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Characterization of *I*-*b*-open sets

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Abstract

In this article we obtain some answers for the open problem stated in [3] by Carpintero et al. If (X,τ,I) is an ideal topological space, under what conditions on the space and/or in the ideal, we can say that if $A \subseteq X$ such that $cl(A)$ is an I-b-open set, then A is an I-b-open.

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1. Introduction

In 2013, Friday Michael [7], studied some open sets with respect to an ideal and Hosny et al. [12], introduced and studied the notions of α-open sets and β-open sets with respect to an ideal. In 2014, Hosny [13], introduced and studied pre open sets with respect to an ideal and gave the existent relationship between the notions described above. In 2014, Carpintero et al. [2], introduced and characterized the notions of weakly semi open set with respect to an ideal. In 2016, Carpintero et al. [3], obtained the relationship between the notions described in [7], [13], [12] with the sets defined in [3]. In 2018, Roy et al. [14], the concept of weakly I_μ-open sets are investigated and in 2020, Tiwari et al. [15], defined the notion of weakly I_μ-continuity in generalize topology on topological spaces via ideal. Sonal Mittal and Rashmi Singh [6] studied similar sets in a fuzzy topology framework to generalized some classical topological notions. More recently, Jamil Mahmoud [5] and Mustafa Hasan Hadi et al. [4] studied analogous sets in a context of generalized topological structures and obtain interesting results. The notion of I-b-open sets was given in [3] and shows the relationships with the sets defined in [2] and [12]. We consider λ as a function defined of $SO(X)$ onto $P(X)$ and is called an s-operation [10] if $V \subseteq \lambda(V)$ for each nonempty semi open set V. Let X be a topological space and $\lambda : SO(X) \to P(X)$ be an s-operation, then a subset A of X is called a λ-open set [10] if for each $x \in A$ there exists a semi open set U such that $x \in U$ and $\lambda(U) \subseteq A$. A λ-open set is called λ_c-open set [10] if for each $x \in A$, there exists a closed set F such that $x \in F \subseteq A$. The family of all λ-open (resp. λ_c-open) subsets of a topological space (X,τ) is denoted by $SO_\lambda(X)$ or $SO_\lambda(X,\tau)$ (resp. $SO_{\lambda_c}(X)$ or $SO_{\lambda_c}(X,\tau)$) [10]

In this article we try to solve the open question stated in [3]. Let (X,τ,I) be an ideal topological space and $A \subseteq X$. If $cl(A)$ is an I-b-open set, under what conditions A is an I-b-open. Finally we study the relationships of *I*-b open set and the weakly *I*-semi open set and study the *I*-b-continuity.

2. Generalized sets with respect to an ideal

Throughout this paper, (X, τ) always mean topological space in which no separation axioms are assumed, unless explicitly stated. If I is and ideal on X, (X, τ, I) mean an ideal topological space. For a subset A of (X, τ), $cl(A)$ and $int(A)$ denote the closure of A with respect to τ and the interior of A with respect to τ, respectively.

Definition 2.1 : A subset A of a topological space (X, τ) is called:

(1) Semi open [11] if $A \subseteq cl(int(A))$;

(2) Pre open [8] if $A \subseteq int(cl(A))$;

(3) b-open [1] if $A \subseteq cl(int(A)) \cup int(cl(A))$.

Theorem 2.2 : [9] *For a topological space* (X, τ), $SO_{\lambda_c}(X) \subseteq SO_{\lambda}(X) \subseteq SO(X)$

Definition 2.3 : Let (X, τ, I) be an ideal topological space. A subset A of X is said to be:

(1) Semi open with respect to an ideal I [7], denoted by *I*-semi open, if there exists $U \in \tau$ such that $U \setminus A \in I$ and $A \setminus cl(U) \in I$;

(2) Pre open with respect to an ideal I [13], denoted by *I*-pre open, if there exists $U \in \tau$ such that $A \setminus U \in I$ and $U \setminus cl(A) \in I$;

(3) Weakly Semi open with respect to an ideal I [3], denoted by *I*-weakly semi open, if $A = \varnothing$ or if $A \neq \varnothing$ there exist an open set $U \neq \varnothing$ such that $U \setminus A \in I$.

