Journal of Interdisciplinary Mathematics
ISSN: 0972-0502 (Print), ISSN: 2169-012X (Online)
Vol. 26 (2023), No. 1, pp. 77–94
DOI : 10.47974/JIM-1649

Multiple eulerian integrals with modified multivariable I-function having general arguments

N. Srimannarayana *
Department of Mathematics
Koneru Lakshmaiah Education Foundation
Vaddeswaram
Andhra Pradesh
India

Fredric Ayant †
Six-Fours les Plages 83140
Var
France

Y. Pragathi Kumar §
Department of Mathematics
Chalapathi Institute of Engineering and Technology
Lam
Guntur
Andhra pradesh
India

D. K. Pavan Kumar ‡
Department of Mathematics
Gudlavalleru Engineering College
Gudlavalleru
Andhra pradesh
India

* *E-mail:* `sriman72@gmail.com` (Corresponding Author)
† *E-mail:* `fredericayant@gmail.com`
§ *E-mail:* `pragathi.ys@gmail.com`
‡ *E-mail:* `krish.pav@gmail.com`

B. Satyanarayana [@]
Department of Mathematics
Acharya Nagarjuna University
Nagarjuna Nagar
Andhra Pradesh
India

Abstract

Many authors have recently provided a number of expressions for a general Eulerian integrals about the multi-variable H-functions. Inspired by these works, we evaluated here a general class of multi-variable Eulerian integrals with modified multi-variable I-function (MMIF) with general arguments, which has been defined in this article. Some of the particular cases have been discussed. These results will help to deduce numerous useful integrals.

Subject Classification: *(2010) 33C99, 33C60, 44A20.*

Keywords: *Modified multivariable I and H-functions, Multiple integral contours, Multiple eulerian integral, Sequence of polynomials.*

1. Introduction

Srivastava and Panda[13, 14] established some expansion theorems and generating relations for H-function. Prasad and Singh[4] extended the H-function defined by Srivastava. Later Prasad[3] extended the H-function as I-function. Recently Prathima et. al.[5] defined multivariable I-function as an extension of H-function. In this article, we establish a general class of multiple Eulerian integrals with MMIF. We note I-function as :

$$I[z_1, z_2, ..., z_r] = I^{0,n_2;0,n_3;....;0,n_r:\left|R^1:m^1,n^1;....;m^{(r)},n^{(r)}\right.}_{p_2,q_2;p_3,q_3;....;p_r,n_r:\left|R:p^1,q^1;....;p^{(r)},q^{(r)}\right.}$$

$$\left[\begin{array}{c|c} Z_1 & (a_{2j};\alpha_{2j}^{(1)},\alpha_{2j}^{(2)};A_{2j})_{1,p_2};(\alpha_{3j};\alpha_{3j}^{(1)},\alpha_{3j}^{(2)},\alpha_{3j}^{(3)};A_{3j})_{1,p_3};....;(a_{rj};\alpha_{rj}^{1},....,\alpha_{rj}^{(r)};A_{rj})_{1,p_r}: \\ \vdots & \\ Z_r & (b_{2j};\beta_{2j}^{(1)},\beta_{2j}^{(2)};B_{2j})_{1,q_2};(b_{3j};\beta_{3j}^{(1)},\beta_{3j}^{(2)},\beta_{3j}^{(3)};B_{3j})_{1,q_3};....;(b_{rj};\beta_{rj}^{1},....,\beta_{rj}^{(r)};B_{rj})_{1,q_r}: \end{array}\right.$$

$$I^{0,n_2;0,n_3;....;0,n_r:\left|R^1:m^1,n^1;....;m^{(r)},n^{(r)}\right.}_{p_2,q_2;p_3,q_3;....;p_r,n_r:\left|R:p^1,q^1;....;p^{(r)},q^{(r)}\right.}$$

$$\left[\begin{array}{c|c} Z_1 & (a_{2j};\alpha_{2j}^{(1)},\alpha_{2j}^{(2)};A_{2j})_{1,p_2};(\alpha_{3j};\alpha_{3j}^{(1)},\alpha_{3j}^{(2)},\alpha_{3j}^{(3)};A_{3j})_{1,p_3};....;(a_{rj};\alpha_{rj}^{1},....,\alpha_{rj}^{(r)};A_{rj})_{1,p_r}: \\ \vdots & \\ Z_r & (b_{2j};\beta_{2j}^{(1)},\beta_{2j}^{(2)};B_{2j})_{1,q_2};(b_{3j};\beta_{3j}^{(1)},\beta_{3j}^{(2)},\beta_{3j}^{(3)};B_{3j})_{1,q_3};....;(b_{rj};\beta_{rj}^{1},....,\beta_{rj}^{(r)};B_{rj})_{1,q_r}: \end{array}\right.$$

[@] *E-mail:* `drbsn63@yahoo.co.in`

$$\left.\begin{matrix}(e_j;u'_j g'_j,....,u_j^{(r)}g_j^{(r)};E_j)_{1,R'} : (a_j^{(1)};\alpha_j^{(1)};A_j^{(1)})_{1,p^{(1)}},(a_j^{(r)};\alpha_j^{(r)};A_j^{(r)})_{1,p^{(r)}} \\ : \\ (l_j;U'_j f'_j,....,U_j^{(r)}f_j^{(r)};L_j)_{1,R} : (b_j^{(1)},\beta_j^{(1)};B_j^{(1)})_{1,q^{(1)}},(b_j^{(r)},\beta_j^{(r)};B_j^{(r)})_{1,q^{(r)}}\end{matrix}\right]$$

$$= \frac{1}{(2\pi w)^r}\int_{L_1}....\int_{L_r}\xi(s_1,....,s_r)\prod_{i=1}^{r}\phi(s_i)z_i^{s_i}ds_1....ds_r \tag{1}$$

where $\xi(s_1,....,s_r)$ and $\phi(s_i)$ are explained clearly in [3].

The modified generalized multivariable I-function is analytic if

$$\sum_{k=1}^{p_2}A_{2k}\alpha_{2k}^{(i)}+\sum_{k=1}^{p_3}A_{3k}\alpha_{3k}^{(i)}+....+\sum_{k=1}^{p_s}A_{sk}\alpha_{sk}^{(i)}-\sum_{k=1}^{q_2}B_{2k}\beta_{2k}^{(i)}....-\sum_{k=1}^{q_s}B_{sk}\beta_{sk}^{(i)}-\sum_{j=1}^{R}L_j f_j^{(i)}\leq 0 \tag{2}$$

