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A new solution procedure for multi-objective linear fractional transportation problem with rough parameters

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Abstract

In this article, we are discussing a rough multi-objective linear fractional transportation problem (RMOLFTP). In Day-to-day life, we frequently encounter many situations where the transportation problems are represented by a rough set. The main aim of our research article is to solve the RMOLFTP where the transportation cost and profit, of the product as rough

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interval coefficients. Here a new logic is introduced by us to solve RMOLFTP that diversifies the form of Nomani *et. al.,* (2016). To demonstrate the recommended algorithm, a rough numeric problem with two objective functions is offered. A weighted model is presented in this approach for the solutions.

Subject Classification: *90C29, 90C32, 90B06, 90B99.*

Keywords: *Multi-objective linear fractional transportation problem (MOLFTP), Rough set theory, Fractional transportation problem, Weighted sum approach.*

1. Introduction

Transportation problems (TP) are the most important part of supply chain management and offer important possibilities for reducing transportation costs and different components. Using transportation problem we can optimize the cost of transportation for moving goods from one point to another. The management of delivery series is set off more and more basic in the worldwide economy, and awareness of the transportation process is the basis of the systematic and economic behavior of business owners.

Over the last few years, many experimentation reports on delivery series management to reduce costs and improve services have been published. Hitchock (1941) first of all studied this. If both supply and demand are equal, it is said to be an equalized transportation problem, and if the supply and demand are not equal, it is said to be an un-equalized transportation problem (TP). Many algorithms have been developed in the correct procedure for any decision parameter, but in reality, many goals are associated with the same problem. Since then, all kinds of transportation problems have arisen in the study. A conversation in the fuzzy form of the TP is given by Chanas *et. al.*, (1984). This approach produces fuzzy stock utilities for sources and fuzzy requirements. The theoretical and vague form of multi-area TP was given by Klibi *et. al.*, (2010). This paper provided information on a probabilistic area of TP distinguished by numerous requirements, multiple transport possibilities, and a probabilistic order function. Some aspects of solving transportation problems are proposed by Das *et. al.,* (2019). A class of transportation problems is considered in this approach which arises due to a sampling survey and other areas of statics and mathematics. Cost matrices of transportation problems are associated with special structures. Here North West corner solution holds the optimality for the problem and the cost component is also replaced by a convex function.

Single-objective TPs are converted to multi-objective TPs (MOTPs), which pertain to a unique set of points with an LPP that specifically contains multiple conflicting and different objective functions. Optimizing Transportation Problem with Multiple Objectives given by Lee and Moore (1973). In this issue, they studied a goal-scheduled perspective to handle optimizations for multiple conflicting goals. Ammar and Youness (2005) investigated TPs and MOTPs in numerical format. In this paper, the idea of -fuzzy efficiency was presented. This paper extends the usual systematic solution depending on the alpha level of the fuzzy numerical format. Joshi and Gupta (2012) used the objective matrix to identify more or fewer paradoxes of linear-fractional TP. Nomani *et. al.,* (2016) presented a new solution method for solving the MOTPs. This approach provides a compromise result using a weighted point of view depending on goal programming. The TP has a multi-objective with tariff dependability undergoing certain environments suggested by Maity *et. al.,*(2016). This paper examines the learning of a MOTP in undetermined conditions, taking into account the unreliability of real-world select problems, using the idea of accuracy integrated into real-world phenomena in MOTP. A new scheme for undetermined MOTPs using fuzzy linear integration function given by Uddin *et. al.,*(2021). This approach depends on a fuzzy membership goal program to obtain the required settled solution for the MOTP in undetermined conditions where DM may choose dependence levels for various parameters. Rough parameters were used by Saini *et. al.,* (2021) to solve various types of TP. Midya and Roy (2020) solved MOFCTP utilizing rough programming. In this article, the cost function is given a fixed charge and all calculations depend on this fixed charge. Joshi and Saini (2018) proposed a research paper on solving the MOFTP. This article optimizes the selected function for one or more proportions with few linear restrictions. A new method for solving RMOTP: evolution and possibility given by Garg and Allah (2021). In this paper, a rough set of functions is proposed and solved. An alternative approach to solving multi-level, multi-objective, linear fractional programming problems is given by Joshi *et. al.,*(2022). Joshi and Gupta (2009) solved the fractional TP by an analytical approach.