(4) b-open with respect to an ideal I [3], denoted by *I*-b-open, if there exist two open sets U, V such that $U \setminus A \in I$, $V \setminus cl(A) \in I$ and $A \setminus (V \cup cl(U)) \in I$.

Definition 2.4 : Let I be an ideal on generalized topological space (X, μ) is called weakly I_{μ}-open set [14], if there exists non-empty μ-open set U such that $U \setminus A \in I$. The complement of weakly I_{μ}-open set is weakly I_{μ}-closed set

Remark 2.5 : We point out that in [14], the concept of weakly I_{μ}-open set were defined. Observe that if we take the generalized topology μ equal to the topology τ, then we obtain the definition of *I*-weakly semi open given in [2].

Example 2.6 : Let $X = \{a,b,c,d\}$, $\tau = \{\emptyset, X, \{a,c\}, \{a,c,d\}\}$ and $A = \{b,c\}$. If we chose:

(1) $I = \{\emptyset\}$, then $cl(A) = X$ is an *I*-b-open, also A is an *I*-b-open. Take $U = \emptyset$, $V = X$;

(2) I any ideal, then $cl(A) = X$ is an *I*-b-open and also A is an *I*-b-open. Take $U = \emptyset$, $V = X$.

Example 2.7 : According with Example 2.6, the sets $\{a,c\}, \{a,b,c\}$ and $\{a,c,d\}$ are semi open. Also for any ideal *I*, all this sets are *I*-b open.

Example 2.8 : According with Example 2.6, all subset *A* of *X* distinct of {*b*}, {*d*} and {*b, d*} are pre open.

(1) If $I = \{\emptyset\}$, then all the pre open set are *I*-b-open;

(2) If $I = \{\emptyset, \{c\}\}$, then all the pre open set are *I*-b-open;;

(3) If $I = \{\emptyset, \{a\}\}$, then all the pre open set are *I*-b-open;

(4) If *I* is any ideal, then all the pre open set are *I*-b-open;

Example 2.9 : According with Example 2.6. If we take $A = \{b\}$ and $I = \{\emptyset, \{b\}\}$, we obtain that A is an *I*-b-open set, taking $U = V = \emptyset$. But $cl(A) = \{b,d\}$ is not and *I*-b-open set. This means that if the ideal $I \neq \{\emptyset\}$ and $A \subseteq X$, such that $cl(A) \neq X$, then $cl(A)$ not necessarily is an *I*-b-open.

Remark 2.10 : Observe that if (X, τ, I) is an ideal topological space. If $A \subseteq X$ is *I*-b-open and *A* is a dense set, then $cl(A)$ is an *I*-b-open set. In this case, take $U = \emptyset$ and $V = X$.

Theorem 2.11 : Let (X, τ, I) be an ideal topological space. If $A \subseteq X$ is a dense set and *I* is any ideal, then *A* and $cl(A)$ are *I*-b-open set.

Proof: Take $U = \emptyset$ and $V = X$ and the proof follows. □

Theorem 2.12 : *Let* (X, τ, I) *be an ideal topological space. If* $A \subseteq X$ *is an open set and I is any ideal, then A and* $cl(A)$ *are I-b-open set.*

Proof: Take $U = V = A$ and the proof follows. □

Remark 2.13 : Let (X,τ,I) be an ideal topological space where I is any ideal and $A\subseteq X$.

(1) if A is an open set, then A is an I-b-open set if and only if $cl(A)$ if an I-b-open set;

(2) If A is a dense set, then A is an I-b-open set if and only if $cl(A)$ if an I-b-open set.

