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# Optimal design and adequacy of a pension system : Case study of Kazakhstan

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**Abstract**

Kazakhstan's pension system evolved from a defined benefit pay-as-you-go model to a hybrid framework incorporating both defined contribution and notional defined contribution elements. Despite this structural transition, substantial policy challenges persisted regarding the optimal pension contribution rate and the appropriate allocation between funded and unfunded pillars. The study developed an analytical framework utilising a mean-variance optimisation approach to determine the ex-ante optimal contribution rate and its distribution between defined contribution and notional defined contribution schemes. The model integrated stochastic processes for financial markets, non-financial returns, and wage trajectories, along with diverse career paths and varying contribution densities. Computer simulations, employing Kazakhstan-specific demographic and economic data, identified optimal pension configurations that maximised the expected utility of pension assets while satisfying adequacy criteria. Based on the results obtained, aligning notional returns positively with wage growth and inversely with financial returns is vital for achieving diversification benefits and sufficient growth in pension capital over time. Ensuring Kazakhstan's pension adequacy while easing fiscal pressures requires diversifying defined contribution assets, expanding formal participation among workers, and incentivizing consistent contributions. The findings provided an evidence-based foundation for pension policy formulation, applicable also to other middle-income countries undergoing substantial reforms of their retirement systems.

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## 1. Introduction

The pension system of the Republic of Kazakhstan has undergone significant transformations since the country's independence in 1991. Initially, Kazakhstan operated a public pay-as-you-go (PAYG) defined benefit (DB) system inherited from the Soviet Union, which was ill-suited to the economic and demographic realities of the 1990s post-Soviet market economy. Economic instability, business closings, rising unemployment, and demographic problems like falling birth rates and large numbers of people leaving the country showed that the system was seriously flawed and called for major changes. In response to these challenges, Kazakhstan transitioned in 1998 to a fully funded mandatory pension system based on defined contributions (DC), requiring employees to contribute 10 percent of their income and paving the way for the emergence of private pension funds in the market. The change ceased the accumulation of new DB entitlements and signified a transition towards individual accountability for retirement savings [1].

Further, in 2013, the Unified Accumulative Pension Fund (UAPF) was established as the sole administrator and operator of the funded pension system, consolidating all pension assets into a unified investment portfolio managed by the National Bank of the Republic of Kazakhstan (NBRK) as the default manager. At the same time, since 2023, contributors have been given the opportunity to transfer up to 50% of their pension funds to private asset management firms, which offer a range of investment strategies that may better align with individual risk preferences. Despite this opportunity, participation remains minimal, with only 0.2% of total assets managed privately, indicating insufficient financial understanding and a lack of faith in alternative investment opportunities [2-4].

In 2024, the government strengthened the second pillar by introducing a notional defined contributions (NDC) system with redistributive PAYG elements. This reform mandated an employer contribution of 1.5% for employees born in 1975 and later, with a phased increase to 5% over time. Despite these reforms, Kazakhstan continues to face significant challenges in defining the optimal structure of its pension system. Key issues include setting an appropriate total pension contribution rate, achieving a balanced split between funded and unfunded components, and ensuring the adequacy of retirement income given the lower contribution densities and uncertainties in future investment performance. These challenges highlight a broader scientific problem: designing a pension system that ensures diversification benefits and sufficient growth in pension capital amid less mature economic conditions, particularly in the context of achieving a balance between the system's sustainability and adequacy. Kazakhstan's experience provides a valuable case study for analysing these issues, offering insights applicable to other emerging economies facing similar complexities.

In international practice, countries rarely use only one type of pension system and mostly have various combinations of classic PAYG and funded components. It contributes to ensuring the financial stability of the pension

system and makes it possible to increase the level of pension provision through the appropriate allocation of roles and responsibilities for it between the government, employee, and employer, as well as to mitigate the shortcomings of one component with the comparative strength of the other. The World Bank, having endorsed pension changes in more than 70 nations, constantly promotes diversified pension plans [5]. Indeed, extensive studies highlight a growing trend in developed nations to complement traditional PAYG systems with DC schemes or mandatory funded pillars to enhance individual responsibility for retirement savings [5]. This transition is seen as a necessary adaptation to the fiscal challenges posed by aging populations and the need for more flexible retirement savings options [6].

