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An improved solution approach for solving multi-objective linear and linear fractional transportation problem

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Abstract

This work shows a transportation problem (TP) that can be thought of as the sum of a linear function and a linear fractional function. Here, the linear function shows the total transportation cost, and the linear fractional function shows the proportion of damaged objects and the transport profit. We aim to set up a transportation plan that reduces the total

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transportation cost and the proportion of total damage to the object to the total profit. Here we are discussing three types of objective functions as: linear plus linear fractional, linear-fractional, and linear separately. We formulated a TP using the three objective functions for the solution. We solve all three objective functions for different weights with the proposed method and get a compromise solution.

Subject Classification: *90C32, 90B06, 90C29.*

Keywords: *Linear plus linear fractions, Fractions, Transportation problems, Weighted sum approach, Multi-objective linear plus linear fractional-transportation problem (MOLPLFTP).*

1. Introduction

Transportation problems (TP) having linear plus linear fractional, linear fractional and linear objective functions are broadly used to get solutions in numerous real-life circumstances as a financial survey of transportation of goods, and many other transportation situations where the single or more than single people tackled the difficulty to maintaining goods ratio some crucial and critical variables treated with the conveyance of product from a fixed source to different destinations. We can say that the TP is to minimize cost and ratio of damage cost & profit. In transportation problems, goods are supplied from different origin points to various target points. The supply series operation gets increasingly fundamental in worldwide economics, understanding the transporting structure which is the base of systematic and reasonable operations of the company's management. Current and past times numerous research articles highlighting supply series administration to get lesser costs and enhance services. Hitchcock (1941) gave the TP first time which was the classical TP. The TP having the total sum of supply and demand equal then (TP) is known as a balanced TP if the total sum of supply and demand is not equal then (TP) is known as an unbalanced TP. The theoretical TP has different intervals and situations given by Klibi *et. al.,* (2010). This paper provided information on the stochastic or probabilistic situation of TP specify by numerous requirements with conveyance possibilities, and a probabilistic requirement function. The multi-objective transportation problem (MOTP) shows the further form of the transportation problem (TP) which contains two or more objective functions. Multi-objective transportation problems have different objectives for different priorities. Sufficient research has been done on this topic. Waiel (2001) provided a MOTP in the process of fuzziness circumstances. This approach gave compromise solutions for absolute solutions and a family of distance functions is used for the solutions. Nomani *et. al.,* (2016) presented a new solution method for

solving the MOTPs. This approach provides a compromise result using a weighted point of view depending on goal programming. Roy *et. al.,* (2017) presented a coned scalarization technique to work out multi-choice, MOTPs with the help of goal-set theory. This approach uses a conical scalar function to investigate MOTPS. The programming for linear-fractional TP given by Saini *et. a*l.,(2021) is rough, where a fixed charge is generated for each objective function. The multi-objective theoretical TP for solids presented by Das & Lee (2021) contains contributions, requirements, and capacity in the Weibull issue. A solid MOTP is solved in a theoretical environment with help of Weibull output. The MOTP of fourth-function management presented by Xu *et al* (2021) depends on queuing hypothesis. Joshi *et. al.,* (2022) gave a neutrophil situation to solve the fractional TP having more than one objective function. A MOTP is called a multi-objective fractional transportation problem (MOFTP) when the objective function is in fractional form. Also, the TP has a linear combination with a MOTP, than it represents a multi-objective linear plus linear fractional transportation problem (MOLPFTP). Many researchers gave their sight to MOFTP and MOLPFTP. Joshi *et. al.,* (2020) presented the research article for MOLFTP in a contradictory format named paradox. This article gave a less paradoxical situation to the MOLFTP. Joshi and Sharma (2018) gave a technique as an absolute technique for less paradoxical in MOLFTPs. Sayed and Sinha (2021) presented a completely intuitionistic new technique to the fuzzy MOFTPs. In this article, they diminish the linearized prototype to a well-defined crisp MOTP using the reliability function for each objective function. Joshi and Saini (2018) proposed a research paper on a new technique to solve the MOTPs. This research paper optimizes the goal function for one or more proportions with few linear restrictions. Gupta and Joshi (2009) proposed an analytical technique for superior starting results to the linear-fractional TP. AnuKokila *et. al.,* (2019) found a new technique for goal-programming to get suitable results for MOFTPs with the help of fuzzy techniques. The author has determined exactly the target values for solving the TP of each object. A new technique as an MFL paradox in MOLPLFTP associated with the fuzzy technique given by Saini and Joshi (2022). An alternative solution technique for MOLFTPs is given by Joshi *et. al.,*(2022).