Theorem 2.14 : *Let (X,τ,I) be an ideal topological space. If $A\subseteq X$ is an I-semi open set and I is any ideal, then A is an I-b-open set. The converse is not necessarily true.*

Proof: Since A is an I-semi open set, there exists $U\in\tau$ such that $U\setminus A\in I$ and $A\setminus cl(U)\in I$. Take $U_1=V_1=U$ and used the hereditary property of the ideal. □

Theorem 2.15 : *Let (X,τ,I) be an ideal topological space. If $A\subseteq X$ is an I-pre open set and I is any ideal, then A is an I-b-open set. The converse is not necessarily true.*

Proof: Since A is an I-pre open set, there exists $U\in\tau$ such that $A\setminus U\in I$ and $U\setminus cl(A)\in I$. Take $U_1=\varnothing$ and $V_1=U$ and obtain that: $U_1\setminus A\in I$, $V_1\setminus cl(A)=U\setminus cl(A)\in I$ and $A\setminus(V_1\cup cl(U_1)=A\setminus U\in I$. In consequence A is an I-b-open set. □

Example 2.16 : Let $X=\mathbb{R}$ the real line with the usual topology and $I=\{\varnothing\}$. Take $A=[0,1]\cup((1,2)\cap\mathbb{Q}^c)$. It is easy to see that A is an I-b-open set but neither I-semi open nor I-pre open

Theorem 2.17 : *Let (X,τ,I) be an ideal topological space. If $A\subseteq X$ is a semi open set, then A is an I-b-open set if and only if $cl(A)$ if an I-b-open set.*

Proof: Since each semi open set is an I-semi open set, then the proof follows directly. □

Corollary 2.18 : *Let (X,τ,I) be an ideal topological space. If $A\subseteq X$ is a λ-open set, then A is an I-b-open set if and only if $cl(A)$ if an I-b-open set.*

Proof: Follows from Theorem 2.17 and Proposition 2.2. □

Corollary 2.19 : *Let (X,τ,I) be an ideal topological space. If $A\subseteq X$ is a λ_c-open set, then A is an I-b-open set if and only if $cl(A)$ if an I-b-open set.*

Proof: Follows from Theorem 2.17 and Proposition 2.2. □

Theorem 2.20 : *Let (X, τ, I) be an ideal topological space. If $A \subseteq X$ is a pre open set, then A is an I-b-open set if and only if $cl(A)$ if an I-b-open set.*

Proof: Since each pre open set is an I-pre open set, then the proof follows directly. □

Example 2.21 : Let $X = \{a, b, c, d\}$, $\tau = \{\varnothing, X, \{a, b, c\}\}$, $I = \{\varnothing, \{c\}\}$ and $A = \{c\} \in I$, then A is a dense set. Observe that $cl(A)$ is an *I-b*-open, taking $U = \varnothing$ and $V = X$. Also A is *I-b*-open. Observe that $\tau \cap I = \{\varnothing\}$.

Example 2.22 : Let $X = \{a, b, c, d\}$, $\tau = \{\varnothing, X, \{a\}, \{a, b\}, \{a, b, c\}\}$, $I = \{\varnothing, \{a\}\}$ and $A = \{b, c\}$, then $cl(A) = \{b, c, d\}$ is an *I-b*-open and A is an *I-b*-open.

Theorem 2.23 : *Let (X, τ, I) be an ideal topological space such that $I \neq \{\varnothing\}$ and there exist an open and dense subset A in I. If $B \subseteq X$, such that $cl(B)$ is an I-b-open set then B if an I-b-open set.*

Proof: Let $B \in X$ such that $cl(B)$ is an *I*-b-open, then there exist *U, V* open subsets of *X* such that $U \setminus cl(B) \in I$, $V \setminus cl(B) \in I$ and $cl(B) \setminus (V \cup cl(U)) \in I$. If we take $U_1 = A$, $V_1 = V$, we obtain that:

(1) $U_1 \setminus B = A \setminus B \subseteq A \in I$;

(2) $V_1 \setminus cl(B) = V \setminus cl(B) \in I$;

(3) $B \setminus (V_1 \cup cl(U_1)) = B \setminus (V \cup X) \in I$.

In consequence, *B* if an *I*-b-open set. □

Example 2.24 : Let $X = \{a, b, c, d\}$, $\tau = \{\varnothing, X, \{a, b\}, \{c\}, \{a, b, c\}, \{b\}, \{b, c\}\}$, $I = \{\varnothing, \{c\}\}$ and $A = \{a\}$. Take $U = \{c\}$, then A is a weakly *I*-semi open set but is not an *I*-b-open set, because for $U \in \tau$, $U \setminus A = \varnothing$ or $\{b, c, d\}$ or $\{b\}$ or $\{c\}$ or $\{b\}$ or $\{b, c\}$; $V \setminus cl(A) = \varnothing$ or $\{b, c\}$ or $\{b\}$ or $\{c\}$. The only possibility is $U = V = \{c\}$, $V \cup cl(U) = \{c, d\}$ and then, $A \setminus \{c, d\} = \{a\} \notin I$.

