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Geo chromatic number of certain graphs

R. Joseph Paul *
U. Mary †
Department of Mathematics
Nirmala College for Women (Autonomous)
Coimbatore 641018
Tamil Nadu
India

Abstract

In this paper, we discussed about the geo chromatic number $\chi_{gc}(G)$ of sunlet graph, prism graph, book graph, banana graph, herschel graph, grotzsch graph and also for tree graph.

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Keywords: *Geodetic, Chromatic number, Geodetic number, Geo chromatic number.*

1. Introduction

Beulah Samli and Robinson Chellathurai [6] introduced the concepts of the geo chromatic number of graph. In this paper, we consider a graph to be connected, finite, simple where V(G) is node set and E(G) is link set [2,7].

The distance between two nodes x_1, x_2 contained in V(G) is the minimum size of $x_1 - x_2$ paths in G. An $x_1 - x_2$ path of the size $d_G(x_1, x_2)$ is known as geodesic. We indicate $I_G(x_1, x_2)$ as the set of nodes which are lies inside some $x_1 - x_2$ geodesics of G. A node is said to be lie on $x_1 - x_2$ geodetic if c is an inner node of P. The bounded interval $I(x_1, x_2)$ includes x_1, x_2 and all nodes lying on some $x_1 - x_2$ geodesic of G. Consider a non- empty set $S \subseteq V(G)$. For a set $I(S) = \bigcup_{x_1, x_2 \in S} I(x_1, x_2)$. If G is connected graph, thus S is

* *E-mail:* **jprmat2020@gmail.com (Corresponding Author)**
† *E-mail:* **marycbe@gmail.com**

a geodetic set with the condition that I(S) = V (G) [1,3]. The minimum cardinality S of G is called geodetic number defined by g(G). A j – node coloring of G is an allotment of j colors to the nodes of G. The coloring is proper if no two joining nodes accept same color such that $\chi(G) = \text{j}$ is said to be j – chromatic, where j ≤ k. a minimum cardinality of a chromatic number of G is called chromatic set [4,5].

In this paper, we discussed about geo chromatic number [6] for some known graph.

2. Preliminaries

Few basic definitions [6, 7] are given below which are related to this work.

3. Geo chromatic number of graphs

Theorem 3.1: *Consider the Sunlet graph S_n, where n ≥ 3, then* $\chi_{gc}(S_n) = n$.

Proof: The Sunlet graph S_n, the set $\{v'_1, v'_2, v'_3, \ldots, v'_n\}$ be the nodes of the cycle C_n, the set $\{u'_1, u'_2, u'_3, \ldots, u'_n\}$ be the pendent nodes which is attached to the cycle C_n. Thus the order of the node of G is $|V(G)| = 2n$. Hence the pendent nodes $\{u'_1, u'_2, u'_3, \ldots, u'_n\}$ are a geodetic set $|g(G)| = n$ is a minimum. Moreover the set S is a minimum chromatic set c of G. Since each pendent nodes of S_n can be assigned with distinct colors $\chi(G) \leq n$. Therefore $\chi_{gc}(S_n) = n$.

Theorem 3.2: *The prism graph Y_n, where n > 2, then* $\chi_{gc}(Y_n) = \begin{cases} 3 \text{ when } n = 3 \\ 4 \text{ when } n > 3 \end{cases}$.

Proof: For G = Y_n, prism graph with the node set V(G) = $\{v'_1, v'_2, v'_3, \ldots, v'_n\}$ $\cup$ $\{u'_1, u'_2, u'_3, \ldots, u'_n\}$, where v'_i's are the nodes of the outer cycle u'_i's are the nodes of inner cycle consider in a cyclic order such that each $v'_i\, u'_i$, where i = 1, 2, 3,…, n are the link joining the two cycles. The prism graph Y_n contains of 2n nodes and the order of G $= |V(G)| = 2n$. Since the set S $= \left\{v_1, v_{\frac{n}{2}+1}, u_{\frac{n}{2}+1}\right\}$ is a minimum geodetic for even and S $= \left\{v_1, u_{\frac{n+3}{2}}, u_{\frac{n+1}{2}}\right\}$ which is a minimum geodetics set for odd.

(i) For n > 3, set S belongs to the same color class say C_1 and C_2, where as S is not a chromatic set of G, that is $\chi(G)$ = 3. Now let us choose another node from G which belongs to the different color class C_3.

Let $u_i \in C_3$, then the set $S_c = S \cup \{u_i\}$ is a geo chromatic set of G. Thus $\chi_{gc}(G)=4$.

(ii) For n = 3 we have the geodetic set S, where as S_c satisfies condition of geo chromatic number. Thus it results $\chi_{gc}(G)=3$.

Theorem 3.3: *Consider any Book graph B_n where n>2, then* $\chi_{gc}(B_n)=n+1$.

Proof: Let G = B_m be the graph which considered with node set $\{x'_1, x'_2, x'_3, \ldots, x'_{2n}, a, b\}$, K_2 is the complete graph on 2 nodes. The maximum degree node of G is labeled as {a, b} and the remaining nodes are labeled as $\{x'_1, x'_2, x'_3, \ldots, x'_{2n}\}$.. Thus the node set $\{x'_{2k}\}$ where k = 1, 2, 3, …., n were considered as color C_1(say) and node set $\{x'_{2k-1}\}$ where k = 1, 2, 3, …., n were considered as color C_2(say). Hence the graph G is 2 – colorable. Thus, set S = $\{x'_{2k}\}$ or S = $\{x'_{2k-1}\}$ where k = 1, 2, 3, …., n be the minimum geodetic set of G where as S is not a chromatic set of G. Now let us choose another node from G which belongs to color class to the set S which forms the geo chromatic set. Therefore, S_c = $\{x'_{2k}\}$ + 1 or S_c = $\{x'_{2k-1}\}$ + 1 where n = 1, 2, 3, …., k satisfied the condition. Thus $\chi_{gc}(B_n)=n+1$.

