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Study of thermoelasticity mathematical modeling via the Adomian decomposition method

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Abstract

This paper explores the mathematical modeling of thermoelastic stress in a perfectly clamped metallic rod using the Adomian Decomposition Method (ADM). The study focuses on formulating and solving the governing partial differential equations (PDEs) that describe the stress distribution within the rod when subjected to thermoelastic deformation. By employing ADM, we decompose these complex PDEs into a series of more manageable components, facilitating an analytical or semi-analytical solution without the need for linearization or perturbation techniques. This approach enhances computational efficiency while maintaining high accuracy in capturing the nonlinear behavior of thermoplastic stress. Furthermore, we examine the broader applicability of ADM in solving PDEs across various engineering and physical sciences domains, emphasizing its effectiveness in handling nonlinear problems and improving solution precision.

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Keywords: *Mathematical modeling, Thermoelasticity, Adomian decomposition methods.*

1. Introduction

The thermoelastic effect, where temperature shifts arise from material expansion and compression, has been recognized for over a hundred years. Despite this long history, the interplay between mechanical strain and heat remains an active research area. Temperature changes induce thermal effects in materials, see [1]. used by heat is one of the things that is produced when there is more strain placed on a material than it can hold. An increase in energy or temperature contributes to an increase in the vibration of the atom and molecules in the substance. The material expands, and in the absence of heat, will reduce in size or become smaller. Thermoelastic stress analysis is a valuable tool employed

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by engineers and scientists to address practical engineering challenges related to structural and material design, see [2]. Most solids undergo volumetric changes when subjected to temperature variations. Typically, temperature distribution (TD) within a solid induces stresses due to boundary or internal constraints. If the temperature variation is significant, these stresses can reach levels that may result in structural failure. Understanding thermal stress analysis is crucial for addressing numerous engineering challenges, particularly those involving substantial temperature variations. The authors in the cited paper [3] presented a novel approach to studying the complex behavior of forced Reiner-Rivlin non-Newtonian fluids under the influence of magnetic fields (Magnetohydrodynamic or MHD flow). Al-Lehaibi [4] investigated the application of a power-law model, combined with ADM, to analyze heat transfer and thermal stresses (TSs) in functionally graded thermoelastic materials. The work presented in [5] explored an iterative numerical approach to solve one-dimensional coupled thermoelasticity problems subject to specified boundary conditions. The modified Adomian decomposition method (MADM) is an advanced version of the ADM, which attempts to improve the effectiveness and precision with which many linear and nonlinear partial differentials equations are solved, see [6]. In [7], the authors conducted a comparative analysis of the variational iteration method (VIM) and the ADM for solving non-linear differential systems encountered in engineering applications. In [8] authors solved the two-dimensional coupled Burgers' system by modified VIM with Genetic Algorithm. Mak, Leung, and Harko [9] offer a comprehensive and accessible introduction to the ADM, along with illustrative examples of its applications. Recent advancements in thermoelasticity include generalized theories for transient TSs, fractional models for anomalous heat conduction, and the Homotopy analysis method (HAM) for nonlinear problems. Numerical improvements such as the smoothed finite element method (S-FEM) enhance stability, while machine learning integration (e.g., I-FENN) accelerates solutions. These developments improve the accuracy and efficiency of modeling thermo-mechanical behaviors in materials [10, 11], and more research examples [12-15]. Adel, Khader, and Assiri [16] analyzed the analytical solution of Burgers' equations using the MADM and the Homotopy Perturbation Transform Method (AHPMTM), comparing the convergence of both methods. In [17], the authors investigated PDEs and solved them numerically using a combination of the Mohand transform and the ADM. The study analyzes the flow of an MHD nanofluid over an extend surface, considering boundary layer effects and various influencing parameters and governing equations are transformed into ODEs and solved using ADM, with an error limit and convergence analysis conducted, see [18] Finite element analysis (FEA) and the ADM are both techniques used for solving differential equations, but they differ significantly in their approach, computational efficiency, and accuracy. The accuracy of FEA depends on the mesh density and the type of elements used, whereas the accuracy of ADM depends on how well its polynomials approximate the actual nonlinear behavior. The rate of convergence of the ADM depends on the problem and the quantity of terms kept in the series. The ADM converges quickly for smooth problems but struggles with stiff or highly nonlinear cases, while FDM and FEA require finer discretization for better convergence at a higher computational cost, and VIM often outperforms ADM in certain nonlinear problems due to the flexibility of Lagrange multipliers. Recently, Adel et al. [19] studied noninstantaneous impulsive non-linear Hilfer fractional stochastic systems of Sobolev-type. In general, to comprehend and elucidate these phenomena, the mathematical models need to be addressed through different numerical methods, including integral transformations; we direct readers to these articles [20-22].