This paper has a new technique (approach) to solving the problem. The work is divided into the following parts or sections: part 1 represents an introduction; part 2 represents the formulation of (MOLPLFTP), (MOLFTP) and Linear transportation problem (TP) and required definitions; In part 3 the weighted sum method and related formulation discussed; part 4 explain the proposed method as our main method; part

5 examines the computational presentation of the algorithm on a problem; part 6 represents the conclusion and remarks; part 7 shows references.

2. Mathematical formulation

2.1 *Multi-objective linear plus linear fractional transportation have following formulation (MOLPLFTP):*

$$\text{minimize} \qquad Z_k(x_{ij}) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} + \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}x_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n} t_{ij}x_{ij}}, \quad k = 1,2,3\ldots..,K$$

$$\text{Subject to}: \ \sum_{i=1}^{m} x_{ij} = a_i, \ \text{for} \ j = 1,2,3,\ldots n, \sum_{j=1}^{n} x_{ij} = b_j, \ \text{for} \ i = 1,2,3,\ldots m$$

$$x_{ij} \geq 0, \quad i = 1,2,3,\ldots.m, \quad j = 1,2,3,\ldots.n \tag{1}$$

Here a_i shows different supply points as $i = 1, 2, 3\ldots., m$, there are m source points which transported commodities. b_j, shows the destination points as $j = 1, 2, 3, \ldots.. n$ there are n destination points where commodities are required. Here c_{ij} shows the cost from i source to j destination. d_{ij}, t_{ij} denotes the damage and profit of transportation of goods from starting point i to target point j, respectively. The variable x_{ij} shows the figure of quantity delivered from the i^{th} starting point to the j^{th} target point. The overall problem is stabilize (total supply from source = total requirement to destination).

2.2 *Mathematical formulation of the multi-objective linear fractional transportation problem (MOLFTP):*

$$\text{Minimize} \qquad Z_k(x_{ij}) = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}x_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n} t_{ij}x_{ij}}, \quad k = 1,2,3,\ldots\ldots K$$

$$\text{Subject to}: \ \sum_{i=1}^{m} x_{ij} = a_i, \ \text{for} \ j = 1,2,3,\ldots n, \ \sum_{j=1}^{n} x_{ij} = b_j, \ \text{for} \ i = 1,2,3\ldots m$$

$$x_{ij} \geq 0, \ i = 1,2,3,\ldots.m, \ j = 1,2,3,\ldots.n \tag{2}$$

2.3 *Mathematical formulation for linear multi-objective transportation problem:*

$$\text{Minimize} \qquad Z_k(x_{ij}) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \qquad k = 1,2,3,\ldots\ldots K$$

$$\text{Subject to}: \quad \sum_{i=1}^{m} x_{ij} = a_i, \text{for } j = 1,2,3,\ldots n, \ \sum_{j=1}^{n} x_{ij} = b_j, \text{for } i = 1,2,3\ldots m$$

$$x_{ij} \geq 0, \qquad i = 1,2,3,\ldots.m, \quad j = 1,2,3,\ldots.n \tag{3}$$

3. Weighted sum method

For different weights, the weighted sum method provides multiple solutions to the MOTP. The already stated technique can be utilized for linear plus linear fractional, linear fractional, and only linear transportation problems.

The formulation of weight sum method for linear plus linear fractional transportation problem given as:

$$\text{Minimize} \quad Z = \sum_{k=1}^{K} w_k Z_k = \sum_{k=1}^{K} w_k \left(\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} + \frac{\sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} x_{ij}}{\sum_{i=1}^{m} \sum_{j=1}^{n} t_{ij} x_{ij}} \right)$$

$$\text{Subject to}: \sum_{i=1}^{m} x_{ij} = a_i, j = 1,2,3,\ldots,n, \ \sum_{j=1}^{n} x_{ij} = b_j, i = 1,2,3,\ldots,m$$

Also, the different weights $w_k, k = 1,2,3,\ldots\ldots,K.$ equivalent objective functions convince the succeeding situation:

$$\sum_{k=1}^{K} w_k = 1, \qquad w_k \geq 0, \quad k = 1,2,3,\ldots..K. \tag{4}$$

4. Proposed method

4.1 *Algorithm of the proposed method:*

Let the successive RMOLFTP:

$$\text{Minimize} \quad Z(x) = [Z_1(x), Z_2(x), \ldots\ldots\ldots Z_k(x)]$$

$$\text{Subject to } x \in S,$$

Where x shows the n-structural resolution variable, S is the set of suitable results. The cause of contradictory goals cannot be optimized for each RMOLFTP at the same time. Here, the aim is to get a balanced solution, not the best one.