This paper is formed such that part 2 described the numerical model of RMOLFTP and part 3 represented the method for solving the RMOLFTP. Part 4 showed the formulation of the weight sum method. A numerical problem having two objective functions is solved in part 5 by using the proposed method, also results are shown in the Table 6. Table 7 represented the comparison results between the suggested process and the weight-sum process. Part 6 and 7 have a conclusion and references.

The major benefits of this paper are summarized as:

- RMOLFTP prototype is generated.
- The expected value operator converts RMOLFTP to crisp format.
- The proposed method gave the solution of RMOLFTP.
- RMOLFTP introduces a strategy to analyze the sensitivity analysis for the parameters.
- This RMOLFTP describes the actual problem model.

2. Mathematical model

Consider two objective functions. One objective function indicates the transportation cost of an individual departure point, and the other indicates the total transportation profit. Suppose the product has m starting points $(i = 1, 2, 3\ldots, m)$ delivered to n targets $(j = 1, 2, 3\ldots, n)$. The issue is to decide the decision variable (which we have to find out) x_{ij}, which is determined from the i^{th} starting point to the j^{th} target so that the estimate of both capabilities is minimized.

2.1 *Notations of RMOLFTP*

The notations which we are using in RMOLFTP:

m	The quantity from the start point.
n	The quantity to the target point.
x_{ij}	The unit quantity of the stuff transported from i^{th} start point to j^{th} target point.
c_{ij}^R	A rough cost variable function that is transported from i^{th} start point to j^{th} target point for the unit part of the stuff.
t_{ij}^R	Profit variable of transportation from i^{th} start point to j^{th} target point.
c_{ij}	Crisp cost variable function that is transported from i^{th} start point to j^{th} target point for the unit part of the stuff.
t^{ij}	Crisp profit of conveyance from i^{th} start point to j^{th} target point.
a_i	Stuff availability at the i^{th} start point.
b_j	Stuff demand at the j^{th} target point.
Z_k	Objective function ($k = 1, 2, 3 \ldots$).

The numeric version of RMOLFTP using coefficients may be expressed as:-

Minimize $$Z_k^R = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^R x_{ij}^R}{\sum_{i=1}^{m}\sum_{j=1}^{n} t_{ij}^R x_{ij}^R},$$

Subject to $$\sum_{i=1}^{m} x_{ij}^R = a_i^R \quad \text{for} \quad j = 1,2,3\ldots,n,$$

$$\sum_{j=1}^{n} x_{ij}^R = b_j^R \quad \text{for} \quad i = 1,2\ldots,m$$

$$\sum_{i=1}^{m} a_i^R = \sum_{j=1}^{n} b_j^R$$

$$x_{ij}^R \geq 0, \quad i = 1,2,\ldots.m, \quad j = 1,2\ldots..n \tag{1}$$

2.2 *A formulation for changing the rough data into crisp data*

If c_{ij}^R is a rough span explained as: $c_{ij}^R = ([c_{ij1}, c_{ij2}],[c_{ij3}, c_{ij4}])$ then

$$E(c_{ij}^R) = \frac{1}{2}[\eta(c_{ij1} + c_{ij2}) + (1-\eta)(c_{ij3} + c_{ij4})], \quad \forall\, i, j \tag{2}$$

Where $0 < \eta < 1$; is the restriction, pre-decided by the arbiter.

3. Method used to solve RMOLFTP

In this paper we are solving RMOLFTP by the proposed method which is described as follows:

3.1 *The suggested technique*

In actual life, RMOLFTP, have an excessive priority over others for some of the objectives and outcomes that are restrictions. There are many ways to provide a balanced result, but this paper describes the way as a suggested or proposed approach. The proposed approach can produce more consistent results for individual tastes or preferences. Preferences are described in the designate of the weights set aside to target the given problem.