However, the experience of developing countries in transitioning to DC pension schemes has been mixed, as these reforms have frequently faced significant challenges rooted in institutional and socioeconomic constraints [7]. For instance, Khai et al. [8] conducted a comparative examination of Ghana, Pakistan, and Myanmar, highlighting that the implementation of DC pensions faces significant challenges due to limited government capacity, low coverage, and insufficient integration with private sector pension arrangements. These instances emphasize the imperative of customizing pension reforms to align with local institutional, economic, and social settings.

In Kazakhstan, similar issues are evident in the operation of the DC pension system. Pension savings can be significantly impacted by systemic inefficiencies, investment dynamics, and social factors, with economic instability playing a particularly strong role [9]. Additionally, the effectiveness of Kazakhstan’s new pension system is likely hindered by a labour market marked by a considerable gap between median and average wages, which negatively impacts the accumulation of pension savings [10]. This observation aligns with findings of Herrerias and Zamarripa [11], which suggest that the design of DC pension policies often fails to adequately account for a country’s labour market characteristics. However, pension reforms are delayed, meaning that any positive outcomes may only become apparent over an extended period. While international experience emphasizes the importance of multi-pillar pension systems, a key challenge lies in determining the optimal mix of components within the system, alongside other critical considerations.

Most of these studies examine pension dynamics in wealthy nations. Guigou, Lovat, and Schiltz [12] evaluate a diversified pension system funded by PAYG and capitalization in Luxembourg's unique socio-economic situation. Efficiency is measured by system longevity. In their portfolio analysis of PAYG integration with a self-funded pension system in Germany, Anders and Groneck [13] compare speculating versus hedging. Westerhout et al. [14] use developed country data to examine how a sustained decline in financial market returns, average economic growth rates, output volatility, and capital market volatility affect PAYG-funding system equilibrium. Boado-Penas et al. [15] study hybrid pension schemes in Europe's demographic and economic environment. A nonlinear optimisation approach is used to assess the impact of adopting an

obligatory funded defined contribution plan alongside a pay-as-you-go scheme on pension benefits. Alonso-García and Devolder [16] present research on optimal allocations between DC and NDC systems, utilizing portfolio theory and building on previous work by Devolder and Melis [17]. Their research compares the model's results to Sweden, Latvia, and Poland.

Theoretical studies that ascertain the optimal pension contribution rate are also significant in the investigation of pension system design. Corsini and Spataro [18] assert that the appropriate contribution rate depends on the appeal of the pension plan and the individual's risk aversion. They specifically illustrate that it is a rising function of wage when relative risk aversion diminishes, and conversely. Ibe and Nkeki [19] investigate the dynamics of time-variable contribution rates within a stochastic framework, facilitating adaptability to an individual's financial situation over the course of their career. This methodology is corroborated by L.'s work. Wang, Wang, and Xue [20] underscore the imperative of instituting a minimum-security system to guarantee sufficient retirement payments, hence complicating the assessment of appropriate contribution rates. Nevertheless, the predominant focus of studies is on assessing fixed contribution rates.

In practice, total pension contribution rates are often relatively high to ensure the sustainability of the system, provide adequate retirement income, and address the demographic challenges posed by aging populations. According to the latest OECD data, total mandatory pension contribution rates averaged 18.2% in 2022 among OECD member countries. The highest rates were recorded in Italy (33.0%), the Czech Republic (28.0%), France (27.8%), and Greece (26.0%). In Kazakhstan, following the successful implementation of the NDC system, the total contribution rate is planned to reach 15%. However, Junusbekova and Zhaumitova [10] argue that to achieve a replacement rate of 40%, as recommended by the International Labour Organization (ILO), the contribution rate in Kazakhstan should be increased to at least 17%. Despite the extensive studies outlined above, the combined analysis of optimal pension contribution rates and the balance between DC and NDC systems remains underutilized, especially in relation to ensuring the adequacy of pension systems. Notably, there is a research gap in examining overall pension design and adequacy within the context of emerging economies.