Several noteworthy contributions further demonstrate the efficiency of ADM and its variants in solving complex physical models. For instance, Alkarawi and Al-Saiq [23] successfully applied the Adomian decomposition method to the Klein–Gordon and nonlinear wave equations, highlighting its robustness in handling nonlinearities. Mohammed and Altaie [24] introduced a coupled Elzaki–Adomian decomposition approach to obtain analytical solutions for engineering applications, showing improved accuracy when combined with integral transforms. Similarly, Talankar, Jadhav, and Muneshwar [25] employed the Laplace transform–Adomian decomposition method to address initial boundary value problems, demonstrating its capability to efficiently solve PDEs arising in applied sciences. These recent advancements further emphasise the relevance of ADM and its coupled forms in thermoelasticity, motivating the present study on thermoelastic stress analysis in metallic rods.

In [26, 27], authors investigated the predator-prey systems and blood ethanol concentration problem by using the VIM in Caputo fractional order sense. Abouelregal et al. [28] investigated a new thermoelastic framework that includes fractional derivatives of a two-temperature approach and dual-phase heat transfer.

Inspired by the previously cited work on the ADM, This work examines the thermoelastic stress response of a fully constrained metallic rod with a length l. The metal rod is exposed to a specified temperature T_2 at the end of the rod where $\tilde{x} = l$. The starting end of the rod at $\tilde{x} = 0$ is thermally insulated. At first, the metal rod has a constant temperature T_1 that is greater than zero, as shown in Figure 1, see [29, 30] for more detail.

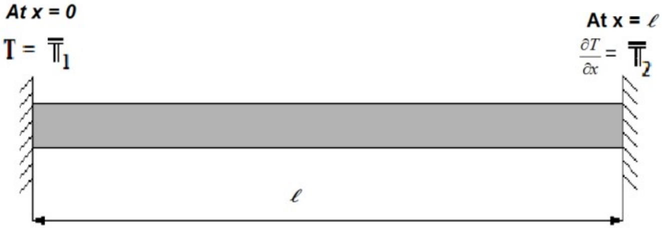

Figure 1
A diagram illustrating the shape and coordinates of the metal rod.

The investigation focuses on determining the TD along the rod and the resulting TSs for materials such as copper, aluminum, and steel. The heat conduction problem is established using the ADM [31], which provides a series-based analytical representation of the TD. The derived solutions for both the temperature profile and the TSs are presented in analytical form. Additionally, the graphical visualization of the solutions to the initial boundary value problems is generated using MATLAB, with T_o as the initial temperature. The thermoelastic stress behavior of a fully clamped metallic rod is examined here using the ADM, a powerful analytical tool.

This document is organized in the following manner: Section 2 offers a comprehensive description of the ADM. Section 3 discusses the thermoelastic stress assessment of a fixed metallic rod with no heat production, while Section 4 examines the case with heat generation. In Section 5, the developed theory is applied to a real engineering problem. Finally, the last section presents concluding remarks.

2. Analysis of the ADM

This part presents the decomposition method as applied to PDEs of the formula:

$$\mathfrak{L}_{\tilde{t}}\mathfrak{U}(\tilde{x},\tilde{t}) + \mathfrak{L}_{\tilde{x}\tilde{x}}\mathfrak{U}(\tilde{x},\tilde{t}) = g(\tilde{x},\tilde{t}), \tilde{t} > 0, \tag{2.1}$$

where $\mathfrak{L}_{\tilde{t}} = \frac{\partial}{\partial \tilde{t}}$ is a linear partial derivative of the first order, and $\mathfrak{L}_{\tilde{x}\tilde{x}} = \frac{\partial^2}{\partial \tilde{x}^2}$ is a linear partial derivative of the second order with spatial variables which might includes other derivatives of lower order. Also, we refer to the above equation as Heat Conduction Equation. Operating the inverse operator $\mathfrak{L}^{-1} = \int_0^{\tilde{t}} (.)d\tilde{t}$, to each side of Eq. (2.1) implies that

$$\mathfrak{U}(\tilde{x},\tilde{t}) = \mathfrak{U}(\tilde{x},0) + \mathfrak{L}_{\tilde{t}}^{-1}g(\tilde{x},\tilde{t}) - \mathfrak{L}_{\tilde{t}}^{-1}[\mathfrak{L}_{\tilde{x}\tilde{x}}\mathfrak{U}(\tilde{x},\tilde{t})]. \tag{2.2}$$

As described in [32-34], the ADM breaks down the solution $\mathfrak{U}(\tilde{x},\tilde{t})$ into an endless series

$$\mathfrak{U}(\tilde{x},\tilde{t}) = \sum_{n=0}^{\infty} \mathfrak{U}_n(\tilde{x},\tilde{t}). \tag{2.3}$$