The multiple contour integral converges absolutely if $|\arg z_i|<\frac{1}{2}\Omega_i\pi$, where

$$\begin{aligned}\Omega_i=&\sum_{k=1}^{n^{(i)}}A_k^{(i)}\alpha_k^{(i)}-\sum_{k=n^{(i)}+1}^{p^{(i)}}A_k^{(i)}\alpha_k^{(i)}+\sum_{k=1}^{m^{(i)}}B_k^{(i)}\beta_k^{(i)}-\sum_{k=m^{(i)}+1}^{q^{(i)}}B_k^{(i)}\beta_k^{(i)}+\sum_{k=1}^{n_2}A_{2k}^{(i)}\alpha_{2k}^{(i)}\\&-\sum_{k=n_2+1}^{p_2}A_{2k}^{(i)}\alpha_{2k}^{(i)}+\sum_{k=1}^{n_3}A_{3k}^{(i)}\alpha_{3k}^{(i)}-\sum_{k=n_3+1}^{p_3}A_{3k}^{(i)}\alpha_{3k}^{(i)}+....+\sum_{k=1}^{n_r}A_{rk}^{(i)}\alpha_{rk}^{(i)}-\sum_{k=n_r+1}^{p_r}A_{rk}^{(i)}\alpha_{rk}^{(i)}\\&-\sum_{k=1}^{q_2}B_{2k}^{(i)}\beta_{2k}^{(i)}-\sum_{k=1}^{q_3}B_{3k}^{(i)}\beta_{3k}^{(i)}....-\sum_{k=1}^{q_r}B_{rk}^{(i)}\beta_{rk}^{(i)}+\sum_{j=1}^{R'}G_j^{(i)}g_j^{(i)}-\sum_{j=1}^{R}F_j^{(i)}f_j^{(i)}>0\\&\forall i=1,...,r\end{aligned} \tag{3}$$

We note

$$A=(a_{2j};\alpha_{2j}^{(1)},\alpha_{2j}^{(2)};A_{2j})_{1,p_2};...;(a_{(r-1)j};\alpha_{(r-1)j}^{(1)},...,\alpha_{(r-1)j}^{(r-1)};A_{(r-1)j})_{1,p_{r-1}} \tag{4}$$

$$B=(b_{2j};\beta_{2j}^{(1)},\beta_{2j}^{(2)};B_{2j})_{1,q_2};....;(b_{(r-1)j};\beta_{(r-1)j}^{(1)},...,\beta_{(r-1)j}^{(r-1)};B_{(r-1)j})_{1,q_{r-1}} \tag{5}$$

$$\mathrm{A}=[(a_{rj};\alpha_{rj}^{(1)},....,\alpha_{rj}^{r};A_{rj})]_{1,p_r};\ \mathrm{E}=[(e_j;u_j^{(1)}g_j^{(1)},....,u_j^{(r)}g_j^{(r)};E_j]_{1,R'} \tag{6}$$

$$\Im=(a_j^{(1)},\alpha_j^{(1)};A_j^{(1)})_{1,p^1};....;a_j^{(r)},\alpha_j^{(r)};A_j^{(r)})_{1,p^{(r)}} \tag{7}$$

$$B=[(b_{rj};\beta_{rj}^{(1)},....,\beta_{rj}^{r};B_{rj})]_{1,q_r};\ L=[(l_j;U_j^{(1)}f_j^{(1)},....,U_j^{(r)}f_j^{(r)};L_j]_{1,R} \tag{8}$$

$$\Re = (b_j^{(1)}, \beta_j^{(1)}; B_j^{(1)})_{1,q^1}; \ldots; b_j^{(r)}, \beta_j^{(r)}; B_j^{(r)})_{1,q^{(r)}} \tag{9}$$

$$U = p_2, q_2; p_3, q_3; \ldots; p_{r-1}, q_{r-1}; \ V = 0, n_2; 0, n_3; \ldots; 0, n_{r-1} \tag{10}$$

$$Y = (p^1, q^1); \ldots; p^{(r)}, q^{(r)}); \ X = (m^1, n^1); \ldots; m^{(r)}, n^{(r)}) \tag{11}$$

The well-known Eulerian Beta integral

$$\int_a^b (z-a)^{\alpha-1}(b-t)^{\beta-1}dt = (b-a)^{\alpha+\beta-1}B(\alpha,\beta); \ (\mathrm{Re}(\alpha)>0, \mathrm{Re}(\beta)>0, b>a) \tag{12}$$

is useful in evaluating various integrals involving special functions defined by Raina and Srivastava[6], Saigo and Saxena[8], Srivastava and Hussain[12], Srivastava and Garg[11], etc. Also by using this integral, we can establish Eulerian integrals involving general class of polynomials like G, H with one and several variables.

The explicit form of the generalized polynomial set [7,p.71,(2.3.4)] is

$$S_n^{\alpha,\beta,\tau}(x) = \sum_{e,p,u,v} C(e,p,u,v) x^R (1-\tau x^v)^{\delta n - v} \tag{13}$$

Where C(e,p,u,v) =

$$\frac{B^{qn}(-1)^p(-p)_e(\alpha)_p(-v)_u(-\alpha-qn)_e\left(-\dfrac{\beta}{\tau}-sn\right)_v}{u!v!e!p!(1-\alpha-p)_e} l^n(-\tau)^v\left(\frac{e+k+cu}{l}\right)_n\left(\frac{A}{B}\right)^b \tag{14}$$

where $\sum\limits_{e,p,u,n} = \sum\limits_{v=0}^{n}\sum\limits_{u=0}^{v}\sum\limits_{p=0}^{n}\sum\limits_{e=0}^{p}$ and $R = ln + v + p$

We recall here the following definition of the general class of polynomials introduced and studied by Srivastava [9]

$$S_V^U(x) = \sum_{\eta=0}^{[V/U]} \frac{(-V)_{U\eta} A_{V,\eta}}{\eta!} x^\eta \tag{15}$$

where V = 0, 1, …. and U is an arbitrary positive integer. By suitable specialization of the coefficients, $A_{V,\eta}$ (V, $\eta \geq 0$) (which are arbitrary constants, real or complex) and $S_V^U(x)$ leads to many polynomials like Jacobi, Laguerre, etc., ([15], p.158-161). The MMIF defined in this paper is a generalized transcendental function of several complex variables and is defined in multiple Mellin-Barnes integral.

In the following section, we shall use the required integral and the notations (16) and (17).

$$X_j=(b_j-a_j)+\rho_j(t_j-a_j)+\sigma_j(b_j-t_j) \tag{16}$$

$$Y_j=\frac{(t_j-a_j)^{\gamma_j}(b_j-t_j)^{\delta_j}X_j^{1-\gamma_j-\delta_j}}{\beta_j(b_j-a_j)+(\beta_j\rho_j+\alpha_j-\beta_j)(t_j-a_j)+\beta_j\sigma_j(b_j-t_j)}\text{ for } j=1,\ldots,s \tag{17}$$

Lemma([2],p.287) is as follows :

$$\int_a^b\frac{(t-a)^{\alpha-1}(b-t)^{\beta-1}}{\{b-a+\lambda(t-a)+\mu(b-t)\}^{\alpha+\beta}}dt=\frac{(1+\lambda)^{-\alpha}(1+\mu)^{-\beta}\Gamma(\alpha)\Gamma(\beta)}{(b-a)\Gamma(\alpha+\beta)} \tag{18}$$

with $t\in[a,b],\ a\neq b,\ Re(\alpha)>0,\ Re(\beta)>0,\ \eta+\lambda(t-a)+\mu(b-t)\neq 0.$

In the following sections, we shall discuss the multiple Eulerian integral of MMIF and some corollaries by specializing some values.