Through several approaches available in the literature, MOTP can be transformed into a TP which has only one objective function. Through this proposed approach, we transform a MOTP into a TP having only one objective function by selecting the objective function as:-

$$\text{Minimize} \qquad \rho' = \sum \rho(1-w_k),$$

Where w_k is weight appoint to the k^{th} function and ρ shows the general deviational variable. Here $Z_k^* + \rho(1-w_k)$ shows the upper class of every objective, where Z_k^* (the ideal objective) can be gotten by the solution of an above problem for every objective Z_k, $k = 1, 2, 3\ldots, K$ is independent of the further objectives.

The MOTP (3) is easily diminished to the next single-objective problem:

$$\text{Minimize} \qquad \rho' = \sum_{k=1}^{K} \rho(1-w_k)$$

$$\text{Subject to} \qquad Z_k \le Z_k^* + \rho(1-w_k)$$
$$x \in S, x_{ij} \ge 0$$

In the present model, rather than utilize the deviation variable, we established the deviation function $\rho(1-w_k)$. However, multiplying by ρ, we can assume that the weight (w_k of the k^{th} objective) is assigned to the priority, but this function reduces the value of the deviation function of w_k. This will give a closer ideal objective of objective which is taken by us. When preferences are defined, this method provides a compromise solution. From the above discussion, we get the conclusion that this method can be used for all types of multi-objective optimization problems, multi-objective linear plus linear-fractional optimization problems, fractional optimization problems, and linear optimization problems.

4.2 *Proposed method for MOLPLFTP:*

The proposed method may be formulated as:

$$\text{Minimize} \quad \rho' = \sum_{k=1}^{K} \rho(1-w_k)$$

$$\text{Subject to}: \ \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} + \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}x_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n} t_{ij}x_{ij}} \le Z_k^* + \rho(1-w_k), \forall \ k = 1,2,3,\ldots,K$$

$$\sum_{i=1}^{m} x_{ij} = a_i, \ j = 1,2,3,\ldots\ldots n, \ \sum_{j=1}^{n} x_{ij} = b_j, \ i = 1,2,3,\ldots..m$$
$$0 \le w_k \le 1, \ k = 1,2,3,\ldots.., K$$
$$x_{ij} \ge 0, \ i = 1,2,3,\ldots\ldots, m, \ j = 1,2,3,\ldots.., n. \qquad (5)$$

5. Numerical Approach

Let us consider an example with three objective functions respectively. The present example shows, that the initial objective function has linear plus linear fractional values which are cost, damage, and profit, and the second objective function has only linear fractional values which show damage and profit, in the last third objective function has only linear values which show cost. Table 1-3 are showing the data for MOLPLFTP, MOLFTP, and MOLTP.

Table 1

Data for the MOLPLFTP where the objective is in the form $\left(c_{ij}, \frac{d_{ij}}{t_{ij}}\right)$

	D_1	D_2	D_3	D_4	Supply
S_1	5 (c_{ij}), 16 (d_{ij}) 19 (t_{ij})	12, 25 36	19, 34 7	26, 18 14	29
S_2	10, 21 13	55, 9 2	28, 8 4	17, 11 22	42
S_3	25, 45 23	44, 41 29	31, 52 47	61, 80 20	11
Demand	25	30	15	12	

Table 2

Data for the MOFTP where the objective is in the form $\left(\frac{d_{ij}}{t_{ij}}\right)$

	D_1	D_2	D_3	D_4	Supply
S_1	18 (d_{ij}) 20 (t_{ij})	25 15	33 45	51 13	29
S_2	23 21	26 75	49 12	17 22	42
S_3	31 41	58 29	50 30	41 16	11
Demand	25	30	15	12	

Table 3

Data for the multi-objective linear TP where the objective is in the form (C_{ij})

	D_1	D_2	D_3	D_4	Supply
S_1	84	70	9	14	29
S_2	66	54	40	46	42
S_3	38	29	72	61	11
Demand	25	30	15	12	

