frac{du_0}{dt}-u_0\frac{du_0}{dx}-v_0\frac{du_0}{dy}+\frac{d^2u_0}{dx^2}+\frac{d^2u_0}{dy^2}\tag{37.c}$$

$$\frac{dv_1}{dt} = -\frac{dv_0}{dt} - u_0\frac{dv_0}{dx} - v_0\frac{dv_0}{dy} + \frac{d^2v_0}{dx^2} + \frac{d^2v_0}{dy^2}$$

$$p^2 : \frac{du_2}{dt} = -u_0\frac{du_1}{dx} - u_1\frac{du_0}{dx} - v_0\frac{du_1}{dy} - v_1\frac{du_0}{dy} + \frac{d^2u_1}{dx^2} + \frac{d^2u_1}{dy^2} \qquad (37.d)$$

$$\frac{dv_2}{dt} = -u_0\frac{dv_1}{dx} - u_1\frac{dv_0}{dx} - v_0\frac{dv_1}{dy} - v_1\frac{dv_0}{dy} + \frac{d^2v_1}{dx^2} + \frac{d^2v_1}{dy^2}$$

$$p^3 : \frac{du_3}{dt} = -u_0\frac{du_2}{dx} - u_1\frac{du_1}{dx} - u_2\frac{du_0}{dx} - v_0\frac{du_2}{dy} - v_1\frac{du_1}{dy} - v_2\frac{du_0}{dy} + \frac{d^2u2}{dx^2} + \frac{d^2u2}{dy^2} \qquad (37.e)$$

$$\frac{dv_3}{dt} = -u_0\frac{dv_2}{dx} - u_1\frac{dv_1}{dx} - u_2\frac{dv_0}{dx} - v_0\frac{dv_2}{dy} - v_1\frac{dv_1}{dy} - v_2\frac{dv_0}{dy} + \frac{d^2v_2}{dx^2} + \frac{d^2v_2}{dy^2}$$

Taking integral from (0) to (t) on Eq. (37.$(a-e)$), we get

$$u_1 = \int_0^t \Big[-\frac{d}{dt}(2x+y) - (2x+y)\frac{d}{dx}(2x+y) - (x-2y)\frac{d}{dy}(2x+y) + \frac{d^2}{dx^2}(2x+y) + \frac{d^2}{dy^2}(2x+y) \Big] dt$$

$$\rightarrow u_1 = -5xt \qquad (38)$$

$$v_1 = \int_0^t \Big[-\frac{d}{dt}(x-2y) - (x-2y)\frac{d}{dy}(x-2y) - (2x+y)\frac{d}{dx}(x-2y) + \frac{d^2}{dx^2}(x-2y) + \frac{d^2}{dy^2}(x-2y) \Big] dt$$

$$\rightarrow v_1 = -5yt \qquad (39)$$

$$u_2 = \int_0^t \Big[-(2x+y)\frac{d}{dx}(-5xt) - (-5xt)\frac{d}{dx}(2x+y) - (x-2y)\frac{d}{dy}(-5xt) - (-5yt)\frac{d}{dy}(2x+y) + \frac{d^2}{dx^2}(-5xt) + \frac{d^2}{dy^2}(-5xt) \Big] dt$$

$$\rightarrow u_2 = 10xt^2 + 5yt^2 \qquad (40)$$

$$v_2=\int_0^t\Big[-(x-2y)\frac{d}{dy}(-5yt)-(-5yt)\frac{d}{dy}(x-2y)-(2x+y)\frac{d}{dx}(-5yt)$$
$$-(-5xt)\frac{d}{dx}(x-2y)+\frac{d^2}{dx^2}(-5yt)+\frac{d^2}{dy^2}(-5yt)\Big]dt$$

$$\rightarrow v_2=5xt^2-10yt^2 \tag{41}$$

$$u_3=\int_0^t\Big[-(2x+y)\frac{d}{dx}(10xt^2+5yt^2)-(-5xt)\frac{d}{dx}(-5xt)$$
$$-(10xt^2+5yt^2)\frac{d}{dx}(2x+y)-(x-2y)\frac{d}{dy}(10xt^2+5yt^2)$$
$$-(-5yt)\frac{d}{dy}(-5xt)-(5xt^2-10yt^2)\frac{d}{dy}(2x+y)$$
$$+\frac{d^2}{dx^2}(10xt^2+5yt^2)+\frac{d^2}{dy^2}(10xt^2+5yt^2)\Big]dt$$

$$\rightarrow u_3=-25xt^3 \tag{42}$$

$$v_3=\int_0^t\Big[-(x-2y)\frac{d}{dx}(5xt^2-10yt^2)-(-5yt)\frac{d}{dy}(-5yt)$$
$$-(5xt^2-10yt^2)\frac{d}{dy}(x-2y)-(2x+y)\frac{d}{dx}(5xt^2-10yt^2)$$
$$-(-5xt)\frac{d}{dx}(-5yt)-(10xt^2+5yt^2)\frac{d}{dx}(x-2y)$$
$$+\frac{d^2}{dx^2}(5xt^2-10yt^2)+\frac{d^2}{dy^2}(5xt^2-10yt^2)\Big]dt$$

$$\rightarrow v_3=-25yt^3 \tag{43}$$

Therefore, the solution obtained is given below:

$$u(x,y,t)=\sum_{n=0}^{\infty}u_n(x,y,t)=u_0+u_1+u_2+u_3+\ldots$$

$$u(x,y,t)=2x+y-5xt+10xt^2+5yt^2-25xt^3 \tag{44}$$

And

$$v(x,y,t)=\sum_{n=0}^{\infty}v_n(x,y,t)=v_0+v_1+v_2+v_3+\ldots$$

$$v(x,y,t)=x-2y-5yt+5xt^2-10yt^2-25yt^3 \tag{45}$$

5. Conclusion

In this paper Homotopy perturbation method has proven successful when used to find the approximate solution to the nonlinear Bateman equation in one and two dimensions.

The results of the Homotopy perturbation method give us evidence that this method is very effective and has a high accuracy to find solutions for the partial differential equations**.**

The solution method has been explained in detail. Hence, this research is an important tool to understand the (HPM) as much as possible.

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