Theorem 3.4: *Consider a graph $B_{2,k}$,* $\chi_{gc}(B_{2,k})=2k-3$ *for k >3.*

Proof: Let G = $B_{2,k}$, consider the set $\{v_1, v_2, v_3, \ldots, v_{k-1}\}$ be the node set of first copy, $\{u_1, u_2, u_3, \ldots, u_{k-1}\}$ be the node set of second copy of the star graph and the center node of the star graph is labeled as v and u for the first and second copy of the star graph. Thus the set S = $\{v_1, v_2, v_3, \ldots, v_{k-2}, u_1, u_2, u_3, \ldots, u_{k-2}\}$ is the minimum geodetic set where n = 1, 2, 3…, k. Since the nodes v_k, u_k and pendant nodes receives the color C_1 (say), v, u and w receives the color C_2 where as G is 2 – Colorable. Thus the geodetic set S receives the color C_1 thereby S is not a chromatic set. Let us choose another node from G which belongs to the different color class. Let $w \in C_i$, $i \neq 1$. Thereby $S_c = S \cup \{w\}$ satisfied the condition. Hence it follows that $\chi_{gc}(B_{2,k})=2k-3$.

Theorem 3.5: *Consider the Herschel graph G admits* $\chi_{gc}(G)=3$.

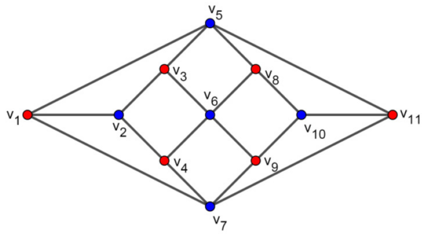

Figure 1

Herschel Graph

Proof: Assume G be the Herschel graph in Figure 1, with 11 nodes and 18 edges. The node set S = {v_1, v_6, v_{11}} be the minimum geodetic set. Let C be the proper color of G. Therefore the minimum chromatic number of G is $\chi(G)=2.$ Hence $\chi_{gc}(G)=3.$

Theorem 3.6: *The Grotzsch Graph is a triangular free graph admits* $\chi_{gc}(G_z)=5.$

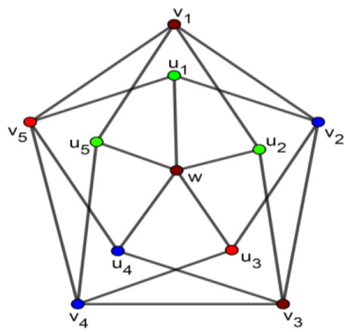

Figure 2

Grotzsch graph

Proof: Assume G_z is the Grotzsch graph is a triangular free graph in Figure 2, with 11 nodes and 20 links. Let V(G_z) = ={v_1, v_2, v_3, v_4, v_5, u_1, u_2, u_3, u_4, u_5, w} be the node set and C be the proper coloring of G_z and chromatic number is 4. Let S = {v_5, u_5, w, u_2, v_2} is the minimum geodetic and chromatic set of G_z. Hence, $\chi_{gc}(G_z)=5.$

4. Geo chromatic Number of Trees

Theorem 4.1: *Consider any tree T with m end edge, then* $\chi_{gc}(T)\leq m+1.$

Proof: Assume T be any tree with m end link. A tree with n vertices has n – 1 links whose chromatic number $\chi(G)=2.$ Every geodetic set of a graph contains its extreme nodes. Here we consider the different cases:

Case (i): Consider S is the set of all pendant nodes of a graph T. Let us assume C be the proper coloring of T. Since, the set S receives different color classes. S is a minimum geodetic set of T. Therefore, S_c is the geo chromatic set of T. Hence, it is result that $\chi_{gc}(T)\leq m+1.$

Case (ii): Assume T has exactly one vertex of degree greater than two is known as Starlike tree. The Set S is the set of all leaves of the T is geodetic set which is not minimum chromatic set of T. Suppose S is not a chromatic set, S has receives the vertex of various color class and also satisfies the condition of the geo chromatic set. Thus its results $\chi_{gc}(T)\leq m+1.$

Remark 1: Consider a tree T with p end edge in which all the vertices are within distance one or two of a central path have $\chi_{gc}(T)\leq p+1.$

Theorem 4.2: *Consider any tree T of order p and diameter q then* $\chi_{gc}(T)\leq p-q+2.$

Proof: Consider T be a non trivial connected graph of order p and diameter q. Assume x_1 and x_2 be the nodes of T for which $d(x_1, x_2) = q$. We consider a $x_1 - x_2$ path of length q, ie., $x_1 = v_0, v_1, v_2, v_3,\ldots, v_q = x_2$ (say). Let S be the geodetic set of a graph T contains its extreme nodes and $S = V(T) - \{v_1, v_2, v_3,\ldots,v_{q-1}\}$ also may receive same or different color class. Thus S not belongs to minimum chromatic set of T. If the set S is not a chromatic set, S has receives the node with various color class. Therefore, S_c becomes the minimum geo chromatic set of T. Hence $\chi_{gc}(T)\leq p-q+2.$

Remark 2: For a tree T with one internal vertex and n pendant vertices of order n+1 of n > 1 with maximum diameter 2, Hence, results $\chi_{gc}(T)\leq n-1.$

5. Conclusion

We defined the geo chromatic number of graph whereas the concept can be extended to some other graphs.

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