2. Main Integral Result :

The following notations are useful in establishing the multiple Eulerian integral of MMIF.

$$A=(a_{2j};\alpha_{2j}^{(1)},\alpha_{2j}^{(2)};A_{2j})_{1,p_2};\ldots;(a_{(r-1)j};\alpha_{(r-1)j}^{(1)},\ldots,\alpha_{(r-1)j}^{(r-1)};A_{(r-1)j})_{1,p_{r-1}} \tag{19}$$

$$\mathrm{A}=[(a_{rj};\alpha_{rj}^{(1)},\ldots.,\alpha_{rj}^{r};A_{rj})]_{1,p_r};\ \mathrm{E}=[(e_j;u_j^{(1)}g_j^{(1)},\ldots.,u_j^{(r)}g_j^{(r)};E_j]_{1,R'} \tag{20}$$

$$\begin{aligned}A_1=&(1-\tau_j-\varsigma_jR:v_j^1,\ldots.,v_j^{(r)},\varsigma_j\nu;1)_{1,s},\\&(-\lambda_j-KS_j-\gamma_j\varsigma_jR-\tau_j;\gamma_jv_j^1,\ldots.,\gamma_jv_j^{(r)},\gamma_j\varsigma_j\nu;1)_{1,s},\\&(-\mu_j-KT_j-\delta_j\varsigma_jR-\tau_j;\delta_jv_j^1,\ldots.,\delta_jv_j^{(r)},\delta_j\varsigma_j\nu:1)_{1,s}\end{aligned} \tag{21}$$

$$\Im=[(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{1,p^1};\ldots.;c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{1,m^{(r)}}];(1-v+\delta\eta,1;1) \tag{22}$$

$$B=(b_{2j};\beta_{2j}^{(1)},\beta_{2j}^{(2)};B_{2j})_{1,q_2};\ldots.;(b_{(r-1)j};\beta_{(r-1)j}^{(1)},\ldots,\beta_{(r-1)j}^{(r-1)};B_{(r-1)j})_{1,q_{r-1}} \tag{23}$$

$$\mathrm{B}=[\tau_{i_r}(b_{rji_r};\beta_{rji_r}^{(1)},\ldots.,\beta_{rji_r}^{r};B_{rji_r})]_{1,q_{i_r}} \tag{24}$$

$$B_1=(\lambda_j-\mu_j-K(S_j+T_j)-\varsigma_j(\gamma_j+\delta_j)R-\tau_j-1;\\ (\gamma_j+\delta_j)v_j^1,....,(\gamma_j+\delta_j)v_j^{(r)},(\gamma_j+\delta_j)\varsigma_j\nu;1)_{1,s},\\ (1-\varsigma_jR;v_j^1,....,v_j^{(r)},\varsigma_j\nu:1)_{1,s} \tag{25}$$

$$\Re=(d_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{1,q^1};....;d_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{1,m^{(r)}};(0,1;1) \tag{26}$$

U, V, X and Y are defined in the previous section. We have the following unified multiple Eulerian integrals.

Theorem 1: *We have the following result*

$$\int_{a_1}^{b_1}....\int_{a_s}^{b_s}\prod_{j=1}^{s}\frac{(t_j-a_j)^{\lambda_j}(b_j-t_j)^{\mu_j}}{X_j^{\lambda_j+\mu_j+2}}S_U^V\left[a\prod_{j=1}^{s}\frac{(t_j-a_j)^{S_j}(b_j-t_j)^{T_j}}{X_j^{S_j+T_j}}\right]$$

$$\times S_n^{\alpha\beta\tau}\left[b\prod_{j=1}^{s}Y^{\varsigma_j};\nu,t,q,A,B,k;l\right]I\begin{pmatrix} z_1\prod_{j=1}^{s}Y_j^{v_j^1}\\ .\\ .\\ .\\ z_r\prod_{j=1}^{s}Y_j^{v_j^{(r)}}\end{pmatrix}dt_1....dt_s$$

$$=\left\{\prod_{j=1}^{s}(b_j-a_j)^{-1}(1+\rho_j)^{-\lambda_j-1}(1+\sigma_j)^{-\mu_j-1}\right\}\sum_{K=0}^{[V/U]}\sum_{e,p,u,v}\sum_{\tau_1,....,\tau_s=0}^{\infty}\frac{(-V)_{UK}A_{V,K}}{K!}$$

$$\times C(e,p,u,v)\left\{\prod_{j=1}^{s}\frac{(\beta_j-\alpha_j)^{\tau_j}(1+\rho_j)^{-K_jS_j-\gamma_j\varsigma_jR-\tau_j}(1+\sigma_j)^{-KT_j-\delta_j\varsigma_jR}}{\tau_j!\beta_j^{\gamma_j+\varsigma_jR}}\right\}a^Kb^R$$

$$\times I_{U;p_r+3s,q_r+2s:|R:Y;1,1}^{V;0,n_r+3s:|R':X;1,1}\left(\begin{matrix} z_1\prod_{j=1}^{s}\{\beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j}\}^{-v_j^1}\\ .\\ .\\ z_r\prod_{j=1}^{s}\{\beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j}\}^{-v_j^{(r)}}\\ b^\nu\prod_{j=1}^{s}\{\beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j}\}^{-\varsigma_j\nu}\end{matrix}\middle|\begin{matrix}A;A_1,\mathrm{A}:E:\Im\\ .\\ .\\ .\\ B;\mathrm{B},B_1:L:\Re\end{matrix}\right) \tag{27}$$

We obtain a modified of I-function of (r+1) variables provided that

$(i)\quad \lambda_j,\mu_j,s_j,t_j,\varsigma_j,v_j^{(i)}>0,\beta_j\neq 0,b_j-a_j\neq 0,\rho_j\neq -1,\sigma_j\neq -1,$
$(b_j-a_j)+\rho_j(t_j-a_j)+\sigma_j(b_j-t_j)\neq 0,t_j\in[a_j,b_j]\,for\,i=1,....,r;j=1,....,s$

$(ii)\quad |(\beta_j-\alpha_j)(t_j-a_j)|<|\beta_j\{(b_j-a_j)+\rho_j(t_j-a_j)+\sigma_j(b_j-t_j)\}|;$
$t_j\in[a_j,b_j]\,for\,j=1,....,s$

(iii) When $\min(S_j,T_j)>0$

$$(a)\quad \mathrm{Re}\left[\lambda_j+\gamma_j\varsigma_j(\ln+p)+\sum_{i=1}^{r}\gamma_j v_j^{(i)}\min_{1\le j\le m^{(i)}}\mathrm{Re}\left[B_j^{(i)}\left(\frac{b_j^{(i)}}{\beta_j^{(i)}}\right)+1\right]\right]>0$$

$$(b)\quad \mathrm{Re}\left[\mu_j+\delta_j\varsigma_j(\ln+p)+\sum_{i=1}^{r}\gamma_j v_j^{(i)}\min_{1\le j\le m^{(i)}}\mathrm{Re}\left[B_j^{(i)}\left(\frac{b_j^{(i)}}{\beta_j^{(i)}}\right)+1\right]\right]>0$$

When $\max(S_j,T_j)<0$

$$(c)\quad \mathrm{Re}\left[\lambda_j+S_j[V/U]+\gamma_j\varsigma_j(\ln+p)+\sum_{i=1}^{r}\gamma_j v_j^{(i)}\min_{1\le j\le m^{(i)}}\mathrm{Re}\left[B_j^{(i)}\left(\frac{b_j^{(i)}}{\beta_j^{(i)}}\right)+1\right]\right]>0$$

$$(d)\quad \mathrm{Re}\left[\mu_j+T_j[V/U]+\delta_j\varsigma_j(\ln+p)+\sum_{i=1}^{r}\gamma_j v_j^{(i)}\min_{1\le j\le m^{(i)}}\mathrm{Re}\left[B_j^{(i)}\left(\frac{b_j^{(i)}}{\beta_j^{(i)}}\right)+1\right]\right]>0$$

When $S_j>0$, $T_j<0$ inequalities (a) and (d) are satisfied. When $S_j<0$, $T_j>0$ inequalities (b) and (c) are satisfied. The multiple series of R.H.S of (27) converges absolutely.