Building on the existing literature and leveraging portfolio theory arguments discussed earlier, this study aims to address this gap by providing new insights through an analysis of Kazakhstan as a case study. Specifically, it extends the framework developed by Alonso-García and Devolder [16] in terms of ex-ante case in several key ways. The study investigates flat optimal values for both the pension contribution rate and the allocation between unfunded and funded components, aligning with practical approaches in which these parameters are age-independent. Financial, non-financial, and wage dynamics are modelled as separate continuous-time stochastic processes, and the financial market model is expanded to allow investment in both a risk-free asset and a single risky asset. Furthermore, the study introduces variability by incorporating

heterogeneous wage profiles, as demonstrated in the work by Alonso-García and Devolder [21], and varying densities of contributions. The latter aspect holds significant importance because the full-career assumption may not align well with developing countries experiencing low rates of participation in pension systems [11]. To illustrate the proposed method, numerical results based on Kazakhstan's data are presented and assessed in terms of pension system adequacy.

## 2. Materials and Methods

To find the optimal pension contribution rate and allocation between DC and NDC systems, an approach within a mean-variance framework is employed. Initially, the efficient frontier is established, comprising mean-variance pairs of relative pension wealth, which minimize risk at specific levels of expectation. Subsequently, the optimal combination along this frontier is determined by maximizing the utility function. This process integrates considerations of risk preferences exhibited by the cohort under examination.

The cohort enters the workforce when aged $x_0$ at time $\tau = 0$ and works until reaching retirement age $x_0 + T$ at time $\tau = T$. The model accounts for career heterogeneity, recognizing that individuals follow different career paths or profiles. The proportion of individuals with a career type k when aged $x_0 + \tau$ at time $\tau$ is denoted by $\alpha(x_0 + \tau, \tau; k)$, where $\tau \in \{0,1, \dots, T-1\}$. It is assumed that there is a range of N different career paths. Each career path has its own average monthly wage $L_{x,k}(t)$, which varies depending on age $x \in \{x_0, x_0 + 1, \dots, x_0 + T - 1\}$ and time $t \in [0, T]$. At the same time, the extent of coverage by the pension system for individuals with a career type k when aged $x_0 + \tau$ at time $\tau$ is determined by such a variable as the annual frequency of pension contributions $d(x_0 + \tau, \tau; k)$.

In this regard, at each moment of time $\tau$, individuals make a total pension contribution from the wage at the fixed rate of $\pi$ in accordance with the specified contribution frequency. Then a fixed share of this contribution in the amount of $\theta$ is allocated to the DC system and the remaining part $1 - \theta$ to the NDC system.

We shall set the admissible range of values for the total pension contribution rate $\pi$ and allocation $\theta$ to funding as follows (formula 1):

$$A = \{\pi, \theta | \pi \in [0, \pi^{sup}], \theta \in [0,1]\}, \quad (1)$$

where $\pi^{sup}$ is the maximum permissible level of pension contribution rate established by the state.

The uncertainty involved by the financial market, non-financial market, and wage dynamics is described by three standard Wiener processes $W_S(t)$, $W_K(t)$, and $W_L(t)$ ($t \in [0, T]$), which are defined on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$ with dependent increments (formula 2):

$$\begin{cases} dW_S(t)dW_K(t) = \rho_{SK}dt, \rho_{SK} \in (-1,1), \\ dW_S(t)dW_L(t) = \rho_{SL}dt, \rho_{SL} \in (-1,1), \\ dW_K(t)dW_L(t) = \rho_{KL}dt, \rho_{KL} \in (-1,1). \end{cases} \quad (2)$$

We suppose that within the DC system the investment manager invests in two kinds of financial assets: a fixed $\alpha$ proportion of the fund in a risk-free asset and the remaining portion $1-\alpha$ in a single risky asset, which represents the index of the stock market. The price of the riskless asset follows the dynamics (formula 3):

$$dS_0(t) = rS_0(t)dt, \tag{3}$$

where r denotes the constant risk-free nominal interest rate. The volatile asset possesses a price S(t) at time t and adheres to the subsequent stochastic differential equation (formula 4):

$$dS(t) = S(t)\big(\mu_S dt + \sigma_S dW_S(t)\big), \tag{4}$$

where $\mu_S$ and $\sigma_S$ are the constant percentage drift (expected return of the stock market) and volatility, respectively.