3.2 *Algorithm of the proposed method*

Let the successive RMOLFTP:

Minimize $$Z(x) = [Z_1(x), Z_2(x), \ldots Z_k(x)]$$

Subject to $$x \in S, \tag{3}$$

Where x shows the n-structural resolution variable, S is the set of suitable results. The cause of contradictory goals cannot be optimized for each RMOLFTP at the same time. Here, the aim is to get a balanced solution, not the best one.

Through several approaches available in the literature, MOTP can be transformed into a TP which has only one objective function. Through this proposed approach, we transform a MOTP into a TP having only one objective function by selecting the objective function as:-

Minimize $\rho' = \sum \rho(1-w_k),$

Where w_k is weight appoint to the k^{th} function and ρ shows the general deviational variable. Here $Z_k^* + \rho(1-w_k)$ shows the upper class of every objective, where Z_k^* (the ideal objective) can be gotten by the solution of an above problem for every objective Z_k, $k = 1, 2, 3 \ldots, K$ is independent of the further objectives.

The MOTP (3) is easily diminished to the next single-objective problem:

Minimize $\rho' = \sum_{k=1}^{K} \rho(1-w_k)$

Subject to $Z_k \le Z_k^* + \rho(1-w_k)$

$$x \in S, x_{ij} \ge 0 \tag{4}$$

In the present model, rather of utilize the deviation variable, we established the deviation function $\rho(1-w_k)$. However, multiplying by ρ, we can assume that the weight (w_k of the k^{th} objective) is assigned to the priority, but this function reduces the value of the deviation function of w_k. This will give a closer ideal objective which is taken by us. When preferences are defined, this method gave a compromise solution. From the above discussion, we conclude that this technique may also be used for all types of optimization problems or rough multi-objective fractional transportation problems.

3.3 *Proposed method for RMOLFTP*

In RMOLFTP goods are shipped from m origin to n target points. The formulation of RMOLFTP is given as:-

The RMOLFTP has the numerical form as:

$$\text{Minimize} \quad Z_k(x_{ij}) = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^{k} x_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n} t_{ij}^{k} x_{ij}}, \quad k = 1,2,3,\ldots\ldots,K$$

$$\text{Subject to} \quad \sum_{i=1}^{m} x_{ij} = a_i,\ j = 1,2,3,\ldots..,n,\ \ \sum_{j=1}^{n} x_{ij} = b_j, \quad i = 1,2,3\ldots\ldots..,m$$

$$x_{ij} \geq 0,\ i = 1,2,3,\ldots.,m \ \text{ and } \ j = 1,2,3,\ldots\ldots,n. \tag{5}$$

To get the above RMOLFTP formulation according to the proposed model, solve the TP individually for each Z_k where $k = 1, 2, 3\ldots K$ and get the perfect objective result Z_k^* where $k = 1, 2, 3\ldots K$. The suggested model has the following mathematical form:-

$$\text{Minimize} \quad \rho' = \sum_{k=1}^{K} \rho(1 - w_k)$$

$$\text{Subject to} \quad \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^{k} x_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n} t_{ij}^{k} x_{ij}} \leq Z_k^* + \rho(1 - w_k), \qquad \forall\ k = 1,2,3,\ldots K$$

$$\sum_{i=1}^{m} x_{ij} = a_i, \quad j = 1,2,\ldots,n, \quad \sum_{j=1}^{n} x_{ij} = b_j, \quad i = 1,2,\ldots,m$$

$$0 \leq w_k \leq 1, \quad k = 1,2,3,\ldots,K$$

$$x_{ij} \geq 0, \qquad i = 1,2,3,\ldots,m \ \text{ and } \ j = 1,2,3,\ldots,n. \tag{6}$$

The step-wise conclusion may be written as:

Step 1: First, we established a rough numerical programming prototype for the TP shown in (1).

Step 2: With the help of (2) a rough model converted into a crisp model (5).

Step 3: Ignore all other objectives and get the solution of all the K-objective functions for the TP having one objective.

Step 4: For the optimality, we get one and all objective functions.