Proof: In L.H.S. of eq.(27), first we make use of series representations for the polynomial sets

$S_V^U(x)$ and $S_n^{\alpha,\beta,\tau}(x)$ respectively by using the equations (14) and (12). Later, by the multiple Mellin-Barnes contour integral representation of MMIF and then interchanging the order of summation and integration, we come across at the following :

$$\text{L.H.S.}\ \sum_{K=0}^{[V/U]} \sum_{e,p,u,n} \frac{(-V)_{UK} A_{V,K}}{K!} a^K b^R C(e,p,u,v) \frac{1}{(2\pi\omega)^r}$$

$$\int_{L_1} \cdots \int_{L_r} \xi(s_1, \ldots, s_r) \prod_{i=1}^{r} \varphi_i(s_i) z_i^{s_i} \int_{a_1}^{b_1} \cdots \int_{a_s}^{b_s} \prod_{j=1}^{s} \frac{(t_j - a_j)^{\lambda_j + KS_j} (b_j - t_j)^{\mu_j + KT_j}}{X_j^{\lambda_j + \mu_j + K(S_j + T_j) + 2}}$$

$$Y_j^{\varsigma_j R + \sum_{i=1}^{r} s_i v_j^{(i)}} \left(1 - \tau x^v \prod_{j=1}^{s} Y_j^{\varsigma_j q}\right)^{\delta n - v} dt_1 \ldots dt_s dS_1 \ldots dS_r \tag{28}$$

Now expressing $(1-\tau x^v \prod_{j=1}^{s} Y_j^{\varsigma_j q})^{\delta n - v}$ in the form of contour integral and changing the order of integration, we obtain

$$\text{L.H.S.}\ \sum_{K=0}^{[V/U]} \sum_{e,p,u,n} \frac{(-V)_{UK} A_{V,K}}{K!} a^K b^R C(e,p,u,v) \frac{1}{(2\pi\omega)^{r+1}} \int_{L_1} \cdots \int_{L_r} \int_{L_{r+1}} \xi(s_1, \ldots, s_r)$$

$$\prod_{i=1}^{r} \varphi_i(s_i) z_i^{s_i} (-\tau b^v)^{s_{r-1}} \Gamma(-s_{r+1}) \Gamma(v - \delta n + s_{r+1}) \left[\int_{a_1}^{b_1} \cdots \int_{a_s}^{b_s} \right.$$

$$\left. \left\{ \prod_{j=1}^{s} \frac{(t_j - a_j)^{\lambda_j + KS_j} (b_j - t_j)^{\mu_j + KT_j}}{X_j^{\lambda_j + \mu_j + K(S_j + T_j) + 2}} Y_j^{\varsigma_j R + \sum_{i=1}^{r} s_i v_j^{(i)} + \varsigma_j v s_{r-1}} \right\} dt_1 \ldots dt_s \right] dS_1 \ldots dS_r ds_{r+1} \tag{29}$$

By the use of Y_j from (17) and performing algebraic simplifications, we get

$$\text{L.H.S.}\ \sum_{K=0}^{[V/U]} \sum_{e,p,u,n} \frac{(-V)_{UK} A_{V,K}}{K!} a^K b^R C(e,p,u,v) \frac{1}{(2\pi\omega)^{r+1}} \int_{L_1} \cdots \int_{L_r} \int_{L_{r+1}} \xi(s_1, \ldots, s_r)$$

$$\prod_{i=1}^{r} \varphi_i(s_i) z_i^{s_i} (-\tau b^v)^{s_{r-1}} \Gamma(-s_{r+1}) \Gamma(v - \delta n + s_{r+1})$$

$$\left[\int_{a_1}^{b_1} \cdots \int_{a_s}^{b_s} \left\{ \prod_{j=1}^{s} \frac{(t_j - a_j)^{\lambda_j + KS_j + \gamma_j \sum_{i=1}^{r} \xi_j v_j^{i)} + \gamma_j \varsigma_j (R + v s_{r-1})}}{X_j^{\lambda_j + \mu_j + K(S_j + T_j) + 2 + (\gamma_j + \delta_j)(R\varsigma_j + \sum_{i=1}^{r} s_i v_j^{(i)} + \varsigma_j v s_{r+1})}} \right. \right.$$

$$\left. \frac{(b_j - t_j)^{\mu_j + KT_j + \delta_j \sum_{i=1}^{r} s_j v_j^{i)} + \gamma_j \varsigma_j (R + v s_{r-1})}}{\beta_j^{(R\varsigma_j + \sum_{i=1}^{r} s_i v_j^{(i)} + \varsigma_j v s_{r+1}}} \left(1 - \frac{(\beta_j - \alpha_j)(t_j - a_j)}{\beta_j X_j}\right)^{-(\varsigma_j R + \sum_{i=1}^{r} s_j v_j^{(i)} + \varsigma_j v s_{r-1})} \right\}$$

$$\left. dt_1 \ldots dt_s \right] dS_1 \ldots dS_r ds_{r+1} \tag{30}$$

If $\frac{(\beta_j - t_j)(t_j - a_j)}{\beta_j X_j} < 1, t_j \in [a_j, b_j]$ for $j = 1, ..., s$

Then by using binomial expansion we thus found that

$$\text{L.H.S} = \sum_{K=0}^{[V/U]} \sum_{e,p,u,n} \frac{(-V)_{UK} A_{V,K}}{K!} a^K b^R C(e,p,u,v) \prod_{j=1}^{s} \frac{(\beta_j - \alpha_j)^{\tau_j}}{\beta_j^{\tau_j} \tau_j !}$$

$$\frac{1}{(2\pi\omega)^{r+1}} \int_{L_1} .... \int_{L_r} \int_{L_{r+1}} \xi(s_1, ..., s_r) \prod_{i=1}^{r} \varphi_i(s_i) z_i^{s_i} (-\tau b^{\nu})^{s_{r+1}} \Gamma(-s_{r+1})$$

$$\Gamma(v - \delta n + s_{r+1}) \left[\int_{a_1}^{b_1} .... \int_{a_s}^{b_s} \left\{ \prod_{j=1}^{s} \frac{(t_j - a_j)^{\lambda_j + KS_j + \gamma_j \sum_{i=1}^{r} \xi_j v_j^{i)} + \gamma_j \varsigma_j (R + \nu s_{r-1}) + \tau_j}}{X_j^{\lambda_j + \mu_j + K(S_j + T_j) + 2 + (\gamma_j + \delta_j)(R\varsigma_j + \sum_{i=1}^{r} s_i v_j^{(i)} + \varsigma_j \nu s_{r+1}) + \tau_j}} \right. \right.$$

$$\left. \prod_{j=1}^{s} \frac{\Gamma(\tau_j + R\varsigma_j + \sum_{i=1}^{r} s_j v_j^{(i)} + \xi_j \nu s_{r+1})}{\Gamma(R\varsigma_j + \sum_{i=1}^{r} s_j v_j^{(i)} + \varsigma_j \nu s_{r+1})} \beta_j^{-(R\varsigma_j + \sum_{i=1}^{r} s_i v_j^{(i)} + \varsigma_j \nu s_{r+1})} \right\}$$

$$\left. (b_j - t_j)^{\mu_j + KT_j + \delta_j \sum_{i=1}^{r} s_j v_j^{i)} + \delta_j \varsigma_j (R + \nu s_{r-1})} dt_1 .... dt_s \right] dS_1 .... dS_r ds_{r+1} \tag{31}$$