If we substitute the decomposition series (2.3) into each side of Eq. (2.2), one has

$$\sum_{n=0}^{\infty} \mathfrak{U}_n(\tilde{x},\tilde{t}) = \mathfrak{U}(\tilde{x},0) + \mathfrak{L}_{\tilde{t}}^{-1}g(\tilde{x},\tilde{t}) - \mathfrak{L}_{\tilde{t}}^{-1}[\mathfrak{L}_{\tilde{x}\tilde{x}}(\sum_{n=0}^{\infty} \mathfrak{U}_n(\tilde{x},\tilde{t}))], \tag{2.4}$$

which yields to the following recursive scheme for determining the iterates:

$$\mathfrak{U}_0(\tilde{x},\tilde{t}) = \mathfrak{U}(\tilde{x},0) + \mathfrak{L}_{\tilde{t}}^{-1}g(\tilde{x},\tilde{t}),$$
$$\mathfrak{U}_1(\tilde{x},\tilde{t}) = -\mathfrak{L}_{\tilde{t}}^{-1}(\mathfrak{L}_{\tilde{x}\tilde{x}}\mathfrak{U}_0),$$
$$\mathfrak{U}_2(\tilde{x},\tilde{t}) = -\mathfrak{L}_{\tilde{t}}^{-1}(\mathfrak{L}_{\tilde{x}\tilde{x}}\mathfrak{U}_1) \text{ and so on.}$$

As a final step, the solution $\mathfrak{U}(\tilde{x},\tilde{t})$ is approximated by the finite series.

$$\psi_N(\tilde{x},\tilde{t}) = \sum_{n=0}^{N-1} \mathfrak{U}_n(\tilde{x},\tilde{t}), \tag{2.5}$$

and

$$\lim_{N\to\infty} \psi_N(\tilde{x},\tilde{t}) = \mathfrak{U}(\tilde{x},\tilde{t}). \tag{2.6}$$

Although a closed-form solution is often possible, the decomposition series solutions usually converge quickly. Convergence is discussed at length by Cherruault in [35].

3. Thermoelastic Stress Examination of a Fixed Metallic Rod without Heat Production

Here, we analyze the TD and stress within a perfectly clamped metallic rod, assuming no internal heat generation

Proposition 3.1: Consider the TD in a clamped metallic rod of length l. The both ends $\tilde{x} = 1$ and $\tilde{x} = 0$ has stay at fixed temperature $\mathcal{T}_1$ and $\mathcal{T}_2$ respectively. At first, the metal rod is at a constant temperature $\mathcal{T}_0$ that is not zero. Find the mathematical expression for TD $T(\tilde{x}, \tilde{t})$, and discuss the stress analysis σ.

Formation of Mathematical Problem: The temperature of the clamped metallic rod for all time $\tilde{t} > 0$ Fig. 1, is gifted by the heat equation

$$\frac{\partial T}{\partial \tilde{t}} = c^2 \frac{\partial^2 T}{\partial \tilde{x}^2}, \tilde{t} > 0, 0 < \tilde{x} < l, \tag{3.1}$$

under initial status

$$T(\tilde{x}, 0) = \mathcal{T}_0, \tag{3.2}$$

and boundary status

$$T(0, \tilde{t}) = \mathcal{T}_1, \tag{3.3}$$

$$\frac{\partial T}{\partial \tilde{x}}(l, \tilde{t}) = \mathcal{T}_2. \tag{3.4}$$

It is initial boundary value problem (IBVP) for heat conduction through metallic rod. The ADM is developed to determine TD in a clamped metallic rod under given conditions. The heat conduction equation (3.1) can be written in operator form

$$\mathfrak{L}_{\tilde{t}}(T) = c^2 \mathfrak{L}_{\tilde{x}\tilde{x}}(T), \tag{3.5}$$

where $\mathfrak{L}_{\tilde{t}} = \frac{\partial}{\partial \tilde{t}}$ and $\mathfrak{L}_{\tilde{x}\tilde{x}} = \frac{\partial^2}{\partial \tilde{x}^2}$ are partial differential operators.

We look for series solution of IBVP (3.1)−(3.4),

$$T(\tilde{x}, \tilde{t}) = \sum_{n=0}^{\infty} T_n(\tilde{x}, \tilde{t}) = T_0 + T_1 + T_2 + T_3 + \ldots. \tag{3.6}$$

Operating the inverse $\mathfrak{L}_{\tilde{t}}^{-1} = \int_0^{\tilde{t}} (.) d\tilde{t}$ on each side of the Eq. (3.5) and utilizing the initial status, one finds

$$T(\tilde{x}, \tilde{t}) - T(\tilde{x}, 0) = c^2 [\mathfrak{L}_{\tilde{t}}^{-1}(\mathfrak{L}_{\tilde{x}\tilde{x}}(T))]. \tag{3.7}$$

$$T_0 + T_1 + T_2 + \ldots = \mathcal{T}_0 + c^2 \mathfrak{L}_{\tilde{t}}^{-1}[\mathfrak{L}_{\tilde{x}\tilde{x}}(T_0) + \mathfrak{L}_{\tilde{x}\tilde{x}}(T_1) + \mathfrak{L}_{\tilde{x}\tilde{x}}(T_2) + \ldots]. \tag{3.8}$$

Now, we compare LHS and RHS terms by term and also proceed to compute $T_0, T_1, T_2, T_3, \ldots$ with the help of following recursive scheme

$$\begin{aligned} T_0 &= \mathcal{T}_0, \\ T_1 &= c^2 \mathfrak{L}_{\tilde{t}}^{-1} \mathfrak{L}_{\tilde{x}\tilde{x}}(\mathcal{T}_0), \\ T_2 &= c^2 \mathfrak{L}_{\tilde{t}}^{-1} \mathfrak{L}_{\tilde{x}\tilde{x}}(T_1), \\ T_3 &= c^2 \mathfrak{L}_{\tilde{t}}^{-1} \mathfrak{L}_{\tilde{x}\tilde{x}}(T_2), \text{ and so on.} \end{aligned} \tag{3.9}$$