5.1 *Solution by the proposed technique*

Table 4
Solutions for different weights

S. No.	Weights assigned (w_1, w_2, w_3)	Z_1, Z_2, Z_3
1	w_1=0.1, w_2 =0.9, w_3 =0.0	1764.40934, 0.99762, 3552.43226
2	w_1=0.2, w_2 =0.8, w_3 =0.0	1734.53002, 1.00137, 3844.02378
3	w_1=0.3, w_2 =0.7, w_3 =0.0	1701.28525, 1.00564, 3874.66477
4	w_1=0.4, w_2 =0.0, w_3 =0.6	1895.15840, 0.98223, 3695.82557
5	w_1=0.5, w_2 =0.0, w_3 =0.5	1791.42839, 0.99431, 3791.50852
6	w_1=0.6, w_2 =0.0, w_3 =0.4	1689.36224, 1.00721, 3885.66302
7	w_1=0.0, w_2 =0.3, w_3 =0.7	2077.44491, 0.96342, 3530.30966
8	w_1=0.0, w_2 =0.2, w_3 =0.8	2143.92263, 0.95678, 3466.40043
9	w_1=0.0, w_2 =0.1, w_3 =0.9	2226.79527, 0.94925, 3389.98752
10	w_1=0.3, w_2 =0.3, w_3 =0.4	1831.12523, 0.98957, 3754.89026
11	w_1=0.3, w_2 =0.4, w_3 =0.3	1791.42839, 0.99431, 3791.50852
12	w_1=0.4, w_2 =0.3, w_3 =0.3	1751.97771, 0.99917, 3827.90071

The result to a single-objective, ignoring the other objectives is:

$$X_1 = (x_{11}=0, x_{12}=29, x_{13}=0, x_{14}=0, x_{21}=25, x_{22}=0, x_{23}=5, x_{24}=12, x_{31}=0, x_{32}=1, x_{33}=10, x_{34}=0)\ Z_1(X^1)=1296.92,\ Z_2(X^1)=1.43025, Z_3(X^1)=5181$$

$$X_2 = (x_{11}=14, x_{12}=0, x_{13}=15, x_{14}=0, x_{21}=0, x_{22}=30, x_{23}=0, x_{24}=12, x_{31}=11, x_{32}=0, x_{33}=0, x_{34}=0), Z_1(X^2)=2485.72,\ Z_2(X^2)=0.5287, Z_3(X^2)=3901$$

$$X_3 = (x_{11}=0, x_{12}=2, x_{13}=15, x_{14}=12, x_{21}=14, x_{22}=28, x_{23}=0, x_{24}=0, x_{31}=11, x_{32}=0, x_{33}=0, x_{34}=0),\ Z_1(X^3)=2578.17,\ Z_2(X^3)=0.68753, Z_3(X^3)=3297$$

Table 5

Comparison of the results of the proposed technique and the weighted sum technique

S. No.	Weights assigned (w_1, w_2, w_3)	Proposed method Z_1, Z_2, Z_3	Weight sum method Z_1, Z_2, Z_3
1	w_1=0.1, w_2 =0.9, w_3 =0.0	1764.40934, 0.99762, 3552.43226	1296.921467, 1.43025, 5181
2	w_1=0.2, w_2 =0.8, w_3=0.0	1734.53002, 1.00137, 3844.02378	1296.921467, 1.43025, 5181
3	w_1=0.3, w_2 =0.7, w_3 =0.0	1701.28525, 1.00564, 3874.66477	1296.921467, 1.43025, 5181
4	w_1=0.4, w_2 =0.0, w_3 =0.6	1895.15840, 0.98223, 3695.82557	2291.86217, 0.94362, 3330
5	w_1=0.5, w_2 =0.0, w_3 =0.5	1791.42839, 0.99431, 3791.50852	1667.31827, 1.01014, 3906
6	w_1=0.6, w_2 =0.0, w_3 =0.4	1689.36224, 1.00721, 3885.66302	1497.16706, 1.29637, 4141
7	w_1=0.0, w_2 =0.3, w_3 =0.7	2077.44491, 0.96342, 3530.30966	2578.17345, 0.68753, 3297
8	w_1=0.0, w_2 =0.2, w_3 =0.8	2143.92263, 0.95678, 3466.40043	2578.17345, 0.68753, 3297
9	w_1=0.0, w_2 =0.1, w_3 =0.9	2226.79527, 0.94925, 3389.98752	2578.17345, 0.68753, 3297
10	w_1=0.3, w_2 =0.3, w_3 =0.4	1831.12523, 0.98957, 3754.89026	2291.86217, 0.94362, 3330
11	w_1=0.3, w_2 =0.4, w_3 =0.3	1791.42839, 0.99431, 3791.50852	1667.31827, 1.01013, 3906
12	w_1=0.4, w_2 =0.3, w_3 =0.3	1751.97771, 0.99917, 3827.90071	1667.31827, 1.01013, 3906

6. Conclusions

This article introduced a new approach to solving the MOLPLFTP. The suggested method is more effective because it presents optimal solutions that depend on preferences. Table 4 shows the results by proposed method with different weights. We found a correlation between the results of the suggested technique and the weight sum technique (Table 5). All the mathematical results are shown in part 5. For all the mathematical calculations in part 5, we used LINGO 17.0 software.

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