Now by equation (16) and then evaluating inner-most integral with the help of lemma (18), we get

$$\text{L.H.S} = \left\{ \prod_{j=1}^{s} (b_j - a_j)^{-1} (1 + \rho_j)^{-\lambda_j - 1} (1 + \sigma_j)^{-\mu_j - 1} \right\} \sum_{K=0}^{[V/U]} \sum_{e,p,u,n} \frac{(-V)_{UK} A_{V,K}}{K!} a^K b^R$$

$$C(e,p,u,v) \left\{ \prod_{j=1}^{s} \frac{(\beta_j - \alpha_j)^{\tau_j} (1 + \rho_j)^{-K_j S_j - \gamma_j \varsigma_j R - \tau_j} (1 + \sigma_j)^{-KT_j - \delta_j \varsigma_j R}}{\tau_j ! \beta_j^{\tau_j + \varsigma_j R}} \right\}$$

$$\frac{1}{(2\pi\omega)^{r+1}} \int_{L_1} .... \int_{L_r} \int_{L_{r+1}} \xi(s_1, ..., s_r) \prod_{i=1}^{r} \varphi_i(s_i) z_i^{s_i} (-\tau b^{\nu})^{s_{r+1}} \Gamma(-s_{r+1})$$

$$\Gamma(v - \delta n + s_{r+1}) \prod_{j=1}^{s} \frac{\Gamma(\tau_j + R\varsigma_j + \sum_{i=1}^{r} \xi_j v_j^{(i)} + \varsigma_j \nu s_{r+1})}{\Gamma(R\varsigma_j + \sum_{i=1}^{r} s_j v_j^{(i)} + \varsigma_j \nu s_{r+1})} \beta_j^{-(R\varsigma_j + \sum_{i=1}^{r} s_i v_j^{(i)} + \varsigma_j \nu s_{r-1})}$$

$$\prod_{j=1}^{s} \frac{\Gamma(\tau_j + \lambda_j + KS_j + \gamma_j \varsigma_j R + \gamma_j \sum_{i=1}^{r} s_j v_j^{(i)} + \gamma_j \varsigma_j \nu s_{r+1} + 1)}{\Gamma(\lambda_j + \mu_j + K(S_j + T_j) + (\gamma_j + \delta_j)(R\varsigma_j + \sum_{i=1}^{r} s_j v_j^{(i)} + \varsigma_j \nu s_{r+1}) + \tau_j + 2)}$$

$$\times\Gamma(-s_{r+1})\Gamma(v-\delta n+s_{r+1})\Gamma(\mu_j+Kt_j+\delta_j\varsigma_j R+\delta_j\sum_{i=1}^{r}s_j v_j^{(i)}+\delta_j s_j \nu\,\varsigma_{r+1}+1)$$

$$\prod_{j=1}^{s}\left\{\frac{(1+\rho_j)^{-\gamma_j}(1+\sigma_j)^{-\delta_j}}{\beta_j}\right\}^{\sum_{i=1}^{r}s_i v_j^{(i)}}\prod_{j=1}^{s}\left\{\frac{(1+\rho_j)^{-\gamma_j\varsigma_j q_j}(1+\sigma_j)^{-\delta_j\varsigma_j q}(-\tau b^{\nu})}{\beta_j^{\varsigma_j \nu}}\right\}^{s_{r+1}}$$

$$dS_1....dS_r ds_{r+1} \tag{32}$$

Finally, re-interpreting the multiple contour Mellin-Barne's integral in terms of MMIF, we obtain (27).

3. Particular cases

The MMIF in (27) can be specialized in a variety of special functions. These can be expressible in terms of E, G, H and I-function of several variables. Again by specializing parameters and variables, leads to orthogonal polynomials and hypergeometric functions. Thus by different special cases of these functions, one can obtain a remarkable number of other integrals with special functions of one and several variables.

By assuming V = 0, U = 1 and $A_{0,0}$ = 1 in (27), the polynomials $S_V^U(x)$ reduces to unity and we arrive at the following :

Corollary 1:

$$\int_{a_1}^{b_1}....\int_{a_s}^{b_s}\prod_{j=1}^{s}\frac{(t_j-a_j)^{\lambda_j}(b_j-t_j)^{\mu_j}}{X_j^{\lambda_j+\mu_j+2}}$$

$$\times S_n^{\alpha\beta\tau}\left[b\prod_{j=1}^{s}Y^{\varsigma_j};\nu,t,q,A,B,k;l\right] I\begin{pmatrix} z_1\prod_{j=1}^{s}Y_j^{v_j^1} \\ . \\ . \\ . \\ z_r\prod_{j=1}^{s}Y_j^{v_j^{(r)}}\end{pmatrix} dt_1....dt_s$$

$$=\left\{\prod_{j=1}^{s}(b_j-a_j)^{-1}(1+\rho_j)^{-\lambda_j-1}(1+\sigma_j)^{-\mu_j-1}\right\}$$

$$\times\sum_{e,p,u,v}\sum_{\tau_1,....,\tau_s=0}^{\infty}C(e,p,u,v)\left\{\prod_{j=1}^{s}\frac{(\beta_j-\alpha_j)^{\tau_j}(1+\rho_j)^{-\gamma_j\varsigma_j R-\tau_j}(1+\sigma_j)^{-\delta_j\varsigma_j R}}{\tau_j!\,\beta_j^{\gamma_j+\varsigma_j R}}\right\}a^K b^R$$

$$\times I^{V;0,n_r+3s:|R':X;1,1}_{U;p_r+3s,q_r+2s:|R:Y;1,1}\left(\begin{array}{c|c} z_1\prod_{j=1}^{s}\{\beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j}\}^{-v_j^1} & A;A_2,\mathrm{A}:E:\mathfrak{I} \\ . & . \\ . & . \\ z_r\prod_{j=1}^{s}\{\beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j}\}^{-v_j^{(r)}} & . \\ b^{\nu}\prod_{j=1}^{s}\{\beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j}\}^{-\varsigma_j\nu} & B;\mathrm{B},B_2:L:\mathfrak{R} \end{array}\right) \tag{33}$$

where

$$\begin{aligned} A_2 = {} & (1-\tau_j-\varsigma_j R: v_j^1,....,v_j^{(r)},\varsigma_j\nu;1)_{1,s}, \\ & (-\lambda_j-\gamma_j\varsigma_j R-\tau_j;\gamma_j v_j^1,....,\gamma_j v_j^{(r)},\gamma_j\varsigma_j\nu;1)_{1,s}, \\ & (-\mu_j-\delta_j\varsigma_j R-\tau_j;\delta_j v_j^1,....,\delta_j v_j^{(r)},\delta_j\varsigma_j\nu:1)_{1,s} \end{aligned} \tag{34}$$

$$\begin{aligned} B_2 = {} & (\lambda_j-\mu_j-\varsigma_j(\gamma_j+\delta_j)R-\tau_j-1; \\ & (\gamma_j+\delta_j)v_j^1,....,(\gamma_j+\delta_j)v_j^{(r)},(\gamma_j+\delta_j)\varsigma_j\nu;1)_{1,s}, \\ & (1-\varsigma_j R;v_j^1,....,v_j^{(r)},\varsigma_j\nu:1)_{1,s} \end{aligned} \tag{35}$$

by following the same validity conditions and notations as that of (27).