We denote with $C_\tau(t)$ the total financial wealth at time $t \in [\tau, T]$, generated from the allocation of a fixed share of pension contribution to the DC fund at time $\tau \in \{0,1,\dots,T-1\}$ and governed by the following stochastic differential equation (formula 5):

$$dC_\tau(t) = C_\tau(t)(\alpha r dt + (1-\alpha)[\mu_S dt + \sigma_S dW_S(t)]). \tag{5}$$

After applying Itô's lemma to the log of $C_\tau(t)$, we can find the explicit solution of formula (5):

$$C_\tau(T) = C_\tau(\tau)\exp\left\{\left(\alpha r + (1-\alpha)\mu_S - \frac{1}{2}(1-\alpha)^2{\sigma_S}^2\right)(T-\tau) + (1-\alpha)\sigma_S\big(W_S(T) - W_S(\tau)\big)\right\}. \tag{6}$$

Let $K_\tau(t)$ denote the notional (non-financial) capital at time $t \in [\tau, T]$, generated from the contribution into the NDC system at time $\tau \in \{0,1,\dots,T-1\}$. The dynamics of this notional capital are described by formula 7:

$$dK_\tau(t) = K_\tau(t)\big(\mu_K dt + \sigma_K dW_K(t)\big), \tag{7}$$

where $\mu_K$ and $\sigma_K$ are the constant drift (expected notional return) and volatility terms, respectively.

After applying Itô's lemma to the log of $K_\tau(t)$, we can derive the explicit solution of formula (7):

$$K_\tau(T) = K_\tau(\tau)\exp\left\{\left(\mu_K - \frac{1}{2}{\sigma_K}^2\right)(T-\tau) + \sigma_K\big(W_K(T) - W_K(\tau)\big)\right\}. \tag{8}$$

The monthly wage at a given age x in the career path k changes over time in accordance with the following dynamics (formula 9):

$$dL_{x,k}(t) = L_{x,k}(t)\big(\mu_L dt + \sigma_L dW_L(t)\big), \tag{9}$$

where $\mu_L$ and $\sigma_L$ are the constant instantaneous mean and volatility of the wage, respectively.

A straightforward application of Itô's lemma leads to the following analytic solution of formula (9):

$$L_{x,k}(t) = L_{x,k}(0)\exp\left\{\left(\mu_L - \frac{1}{2}{\sigma_L}^2\right)t + \sigma_L W_L(t)\right\}. \tag{10}$$

The weighted average (pre-retirement) annual wage (considering all career profiles) at time T is defined as (formula 11):

$$\mathrm{AL}_{x_0+T-1}(T) = 12\sum_{k=1}^{N} \mathrm{L}_{x_0+T-1,k}(T)\alpha(x_0+T-1, T-1; k). \quad (11)$$

Individuals' pension rights at time T are determined by the pension wealth (PW) or capital. The formula for calculating the pension wealth takes into account savings from the DC system as well as virtual or notional savings generated from the NDC system (formula 12):

$$\mathrm{PW}(\pi,\theta) = \sum_{k=1}^{N}\sum_{\tau=0}^{T-1} \pi \mathrm{L}_{x_0+\tau,k}(0)\alpha(x_0+\tau,\tau;k)\mathrm{d}(x_0+\tau,\tau;k)\mathrm{l}(x_0,0)\exp\left\{\left(\mu_L - \frac{{\sigma_L}^2}{2}\right)\tau + \sigma_L \mathrm{W}_L(\tau)\right\}\left[\theta\, {}_T\mathrm{p}_{x_0}(T)\exp\left\{\left(\alpha r + (1-\alpha)\mu_S - (1-\alpha)^2\frac{{\sigma_S}^2}{2}\right)(T-\tau) + (1-\alpha)\sigma_S\left(\mathrm{W}_S(T) - \mathrm{W}_S(\tau)\right)\right\} + (1-\theta)\, {}_\tau\mathrm{p}_{x_0}(\tau)\exp\left\{(\mu_K - \frac{{\sigma_K}^2}{2})(T-\tau) + \sigma_K\left(\mathrm{W}_K(T) - \mathrm{W}_K(\tau)\right)\right\}\right], \quad (12)$$

where $\mathrm{l}(x_0, 0)$ is the population aged $x_0$ at time 0, and ${}_\tau\mathrm{p}_{x_0}(\tau)$ is the deterministic probability of survival from age $x_0$ at time 0 to age $x_0+\tau$ by time $\tau$. Here, unlike the DC system, where pension savings of deceased individuals are normally passed on to heirs, notional pension savings of deceased are not inherited. Instead, these savings are redistributed among the surviving members of the cohort, resulting in an additional survivorship dividend (inheritance gains) for those remaining.