We find $T_1, T_2, T_3, \ldots$

$$\begin{aligned} T_1 &= c^2 \mathfrak{L}_{\tilde{t}}^{-1} \mathfrak{L}_{\tilde{x}\tilde{x}}(\mathcal{T}_0) = c^2 \int_0^{\tilde{t}} \mathfrak{L}_{\tilde{x}\tilde{x}}(\mathcal{T}_0) d\tilde{t}, \\ T_2 &= c^2 \int_0^{\tilde{t}} \mathfrak{L}_{\tilde{x}\tilde{x}}(T_1) d\tilde{t}, \\ T_3 &= c^2 \int_0^{\tilde{t}} \mathfrak{L}_{\tilde{x}\tilde{x}}(T_2) d\tilde{t}, \text{and so on.} \end{aligned} \tag{3.10}$$

Substitute $T_0, T_1, T_2, T_3, \ldots$ in equation (3.6), we obtain

$$T(\tilde{x}, \tilde{t}) = \mathcal{T}_0 + c^2 \int_0^{\tilde{t}} \mathfrak{L}_{\tilde{x}\tilde{x}}(\mathcal{T}_0) d\tilde{t} + c^2 \int_0^{\tilde{t}} \mathfrak{L}_{\tilde{x}\tilde{x}}(T_1) d\tilde{t} + c^2 \int_0^{\tilde{t}} \mathfrak{L}_{\tilde{x}\tilde{x}}(T_2) d\tilde{t} + \ldots. \tag{3.11}$$

This gives the TD in the clamped metallic rod under the initial boundary status.

Stress Analysis: Now, we discuss stress analysis of clamped metallic rod which It is presented as

$$\sigma = \alpha E \int_0^l T(\tilde{x}, \tilde{t}) d\tilde{t}, \tag{3.12}$$

where $\alpha =$ signifies the metallic coefficient of thermal expansion, and $E =$ denotes Young's modulus.

4. Thermoelastic Stress Examination of a Fixed Metallic Rod with Heat Production

In this section, we study TD and stress examination of perfectly fixed metallic rod with heat production.

Proposition 4.1: Consider the TD in a clamped metallic rod of length l with heat generation. The both ends $\tilde{x} = 1$ and $\tilde{x} = 0$ has stay at fixed temperature $\mathcal{T}_1$ and $\mathcal{T}_2$ respectively. At first, the metallic rod be a non-zero at temperature $\mathcal{T}_0$. Find the mathematical expression for TD $T(\tilde{x}, \tilde{t})$, and discuss the stress analysis σ.

Formation of mathematical problem: If the temperature of the clamped metallic rod with internal heat source for all time $\tilde{t} > 0$ Fig. 1, is gifted by heat Eq. [36], then

$$\frac{\partial T}{\partial \tilde{t}} = c^2 \frac{\partial^2 T}{\partial \tilde{x}^2} + \frac{g}{k} \; 0 < \tilde{x} < l, \tilde{t} > 0, \tag{4.1}$$

under initial status

$$T(\tilde{x}, 0) = \mathcal{T}_0, \tag{4.2}$$

and boundary statuses

$$T(0, \tilde{t}) = \mathcal{T}_1. \tag{4.3}$$

$$\frac{\partial T}{\partial \tilde{x}}(l, \tilde{t}) = \mathcal{T}_2. \tag{4.4}$$

It is initial boundary value problem (IBVP). The ADM is developed to determine TD of clamped metallic rod with internal heat source under given conditions. The heat conduction equation (4.1) can be expressed in operator format

$$\mathfrak{L}_{\tilde{t}}(T) = c^2 \mathfrak{L}_{\tilde{x}\tilde{x}}(T) + \frac{g}{k}, \tag{4.5}$$

where $\mathfrak{L}_{\tilde{t}} = \frac{\partial}{\partial \tilde{t}}$ and $\mathfrak{L}_{\tilde{x}\tilde{x}} = \frac{\partial^2}{\partial \tilde{x}^2}$ are partial differential operators. We look for series solution of IBVP (4.1)−(4.4)

$$T(\tilde{x}, \tilde{t}) = \sum_{n=0}^{\infty} T_n(\tilde{x}, \tilde{t}) = T_0 + T_1 + T_2 + T_3 + \ldots. \tag{4.6}$$

Operating the inverse $\mathfrak{L}_{\tilde{t}}^{-1} = \int_0^{\tilde{t}} (.) d\tilde{t}$ on each side of the Eq. (4.5) and by utilizing initial status, one obtains

$$T(\tilde{x}, \tilde{t}) - T(\tilde{x}, 0) = c^2 [\mathfrak{L}_{\tilde{t}}^{-1}(\mathfrak{L}_{\tilde{x}\tilde{x}}(T))] + \mathfrak{L}_{\tilde{t}}^{-1} \frac{g}{k}. \tag{4.7}$$

$$T_0 + T_1 + T_2 + \ldots = \mathcal{T}_0 + \frac{g}{k}\tilde{t} + c^2 \mathfrak{L}_{\tilde{t}}^{-1} [\mathfrak{L}_{\tilde{x}\tilde{x}}(T_0) + \mathfrak{L}_{\tilde{x}\tilde{x}}(T_1) + \mathfrak{L}_{\tilde{x}\tilde{x}}(T_2) + \ldots]. \tag{4.8}$$