Assuming, s = 1 in (27), the following integral form is obtained :

Corollary 2:

$$\int_{a_1}^{b_1}\frac{(t-a_1)^{\lambda}(b_1-t)^{\mu}}{X^{\lambda+\mu+2}}S_U^V\left[a\frac{(t-a_1)^S(b_1-t)^T}{X^{S+T}}\right]$$

$$\times S_n^{\alpha\beta\tau}\left[bY^{\varsigma};\nu,t,q,A,B,k;l\right]I\begin{pmatrix} z_1Y^{v^1} \\ . \\ . \\ . \\ z_rY^{v^r}\end{pmatrix}dt_1....dt_s$$

$$=(b_1-a_1)^{-1}(1+\rho_j)^{-\lambda-1}(1+\sigma_j)^{-\mu-1}\sum_{K=0}^{[V/U]}\sum_{e,p,u,v}\sum_{\tau_1,....,\tau_s=0}^{\infty}\frac{(-V)_{UK}A_{V,K}}{K!}$$

$$\times C(e,p,u,v)\left\{\frac{(\beta-\alpha)^{\tau}(1+\rho)^{-KS-\gamma-\tau}(1+\sigma)^{-KT-\delta\varsigma R}}{\tau!\beta^{\tau+\varsigma R}}\right\}a^K b^R$$

$$\times I^{V;0,n_r+3:|R':X;1,1}_{U;p_r+3,q_r+2s:|R:Y;1,1}\left(\begin{array}{c|c} z_1\{\beta(1+\rho)^{\gamma}(1+\sigma)^{\delta}\}^{-v^1} & A;A_3,\mathrm{A}:E:\mathfrak{I} \\ . & . \\ . & . \\ z_r\{\beta(1+\rho)^{\gamma}(1+\sigma)^{\delta}\}^{-v^{(r)}} & . \\ b^{\nu}\{\beta(1+\rho)^{\gamma}(1+\sigma_j)^{\delta}\}^{-\varsigma\nu} & B;\mathrm{B},B_3:L:\mathfrak{R} \end{array}\right) \tag{36}$$

where

$$A_3=(1-\tau_1-\varsigma R:v^1,....,v^{(r)},\varsigma\nu;1)_{1,s},(-\lambda-KS-\gamma\varsigma R-\tau_1;\gamma v^1,....,\gamma v^{(r)},\gamma\varsigma\nu;1),$$
$$(-\mu-KT-\delta\varsigma R-\tau;\delta v^1,....,\delta v^{(r)},\delta\varsigma\nu:1) \tag{37}$$

$$B_2=(\lambda-\mu-K(S+T)-\varsigma(\gamma+\delta)R-\tau_1-1;(\gamma+\delta)v^1,....,(\gamma+\delta)v^{(r)},(\gamma+\delta)\varsigma\nu;1),$$
$$(1-\varsigma R;v^1,....,v^{(r)},\varsigma\nu:1) \tag{38}$$

by following the same validity conditions and notations as that of (27).

Putting $t_j-b_j=v_j,(b_j-a_j)=1;j=1,....,s$ in (27), we obtain the following result :

Corollary 3:

$$\int_0^1....\int_0^1\prod_{j=1}^{s}\frac{(1-v_j)^{\lambda_j}v_j^{\mu_j}}{X'^{\lambda_j+\mu_j+2}_j}S_U^V\left[a\prod_{j=1}^{s}\frac{(1-v_j)^{S_j}v_j^{T_j}}{X'^{S_j+T_j}_j}\right]$$

$$\times S_n^{\alpha,\beta,\tau}\left[b\prod_{j=1}^{s}Y^{\varsigma_j};\nu,t,q,A,B,k;l\right]I\left(\begin{array}{c} z_1\prod_{j=1}^{s}Y_j^{v_j^1} \\ . \\ . \\ . \\ z_r\prod_{j=1}^{s}Y_j^{v_j^{(r)}} \end{array}\right)dt_1....dt_s$$

$$=\left\{\prod_{j=1}^{s}(b(1+\rho_j)^{-\lambda_j-1}(1+\sigma_j)^{-\mu_j-1}\right\}\sum_{K=0}^{[V/U]}\sum_{e,p,u,v}\sum_{\tau_1,....,\tau_s=0}^{\infty}\frac{(-V)_{UK}A_{V,K}}{K!}$$

$$\times C(e,p,u,v)\left\{\prod_{j=1}^{s}\frac{(\beta_j-\alpha_j)^{\tau_j}(1+\rho_j)^{-K_jS_j-\gamma_j\varsigma_jR-\tau_j}(1+\sigma_j)^{-KT_j-\delta_j\varsigma_jR}}{\tau_j!\beta_j^{\gamma_j+\varsigma_jR}}\right\}a^Kb^R$$

$$\times I^{V;0,n_r+3s:|R':X;1,1}_{U;p_r+3s,q_r+2s:|R:Y;1,1}\left(\begin{matrix} z_1\prod_{j=1}^{s}\{\beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j}\}^{-v_j^1} \\ \cdot \\ \cdot \\ z_r\prod_{j=1}^{s}\{\beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j}\}^{-v_j^{(r)}} \\ b^{\nu}\prod_{j=1}^{s}\{\beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j}\}^{-\varsigma_j\nu} \end{matrix}\left|\begin{matrix} A;A_1,\mathrm{A}:E:\mathfrak{I} \\ \cdot \\ \cdot \\ \cdot \\ B;\mathrm{B},B_1:L:\mathfrak{R} \end{matrix}\right.\right) \tag{39}$$

where

$$X'_j = v_j(\rho_j-\sigma_j)+\rho_j+1 \tag{40}$$

$$Y_j = \frac{(1-v_j)^{\lambda_j}v_j^{\delta_j}(X'_j)^{1-\gamma_j-\delta_j}}{(\alpha_j+\beta_j\rho_j)(1-v_j)+(1+\sigma_j)\beta_jv_j} \text{ for } j=1,\ldots,s \tag{41}$$

by following the same validity conditions and notations as that of (27).