Step 5: In this step, we define the weights of objectives.

Step 6: Proposed model (6) developed and solved.

Step 7: A compromise result can be acquired by working out one and all objective functions of the solution obtained in step (4).

4. Weighted sum method

For different weights, the weighted sum method provides multiple solutions to the MOTP. This method may be used for linear plus linear fractional, linear fractional, and only linear transportation problems.

The optimization problem having one objective function with the help of weighted-sum technique acquire as:

$$\text{Minimize} \qquad Z = \sum_{k=1}^{K} w_k Z_k = \sum_{k=1}^{K} w_k \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^{k} x_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n} t_{ij}^{k} x_{ij}},$$

$$\text{Subject to}: \quad \sum_{i=1}^{m} x_{ij} = a_i, \quad j = 1,2,3,\ldots..,n, \ \sum_{j=1}^{n} x_{ij} = b_j, \quad i = 1,2,3,\ldots,m$$

Also, the different weights $w_k, k = 1,2,3,\ldots,K$ equivalent objective-functions convince the succeeding situation:

$$\sum_{k=1}^{K} w_k = 1, \quad w_k \geq 0, \quad k = 1,2,3,\ldots..K.$$

5. Numerical illustration

Let us consider an example with two objectives. The suggested technique gave the solution of this illustration for several different weights. Let's look at the costs and profit connected with RMOLFTP in the table below:

Table 1

Rough data for the RMOLFTP (C_1^R)

	D_1	D_2	D_3	D_4
S_1	([1,2], [0,3])	([1,3], [0,4])	([5,9], [4,10])	([4,8], [3,9])
S_2	([1,2], [0,3])	([7,10], [6,11])	([2,6], [1,7])	([3,5], [2,6])
S_3	([3,5], [2,6])	([3,5], [2,6])	([3,5], [2,6])	([5,7], [4,8])

Table 2

Rough data for the RMOLFTP (t_1^R)

	D_1	D_2	D_3	D_4
S_1	([13,21], [3,2])	([19,4], [7,5])	([25,10], [5,19])	([5,9],[10,10])
S_2	([11,3], [17,17])	([17,11], [28,3])	([3,7], [17,8])	([12,6],[27,7])
S_3	([8,10], [6,25])	([19,2], [13,13])	([4,19], [3,23])	([6,8],[9,9])

Table 3

Rough data for the RMOLFTP (C_2^R)

	D_1	D_2	D_3	D_4
S_1	([3,4], [2,5])	([3,5], [2,6])	([7,11], [6,12])	([6,10], [5,11])
S_2	([3,4], [2,5])	([9,12], [8,13])	([4,8], [3,9])	([5,7], [4,8])
S_3	([9,11], [8,12])	([9,13], [8,14])	([5,7], [4,8])	([7,9], [6,10])

Table 4

Rough data for the RMOLFTP (t_2^R)

	D_1	D_2	D_3	D_4
S_1	([1,1], [1,6])	([9,16], [3,11])	([35,14], [7,7])	([29,23], [8,40])
S_2	([35,39], [19,31])	([41,45], [24,7])	([31,43], [21,46])	([4,6], [5,19])
S_3	([48,42], [2,37])	([7,4], [9,21])	([44,9], [11,21])	([24,12], [7,7])

In Table 1-4, rough data provided for RMOLFTP, and Table 5 shows converted data from rough to crisp. In addition, the rough starting point limitations a^R, and rough target point limitations b^R are explained as follows:

$$a_1^R = ([7,9],[6,10]),\ a_2^R = ([17,21],[16,22]),\ a_3^R = ([16,18],[15,19]),$$
$$b_1^R = ([10,12],[9,13]),\ b_2^R = ([2,4],[1,5]),\ b_3^R = ([13,15],[12,16]),$$
$$b_4^R = ([15,17],[15,17])$$

Interchanging the rough values into crisp values with the help of formulation shown in 2.2:

Table 5

Interchange the rough values with the crisp values:

	D_1	D_2	D_3	D_4	Supplies
S_1	1.5, 3.5 (c_{ij}^k) 9.75, 2.25 (t_{ij}^k)	2, 4 8.75,9.75	7, 9 14.75, 15.75	6, 8 8.5, 25	8
S_2	1.5, 3.5 12, 31	8.5,10.5 14.75, 29.25	4, 6 8.75, 35.25	4, 6 13, 8.5	19
S_3	8, 10 12.25, 32.25	9, 11 11.75, 10.25	4, 6 12.25, 21.25	6, 8 8, 17.5	17
Demand	11	3	14	16	

5.1 *Solutions based on the Suggested Method*

The results to a single objective, ignoring other objectives, are:

$$X_1 = (x_{11} = 5, x_{12} = 3, x_{13} = 0, x_{14} = 0, x_{21} = 6, x_{22} = 0, x_{23} = 0, x_{24} = 13, \quad x_{31} = 0,$$
$$x_{32} = 0, x_{33}, = 14, x_{34} = 3),\ Z_1(X^1) = 0.290323,\ Z_2(X^1) = 0.34425$$

$$X_2 = (x_{11} = 0, x_{12} = 3, x_{13} = 0, x_{14} = 5, x_{21} = 11, x_{22} = 0, x_{23} = 8, x_{24} = 0, x_{31} = 0,$$
$$x_{32} = 0, x_{33} = 6, x_{34} = 11), Z_1(X^2) = 0.403702, Z_2(X^2) = 0.239234$$

The suggested prototype (6) shows the results, which are shown in below Table 6.

Table 6

Balanced objective results that are equivalent to preference:

S. No.	Weighted appoint (w_1, w_2)	Z_1, Z_2 Non-integer
1	w_1=0.1, w_2=0.9	0.37176, 0.248283
2	w_1=0.2, w_2=0.8	0.352569, 0.254796
3	w_1=0.3, w_2=0.7	0.338908, 0.260055
4	w_1=0.4, w_2=0.6	0.32831, 0.264559
5	w_1=0.5, w_2=0.5	0.319644,0.268556
6	w_1=0.6, w_2=0.4	0.314325, 0.275238
7	w_1=0.7, w_2=0.3	0.309981, 0.285102
8	w_1=0.8, w_2=0.2	0.304949, 0.297738
9	w_1=0.9, w_2=0.1	0.298779, 0.315341

Table 7

Comparison of the results obtained from the suggested and weight-sum technique

S. No.	**Weight appoint W_1, W_2**	**proposed method Z_1, Z_2**	**Weight sum method Z_1, Z_2**
1	w_1=0.1, w_2=0.9	0.37176, 0.248283	0.403702, 0.239234
2	w_1=0.2, w_2=0.8	0.352569, 0.254796	0.403702, 0.239234
3	w_1=0.3, w_2=0.7	0.338908, 0.260055	0.332528, 0.262718
4	w_1=0.4, w_2=0.6	0.32831, 0.264559	0.316842, 0.269915
5	w_1=0.5, w_2=0.5	0.319644,0.268556	0.316842, 0.269915
6	w_1=0.6, w_2=0.4	0.314325, 0.275238	0.316842, 0.269915
7	w_1=0.7, w_2=0.3	0.309981, 0.285102	0.310833, 0.283095
8	w_1=0.8, w_2=0.2	0.304949, 0.297738	0.290323, 0.34425
9	w_1=0.9, w_2=0.1	0.298779, 0.315341	0.290323, 0.34425

6. Conclusions

In the current paper, we suggest a goal programming model with new weights. We get results with the help of the proposed method and gave comparison results with the weighted sum method. The outcomes are appearing in Table 6 and Table 7. We can use this technique to get numerous solution locations for RMOLFTP by setting weights. We can use this method to get a single solution point that reflects preferences assumed to have been incorporated to choose the single set of weights. LINGO 17.0 software is used to solve all the mathematical models. It can be used in all types of MOTP (RMOLFTP) with certain parameters. This method (the proposed method) can provide a non-priority compromise solution as well as a priority-based outcome. We are planning to diversify this technique to other types of RMOLFTPs and MOTPs.

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