Now, we compare LHS and RHS terms by terms and also proceed to compute $T_0, T_1, T_2, T_3, \ldots$ with the help of following recursive scheme

$$\begin{aligned} T_0 &= \mathcal{T}_0 + \frac{g}{k}\tilde{t}, \\ T_1 &= c^2 \mathfrak{L}_{\tilde{t}}^{-1} \mathfrak{L}_{\tilde{x}\tilde{x}}(T_0), \\ T_2 &= c^2 \mathfrak{L}_{\tilde{t}}^{-1} \mathfrak{L}_{\tilde{x}\tilde{x}}(T_1). \\ T_3 &= c^2 \mathfrak{L}_{\tilde{t}}^{-1} \mathfrak{L}_{\tilde{x}\tilde{x}}(T_2), \text{ and so on.} \end{aligned} \tag{4.9}$$

We find $T_1, T_2, T_3, \ldots$

$$\begin{aligned} T_1 &= c^2 \mathfrak{L}_{\tilde{t}}^{-1} \mathfrak{L}_{\tilde{x}\tilde{x}}(T_0) = c^2 \int_0^{\tilde{t}} \mathfrak{L}_{\tilde{x}\tilde{x}}(T_0) d\tilde{t} = c^2 \int_0^{\tilde{t}} \mathfrak{L}_{\tilde{x}\tilde{x}}[\mathcal{T}_0 + \tfrac{g}{k}\tilde{t}] d\tilde{t} \\ T_2 &= c^2 \int_0^{\tilde{t}} \mathfrak{L}_{\tilde{x}\tilde{x}}(T_1) d\tilde{t} \\ T_3 &= c^2 \int_0^{\tilde{t}} \mathfrak{L}_{\tilde{x}\tilde{x}}(T_2) d\tilde{t}, \text{and so on.} \end{aligned} \tag{4.10}$$

Substitute $T_0, T_1, T_2, T_3, \ldots$ in equation (4.6), we have

$$T(\tilde{x}, \tilde{t}) = \mathcal{T}_0 + c^2 \int_0^{\tilde{t}} \mathfrak{L}_{\tilde{x}\tilde{x}}(T_0) d\tilde{t} + c^2 \int_0^{\tilde{t}} \mathfrak{L}_{\tilde{x}\tilde{x}}(T_1) d\tilde{t} + c^2 \int_0^{\tilde{t}} \mathfrak{L}_{\tilde{x}\tilde{x}}(T_2) d\tilde{t} + \ldots. \tag{4.11}$$

It is the TD in the clamped metallic rod under given initial and boundary conditions.

Stress Analysis: Now, we discuss stress analysis of clamped metallic rod, which is presented as

$$\sigma = \alpha E \int_0^{l} T(\tilde{x}, \tilde{t}) d\tilde{t}, \tag{4.12}$$

where α is the thermal expansion coefficient of metal and E is the Young's modulus.

Remark 4.1: If source of heat generation g is zero then IBVP, (4.1)−(4.4), reduces to the IBVP (3.1)−(3.4) which is also solved by Laplace integral transform technique [29] by Ghugal Y. M. and Gandhe G. R.

5. Applications

This section applies theory developed in the earlier section to real engineering problem of practical interest as follows. The material properties for steel, copper and aluminium are used.

Example 5.1: The temperature of the clamped metallic rod with internal constant heat source for all time $\tilde{t} > 0$, is gifted by heat Eq.

$$\frac{\partial T}{\partial \tilde{t}} = c^2 \frac{\partial^2 T}{\partial \tilde{x}^2} + \frac{g}{k} \; 0 < \tilde{x} < \pi, \tilde{t} > 0. \tag{5.1}$$

with initial status

$$T(\tilde{x}, 0) = \sin(\tilde{x}). \tag{5.2}$$

and boundary status

$$T(0, \tilde{t}) = \frac{g}{k}[(c^2(e^{\tilde{t}} - \tilde{t} - 1) + \tilde{t})]. \tag{5.3}$$

For the other end $(\tilde{x} = \pi)$ of the rod,

$$\frac{\partial T}{\partial \tilde{x}}|_{\tilde{x}=\pi} = c^2(1 - e^{-\tilde{t}}) - 1, \tag{5.4}$$

where c^2 is the thermal diffusivity which is treated as constant, k is the thermal conductivity which is treated as constant and g is the heat generation, then find the TD and stress analysis.