In the following corollary, the MMIF we considered here reduces to modified one of multivariable H-function [4] with the conditions,

$$(A_{ij}=B_{ij}(i=2,....,r;j=1,....,n^{(i)})=A_j^{(i)}=B_j^{(i)}(i=1,....,r;j=1,....,n^{(i)})=1)$$

and $A = B = U = V = 0$

Corollary 4:

$$\int_{a_1}^{b_1}....\int_{a_s}^{b_s}\prod_{j=1}^{s}\frac{(t_j-a_j)^{\lambda_j}(b_j-t_j)^{\mu_j}}{X_j^{\lambda_j+\mu_j+2}}S_U^V\left[a\prod_{j=1}^{s}\frac{(t_j-a_j)^{S_j}(b_j-t_j)^{T_j}}{X_j^{S_j+T_j}}\right]$$

$$\times S_n^{\alpha,\beta,\tau}\left[b\prod_{j=1}^{s}Y^{\varsigma_j};\nu,t,q,A,B,k;l\right]H\left(\begin{matrix} z_1\prod_{j=1}^{s}Y_j^{v_j^1} \\ \cdot \\ \cdot \\ \cdot \\ z_r\prod_{j=1}^{s}Y_j^{v_j^{(r)}} \end{matrix}\right)dt_1....dt_s$$

$$= \left\{ \prod_{j=1}^{s} (b_j - a_j)^{-1} (1+\rho_j)^{-\lambda_j - 1} (1+\sigma_j)^{-\mu_j - 1} \right\} \sum_{K=0}^{[V/U]} \sum_{e,p,u,v} \sum_{\tau_1,\ldots,\tau_s=0}^{\infty} \frac{(-V)_{UK} A_{V,K}}{K!}$$

$$\times C(e,p,u,v) \left\{ \prod_{j=1}^{s} \frac{(\beta_j - \alpha_j)^{\tau_j} (1+\rho_j)^{-K_j S_j - \gamma_j \varsigma_j R - \tau_j} (1+\sigma_j)^{-KT_j - \delta_j \varsigma_j R}}{\tau_j ! \beta_j^{\gamma_j + \varsigma_j R}} \right\} a^K b^R$$

$$\times H^{0,n_r+3s:|R':X;1,1}_{p_r+3s,q_r+2s:|R:Y;1,1} \left(\begin{array}{c|c} z_1 \prod_{j=1}^{s} \{\beta_j (1+\rho_j)^{\gamma_j} (1+\sigma_j)^{\delta_j}\}^{-v_j^1} & A; A_1', \mathrm{A} : E : \mathfrak{I} \\ \cdot & \cdot \\ \cdot & \cdot \\ z_r \prod_{j=1}^{s} \{\beta_j (1+\rho_j)^{\gamma_j} (1+\sigma_j)^{\delta_j}\}^{-v_j^{(r)}} & \cdot \\ b^{\nu} \prod_{j=1}^{s} \{\beta_j (1+\rho_j)^{\gamma_j} (1+\sigma_j)^{\delta_j}\}^{-\varsigma_j \nu} & B; \mathrm{B}, B_1' : L : \mathfrak{R} \end{array} \right) \tag{42}$$

under the conditions and notation of the theorem in (27)

$$\begin{aligned} A_1' = {} & (1-\tau_j - \varsigma_j R : v_j^1, \ldots, v_j^{(r)}, \varsigma_j \nu)_{1,s}, \\ & (-\lambda_j - KS_j - \gamma_j \varsigma_j R - \tau_j ; \gamma_j v_j^1, \ldots, \gamma_j v_j^{(r)}, \gamma_j \varsigma_j \nu)_{1,s}, \\ & (-\mu_j - KT_j - \delta_j \varsigma_j R - \tau_j ; \delta_j v_j^1, \ldots, \delta_j v_j^{(r)}, \delta_j \varsigma_j \nu)_{1,s} \end{aligned} \tag{43}$$

$$\begin{aligned} B_1' = {} & (\lambda_j - \mu_j - K(S_j + T_j) - \varsigma_j (\gamma_j + \delta_j) R - \tau_j - 1; \\ & (\gamma_j + \delta_j) v_j^1, \ldots, (\gamma_j + \delta_j) v_j^{(r)}, (\gamma_j + \delta_j) \varsigma_j \nu)_{1,s}, \\ & (1 - \varsigma_j R ; v_j^1, \ldots, v_j^{(r)}, \varsigma_j \nu)_{1,s} \end{aligned} \tag{44}$$

By assuming A = B = U = V = 0 in (27), it reduces to I-function defined by Prathima et al.[5] and we arrive at the following corollary :

Corollary 5:

$$\int_{a_1}^{b_1} \cdots \int_{a_s}^{b_s} \prod_{j=1}^{s} \frac{(t_j - a_j)^{\lambda_j} (b_j - t_j)^{\mu_j}}{X_j^{\lambda_j + \mu_j + 2}} S_U^V \left[a \prod_{j=1}^{s} \frac{(t_j - a_j)^{S_j} (b_j - t_j)^{T_j}}{X_j^{S_j + T_j}} \right.$$

$$\times S_n^{\alpha\beta\tau}\left[b\prod_{j=1}^{s}Y^{\varsigma_j};v,t,q,A,B,k;l\right] I\begin{pmatrix} z_1\prod_{j=1}^{s}Y_j^{v_j^1} \\ \cdot \\ \cdot \\ \cdot \\ z_r\prod_{j=1}^{s}Y_j^{v_j^{(r)}} \end{pmatrix} dt_1....dt_s$$

$$= \left\{\prod_{j=1}^{s}(b_j-a_j)^{-1}(1+\rho_j)^{-\lambda_j-1}(1+\sigma_j)^{-\mu_j-1}\right\}\sum_{K=0}^{[V/U]}\sum_{e,p,u,v}\sum_{\tau_1,....,\tau_s=0}^{\infty}\frac{(-V)_{UK}A_{V,K}}{K!}$$

$$\times C(e,p,u,v)\left\{\prod_{j=1}^{s}\frac{(\beta_j-\alpha_j)^{\tau_j}(1+\rho_j)^{-K_jS_j-\gamma_j\varsigma_jR-\tau_j}(1+\sigma_j)^{-KT_j-\delta_j\varsigma_jR}}{\tau_j!\beta_j^{\gamma_j+\varsigma_jR}}\right\}a^Kb^R$$

$$\times I^{0,n_r+3s:|R':X;1,1}_{p_r+3s,q_r+2s:|R:Y;1,1}\left(\begin{array}{c|c} z_1\prod_{j=1}^{s}\{\beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j}\}^{-v_j^1} & A_1, \mathrm{A}:E:\mathfrak{I} \\ \cdot & \cdot \\ \cdot & \cdot \\ z_r\prod_{j=1}^{s}\{\beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j}\}^{-v_j^{(r)}} & \cdot \\ b^v\prod_{j=1}^{s}\{\beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j}\}^{-\varsigma_j v} & \mathrm{B}, B_1:L:\mathfrak{R} \end{array}\right) \qquad (45)$$

under the same notations and conditions that in theorem (27).

Take $(A_{ij}=B_{ij}(i=2,....,r;j=1,....,n^{(i)})=A_j^{(i)}=B_j^{(i)}(i=1,....,r;j=1,....,n^{(i)}$ $=1)$ in (27), we get result involving multivariable I-function defined by Prasad[3].