Example 2.25 : In the Example 2.6, If $I = \{\varnothing\}$ and $A = \{c\}$, then A is a dense set, $cl(A)$ is an *I* b-open and *A* is an *I*-b-open. Also observe that *A* is not an *I*-weakly semi open set.

Example 2.26 : Let $X = \{a, b, c, d\}$, $\tau = \{\varnothing, X, \{a\}, \{a, b\}, \{a, b, c\}\}$, $I = \{\varnothing, \{b\}\}$ and $A = \{b\}$. Observe that A is neither an *I*-weakly semi open set nor an *I*-b-open set.

Theorem 2.27 : *Let (X,τ,I) be an ideal topological space where $I \neq \{\emptyset\}$ and the collection of nonempty open sets satisfies the finite intersection property. If $A \subseteq X$ such that $cl(A) \neq X$ is an I-b-open set, then A is an I-weakly semi open set.*

Proof: Since A is an I-b-open, there exist two open sets U, V such that $U \setminus A \in I$, $V \setminus cl(A) \in I$ and $A \setminus (V \cup cl(U)) \in I$. Take $V_1 = V \setminus cl(A)$, then $V_1 \neq \emptyset$ and $V_1 \setminus A = (V \setminus cl(A)) \setminus A = V \setminus cl(A) \in I$. In consequence, A if an I-weakly semi open set. □

Remark 2.28 : Is the converse of the above theorem true?

3. *I-b* continuous functions

Definition 3.1 : Let (X,τ,I) be an ideal topological space and (Y,σ) a topological space. A function $f:(X,\tau,I)\to(Y,\sigma)$ is said to be I-b-continuous if for each open set V in Y, $f^{-1}(V)$ is an I-b-open set in X.

Theorem 3.2 : *Let (X,τ,I) be an ideal topological space, (Y,σ) a topological space and $f:(X,\tau,I)\to(Y,\sigma)$ a function:*

(1) *If f is a continuous function, then f is an I-b-continuous functions;*

(2) *If f is a semi-continuous function [11], then f is an I-b-continuous functions;*

(3) *If f is a I-semi-continuous function, then f is I-b-continuous functions.*

The following examples show that the converse of the above Theorem is not necessarily true.

Example 3.3 : Let $X = \{a,b,c\}$, $\tau = \{\emptyset, X, \{a\}\}$, $Y = \{a,b\}$, $\sigma = \{\emptyset, \{a\}\}$. I any ideal. Define $f: X \to Y$ as follows: $f(a) = f(b) = a$ and $f(c) = c$. It is easy to see that *f* is *I*-b-continuous but is not continuous.

Example 3.4 : Let $X = Y = \{a,b,c,d\}$, $\tau = \sigma = \{\emptyset, X, \{a,b,c\}\}$ and *I* any ideal. Define $f: X \to Y$ as follows: $f(a) = b, f(b) = d, f(c) = a$ and $f(d) = c$. It is easy to see that *f* is *I-b*-continuous but is not semi-continuous.

Theorem 3.5 : *Let (X,τ,I) be an ideal topological spaces, (Y,σ) a topological space and $f:(X,\tau,I)\to(Y,\sigma)$, be a function. The following properties are equivalents:*

(1) *f is I-b-continuous;*

(2) $f^{-1}(V)$ *is I-b-open in X for each open set V in Y;*

(3) $f^{-1}(K)$ *is I-b-closed in X for each closed set K in Y;*

(4) $f(I-b-cl(A)) \subseteq cl(f(A))$ *for every subset A of X;*

(5) $I-b-cl(f^{-1}(B)) \subseteq f^{-1}(cl(B)$ *for every subset B of Y.*

where $I-b-cl(A)$ *is the intersection of all I-b-closed sets in X containing A.*

Proof: The proof follows directly from the definition and using the fact that the union of *I*-b-open sets is *I*-b-open. ☐

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