The ADM is developed to determine TD of clamped metallic rod with internal heat source under given conditions. The heat conduction equation (5.1) can be expressed in operator format

$$\mathfrak{L}_{\tilde{t}}(T) = c^2 \mathfrak{L}_{\tilde{x}\tilde{x}}(T) + \frac{g}{k}, \tag{5.5}$$

where $\mathfrak{L}_{\tilde{t}} = \frac{\partial}{\partial \tilde{t}}$ and $\mathfrak{L}_{\tilde{x}\tilde{x}} = \frac{\partial^2}{\partial \tilde{x}^2}$ are partial differential operators. We look for series solution of IBVP (5.1)−(5.4)

$$T(\tilde{x}, \tilde{t}) = \sum_{n=0}^{\infty} T_n(\tilde{x}, \tilde{t}) = T_0 + T_1 + T_2 + T_3 + \ldots \tag{5.6}$$

By operating the inverse $\mathfrak{L}_{\tilde{t}}^{-1} = \int_0^{\tilde{t}} (.) d\tilde{t}$ on each side of the Eq. (5.5) and by utilizing initial status, one finds

$$T(\tilde{x}, \tilde{t}) - T(\tilde{x}, 0) = c^2[\mathfrak{L}_{\tilde{t}}^{-1}(\mathfrak{L}_{\tilde{x}\tilde{x}}(T))] + \mathfrak{L}_{\tilde{t}}^{-1} \frac{g}{k}.$$

$$T_0 + T_1 + T_2 + \ldots = \sin(\tilde{x}) + \frac{g}{k}\tilde{t} + c^2 \mathfrak{L}_{\tilde{t}}^{-1}[\mathfrak{L}_{\tilde{x}\tilde{x}}(T_0) + \mathfrak{L}_{\tilde{x}\tilde{x}}(T_1) + \mathfrak{L}_{\tilde{x}\tilde{x}}(T_2) + \ldots].$$

Now, we compare LHS and RHS terms by terms and proceed to compute $T_0, T_1, T_2, T_3, \ldots$ with the help of recursive scheme

$T_0 = \sin(\tilde{x}) + \frac{g}{k}\tilde{t}$,

$T_1 = c^2\mathfrak{L}_{\tilde{t}}^{-1}\mathfrak{L}_{\tilde{x}\tilde{x}}(T_0) = c^2\int_0^{\tilde{t}} \mathfrak{L}_{\tilde{x}\tilde{x}}[\sin(\tilde{x}) + \frac{g}{k}\tilde{t}]d\tilde{t} = -c^2\tilde{t}\sin(\tilde{x}) + c^2\frac{g}{k}\frac{\tilde{t}^2}{2!}$,

$T_2 = c^2\mathfrak{L}_{\tilde{t}}^{-1}\mathfrak{L}_{\tilde{x}\tilde{x}}(T_1) = c^2\frac{\tilde{t}^2}{2!}\sin(\tilde{x}) + c^2\frac{g}{k}\frac{\tilde{t}^3}{3!}$,

$T_3 = c^2\mathfrak{L}_{\tilde{t}}^{-1}\mathfrak{L}_{\tilde{x}\tilde{x}}(T_2) = -c^2\frac{\tilde{t}^3}{3!}\sin(\tilde{x}) + c^2\frac{g}{k}\frac{\tilde{t}^4}{4!}$, and so on.

Substitute $T_0, T_1, T_2, T_3, \ldots$ in Eq. (5.6), we have

$$\begin{aligned}
T(\tilde{x},\tilde{t}) &= \sin(\tilde{x}) + \frac{g}{k}\tilde{t} - c^2\tilde{t}\sin(\tilde{x}) + c^2\frac{g}{k}\frac{\tilde{t}^2}{2!} + c^2\frac{\tilde{t}^2}{2!}\sin(\tilde{x}) + c^2\frac{g}{k}\frac{\tilde{t}^3}{3!} - \\
&\quad c^2\frac{\tilde{t}^3}{3!}\sin(\tilde{x}) + c^2\frac{g}{k}\frac{\tilde{t}^4}{4!} + \ldots \\
&= \sin(\tilde{x}) - c^2\tilde{t}\sin(\tilde{x}) + c^2\frac{\tilde{t}^2}{2!}\sin(\tilde{x}) - c^2\frac{\tilde{t}^3}{3!}\sin(\tilde{x}) + \ldots + \frac{g}{k}\tilde{t} + \\
&\quad c^2\frac{g}{k}\frac{\tilde{t}^2}{2!} + c^2\frac{g}{k}\frac{\tilde{t}^3}{3!} + c^2\frac{g}{k}\frac{\tilde{t}^4}{4!} + \ldots \\
&= \sin(\tilde{x}) + c^2\sin(\tilde{x})[(1 - \tilde{t} + \frac{\tilde{t}^2}{2!} - \frac{\tilde{t}^3}{3!} + \ldots) - 1] + \frac{g}{k}\tilde{t} + \\
&\quad c^2\frac{g}{k}[(1 + \tilde{t} + \frac{\tilde{t}^2}{2!} + \frac{\tilde{t}^3}{3!} + \frac{\tilde{t}^4}{4!} + \ldots) - 1 - \tilde{t}] \\
&= \sin(\tilde{x}) + c^2\sin(\tilde{x})(e^{-\tilde{t}} - 1) + \frac{g}{k}\tilde{t} + \\
&\quad c^2\frac{g}{k}[((1 + \tilde{t} + \frac{\tilde{t}^2}{2!} + \frac{\tilde{t}^3}{3!} + \frac{\tilde{t}^4}{4!} + \ldots) - 1 - \tilde{t})] \\
&= \sin(\tilde{x})[1 + c^2(e^{-\tilde{t}} - 1)] + \frac{g}{k}\tilde{t} + c^2\frac{g}{k}[(e^{\tilde{t}} - 1 - \tilde{t})] \\
T(\tilde{x},\tilde{t}) &= \sin(\tilde{x})[c^2(e^{-\tilde{t}} - 1) + 1] + \frac{g}{k}[(c^2(e^{\tilde{t}} - 1 - \tilde{t}) + \tilde{t})].
\end{aligned}$$