Corollary 6:

$$\int_{a_1}^{b_1}....\int_{a_s}^{b_s}\prod_{j=1}^{s}\frac{(t_j-a_j)^{\lambda_j}(b_j-t_j)^{\mu_j}}{X_j^{\lambda_j+\mu_j+2}}S_U^V\left[a\prod_{j=1}^{s}\frac{(t_j-a_j)^{S_j}(b_j-t_j)^{T_j}}{X_j^{S_j+T_j}}\right]$$

$$\times S_n^{\alpha\beta\tau}\left[b\prod_{j=1}^{s}Y^{\varsigma_j};v,t,q,A,B,k;l\right] I\begin{pmatrix} z_1\prod_{j=1}^{s}Y_j^{v_j^1} \\ . \\ . \\ . \\ z_r\prod_{j=1}^{s}Y_j^{v_j^{(r)}} \end{pmatrix} dt_1....dt_s$$

$$= \left\{\prod_{j=1}^{s}(b_j-a_j)^{-1}(1+\rho_j)^{-\lambda_j-1}(1+\sigma_j)^{-\mu_j-1}\right\}\sum_{K=0}^{[V/U]}\sum_{e,p,u,v}\sum_{\tau_1,....,\tau_s=0}^{\infty}\frac{(-V)_{UK}A_{V,K}}{K!}$$

$$\times C(e,p,u,v)\left\{\prod_{j=1}^{s}\frac{(\beta_j-\alpha_j)^{\tau_j}(1+\rho_j)^{-K_jS_j-\gamma_j\varsigma_jR-\tau_j}(1+\sigma_j)^{-KT_j-\delta_j\varsigma_jR}}{\tau_j!\beta_j^{\gamma_j+\varsigma_jR}}\right\}a^Kb^R$$

$$\times I_{U;p_r+3s,q_r+2s:|R:Y;1,1}^{V;0,n_r+3s:|R':X;1,1}\left(\begin{matrix} z_1\prod_{j=1}^{s}\{\beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j}\}^{-v_j^1} \\ . \\ . \\ z_r\prod_{j=1}^{s}\{\beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j}\}^{-v_j^{(r)}} \\ b^v\prod_{j=1}^{s}\{\beta_j(1+\rho_j)^{\gamma_j}(1+\sigma_j)^{\delta_j}\}^{-\varsigma_jv} \end{matrix}\left|\begin{matrix} A;A_1',\mathrm{A}:E:\Im \\ . \\ . \\ B;\mathrm{B},B_1':L:\Re \end{matrix}\right.\right) \quad (46)$$

under the same notations and conditions that in Theorem 1 and Corollary 4.

Remarks: If

$$(A_{ij}=B_{ij}(i=2,....,r;j=1,....,n^{(i)})=A_j^{(i)}=B_j^{(i)}(i=1,....,r;j=1,....,n^{(i)})=1)$$

and $R = R' = E_j = L_j = 0$, we can obtain similar type integral results of I-function defined by Prasad[3]. And if $A = B = U = V = 0$ and $R = R' = E_j = L_j = 0$, we can obtain the results with I-function defined by Prathima et al.[5].

By following the similar lines, the results of this article can be extended to a product of multi variable polynomials by Srivastava[10] and Srisvastava and Garg[11].

4. Conclusion

By utilizing the general arguments and a general class of polynomials in this study, we can obtain many single, double and multiple Eulerian integrals. Secondly, by specializing the various parameters and variables involved in MMIF, we get a remarkable number of special functions or a product of such functions. Hence, the results derived in this article have a wide range of applications in mathematics and mathematical physics. We can also have applications of these multiple integrals with H-function and hypergeometric functions as discussed by the author [16,17,18].

References

[1] AlokBhargava, Amber Srivastava and Rohit Mukherjee, On a General Class of Multiple Eulerian Integrals, *IJLTEMAS*, 3(8), 57-64 (2014).

[2] I.S. Gradsteyn and I.M. Ryxhik, Table of integrals, series and products: Academic New York (1980).

[3] Y. N. Prasad, Multivariable I-function,*Vijnana Parishad Anusandhan Patrika* 29, 231-237 (1986).

[4] Y.N. Prasad and A.K. Singh, Basic properties of the transform involving and H-function of r-variables as kernel. *Indian Acad Math*, no 2, page 109-115 (1982).

[5] J. Prathima, V. Nambisan and S.K. Kurumujji, A Study of I-function of Several Complex Variables, *International Journal of Engineering Mathematics* Vol, 1-12 (2014).

[6] R.K. Raina and H.M. Srivastava, Evaluation of certain class of Eulerian integrals. *J. phys. A: Math.Gen.* 26, 691-696 (1993).

[7] S.K. Raizada, A study of unified representation of special functions of mathematical physics and their use instatistical and boundary value problems, Ph.D. Thesis, Bundelkhand University, Jhansi, India (1991).

[8] M. Saigo, and R.K. Saxena, Unified fractional integral formulas forthe multivariable H-function. *J. Fractional Calculus* 15, 91-107 (l999).

[9] H.M. Srivastava, A contour integral involving Fox's H-function, *Indian. J. Math.* 14, 1-6 (1972).

[10] H.M. Srivastava, A multilinear generating function for the Konhauser set of biorthogonal polynomials suggested by Laguerre polynomial, *Pacific. J. Math.* 177, page 183-191 (1985).

[11] H.M. Srivastava and M. Garg, Some integrals involving general class of polynomials and the multivariable H-function. *Rev. Roumaine. Phys.* 32, 685-692 (1987).

[12] H.M. Srivastava and M.A. Hussain, Fractional integration of the H-function of several variables. Comput. Math. Appl. 30 (9), 73-85 (1995).

[13] H.M. Srivastava and R. Panda, Some expansion theorems and generating relations for the H-function of severalcomplex variables. Comment.Math. Univ. St. Paul. 24, 119-137 (1975).

[14] H.M. Srivastava and R. Panda, Some expansion theorems and generating relations for the H-function of several complex variables II.Comment. Math. Univ. St. Paul. 25, 167-197 (1976).

[15] H.M. Srivastava and N.P. Singh, The integration of certains products of the multivariable H-function with a general class of polynomials, *Rend. Circ. Mat. Palermo.* 32(2), 157-187 (1983).

[16] Devendra Kumar, Manish Kumar Bansal, Kottakkaran Sooppy Nisar and Jagdev Singh,Mathematical Modeling of internal blood pressure involving incomplete H-functions, *Journal of Interdisciplinary Mathematics,* V.22(7), 1213-1221 (2019). https://doi.org/10.1080/09720502.2019.1706842.

[17] Manish Kumar Bansal, Devendra Kumar, Kottakkaran Sooppy Nisar and Jagdev Singh, Applications of incomplete H-functions in determination of Lambert's law, *Journal of Interdisciplinary Mathematics,* V. 22(7), 1205-1212 (2019). https://doi.org/10.1080/09720502.2019.1709319.

[18] Mohammad Idris Qureshi, Saima Jabee and Mohammad Shadab, General series identities, Some additive theorems on hypergeometric functions and their applications, *Journal of Interdisciplinary Mathematics,* V.25(2), 305-322 (2022). https://doi.org/10.1080/09720502.2021.1970946.