It is the TD in the clamped metallic rod under given initial and boundary conditions.

Stress Analysis: Now we discuss stress analysis of above example. It is given by

$$\begin{aligned}
\sigma &= \alpha E\int_0^{\pi} T(\tilde{x},\tilde{t})d\tilde{x} \\
&= \alpha E\int_0^{\pi} [\sin(\tilde{x})[c^2(e^{-\tilde{t}} - 1) + 1] + \frac{g}{k}[(c^2(e^{\tilde{t}} - \tilde{t} - 1) + \tilde{t})]]d\tilde{x} \\
\sigma &= \alpha E\{2[c^2(e^{-\tilde{t}} - 1) + 1] + \frac{g\pi}{k}[c^2(e^{\tilde{t}} - \tilde{t} - 1) + \tilde{t}]\}.
\end{aligned}$$

Material Properties: The material properties for steel, copper and aluminium which are used for solving test problem are as given below.

5.1 *Regarding Steel*

1. Elastic modulus $E = 2 \times 10^5\ N/mm^2$.
2. Thermal expansion coefficient $\alpha = 13 \times 10^{-6}\ /^0c$.
3. Thermal diffusivity $c^2 = 14.74 \times 10^{-6}\ m^2/s$.

Remark 5.1: Figure 2 shows that, the temperature increases gradually from initial edge towards the edge in $\tilde{x}$ direction for steel. This rise in temperature gives increase in strains of (steel) material which in turn increases the stress in the material. This increase in stress are observed by the graphs plotted in Figure 3.

In present work, the attempt has made for the thermoelastic stress examination of perfectly fixed metallic steel rod subject to constant heat source. The ADM is first time successfully applied and analytical solution of stress distribution and TD within clamped metallic steel rod is obtained.

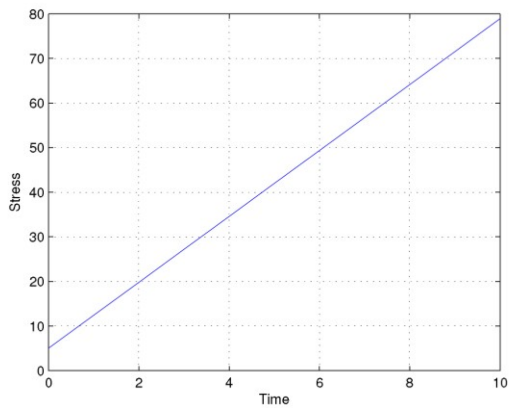

Figure 2
Variation of temperature with distance $\tilde{x}$ for steel rod

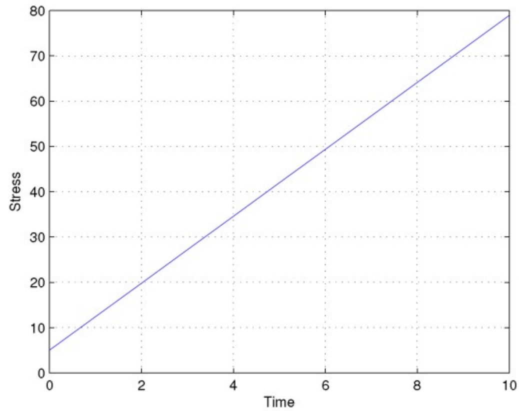

Figure 3
Variation of temperature stress with time for steel rod

5.2 *Regarding Copper*

1. Elastic modulus $E = 1.17 \times 10^5\ N/mm^2$.
2. Thermal expansion coefficient $\alpha = 17 \times 10^{-6}\ /^0c$.
3. Thermal diffusivity $c^2 = 113 \times 10^{-6}\ m^2/s$.

Remark 5.2: Figure 4 shows that, the temperature increases gradually from initial edge towards the edge in $\tilde{x}$ direction for copper. This rise in temperature gives increase in strains of (copper) material which in turn increases the stress in the material. This increase in stress are observed by the graphs plotted in Figure 5.

In the present work, an attempt has been made for the thermoelastic stress examination of a perfectly fixed metallic copper rod subject to a constant heat source. The ADM is the first time successfully applied and an analytical solution of stress distribution and TD within a clamped metallic copper rod is obtained.

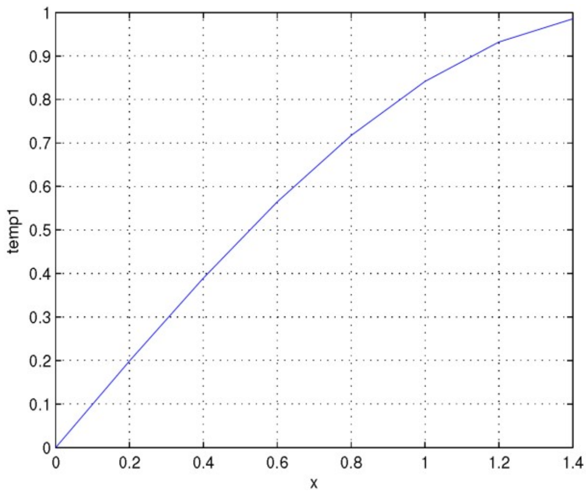

Figure 4
Variation of temperature with distance $\tilde{x}$ for copper rod

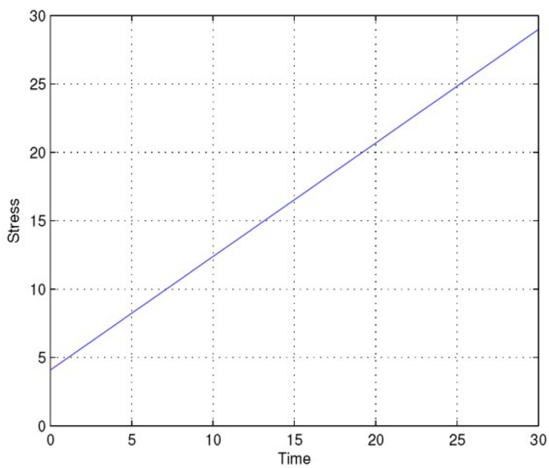

Figure 5
Variation of temperature stress with time for copper rod

5.3 *Regarding Aluminium*

1. Elastic modulus $E = 68.9 \times 10^3 \ N/mm^2$.
2. Thermal expansion coefficient $\alpha = 24 \times 10^{-6} \ /^0c$.
3. Thermal diffusivity $c^2 = 97.50 \times 10^{-6} \ m^2/s$.

Remark 5.3: Figure 6 shows that, temperature increases gradually from initial edge towards the edge in x̃ direction for aluminium. This rise in temperature gives increase in strains of (aluminium) material which in turn increases the stress in the material. This increase in stress are observed by the graphs plotted in Figure 7.

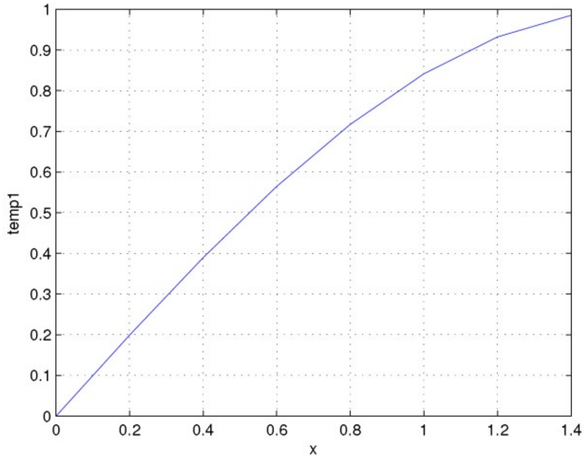

Figure 6
Variation of temperature with distance x̃ for Aluminimum rod

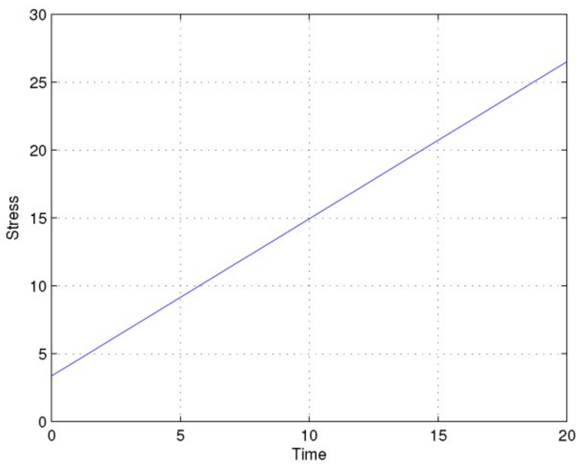

Figure 7
Variation of temperature stress with time for Aluminium rod

In the present work, an attempt has been made for the thermoelastic stress examination of a completely fixed metallic aluminum rod subject to a constant heat source. The ADM method is the first time successfully applied and an analytical solution of TD and stress distribution within clamped metallic aluminum rod is obtained.

6. Conclusions

This paper applies the Adomian decomposition method to analyze heat conduction in metallic rods (steel, copper, and aluminum) under suitable initial and boundary conditions, discussing TD and stress. The developed theory in this paper is illustrated with a suitable test problem. The graphical representation of the solution is given using MATLAB. This kind of study has many practical applications in industry.

Conflict of interest

The authors declare no competing interests.

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No finding.

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