In order to determine the optimal pension contribution rate and the allocation between the DC and NDC systems, the (ex-ante) expected utility of the relative pension wealth (RW) is maximized by utilizing its expected value and variance.

We define RW as the ratio between the pension wealth and the weighted average (pre-retirement) annual wage, as was done similarly in the articles of Devolder and Melis [17], and Alonso-García and Devolder [16] (formula 13):

$$\mathrm{RW}(\pi,\theta) = \frac{\mathrm{PW}(\pi,\theta)}{\mathrm{AL}_{x_0+T-1}(T)\mathrm{l}(x_0+T,T)} = \sum_{k=1}^{N}\sum_{\tau=0}^{T-1}\pi[\theta A(x_0+\tau,\tau;k)\exp\{\mathrm{I}_1(\tau,T)\} + (1-\theta)B(x_0+\tau,\tau;k)\exp\{\mathrm{I}_2(\tau,T)\}], \quad (13)$$

where $\mathrm{I}_1(\tau,T) = (1-\alpha)\sigma_S\left(\mathrm{W}_S(T) - \mathrm{W}_S(\tau)\right) - \sigma_L\left(\mathrm{W}_L(T) - \mathrm{W}_L(\tau)\right)$, $\mathrm{I}_2(\tau,T) = \sigma_K\left(\mathrm{W}_K(T) - \mathrm{W}_K(\tau)\right) - \sigma_L\left(\mathrm{W}_L(T) - \mathrm{W}_L(\tau)\right)$, and $A(x_0+\tau,\tau;k)$ and $B(x_0+\tau,\tau;k)$ are deterministic functions of the following form (formulas 14, 15):

$$A(x_0+\tau,\tau;k) = \left(\frac{\mathrm{L}_{x_0+\tau,k}(0)\alpha(x_0+\tau,\tau;k)\mathrm{d}(x_0+\tau,\tau;k)}{12\sum_{k=1}^{N}\mathrm{L}_{x_0+T-1,k}(0)\alpha(x_0+T-1,T-1;k)}\right)\exp\left\{(\alpha r + (1-\alpha)\mu_S - \mu_L - (1-\alpha)^2\frac{{\sigma_S}^2}{2} + \frac{{\sigma_L}^2}{2})(T-\tau)\right\}, \quad (14)$$

$$B(x_0+\tau,\tau;k) = \left(\frac{\mathrm{L}_{x_0+\tau,k}(0)\alpha(x_0+\tau,\tau;k)\mathrm{d}(x_0+\tau,\tau;k)}{{}_{T-\tau}\mathrm{p}_{x_0+\tau}(T)12\sum_{k=1}^{N}\mathrm{L}_{x_0+T-1,k}(0)\alpha(x_0+T-1,T-1;k)}\right)\exp\left\{(\mu_K - \mu_L - \frac{{\sigma_K}^2}{2} + \frac{{\sigma_L}^2}{2})(T-\tau)\right\}. \quad (15)$$

To derive analytical expressions for the expected value and variance of the relative wealth according to the filtration at time $t = 0$, one can utilize well-known properties of the operators $\mathbb{E}[\cdot\,|\mathcal{F}_0]$ and $\mathrm{Var}[\cdot\,|\mathcal{F}_0]$, as well as the results from the propositions below.

The following proposition presents analytic expressions for the first two moments and the covariance of the random processes $I_1(t,T)$ and $I_2(t,T)$ at a fixed time t.

**Proposition 3.1:** Random processes $I_1(t,T) = (1-\alpha)\sigma_S\big(W_S(T) - W_S(t)\big) - \sigma_L\big(W_L(T) - W_L(t)\big)$ and $I_2(t,T) = \sigma_K\big(W_K(T) - W_K(t)\big) - \sigma_L\big(W_L(T) - W_L(t)\big)$, where $0 \le t < T$, are Gaussian with the following expressions for their conditional expectation, variance, and covariance, given a σ-algebra $\mathcal{F}_0$ (formulas 16-19):

$$\mathbb{E}[I_1(t,T)|\mathcal{F}_0] = \mathbb{E}[I_2(t,T)|\mathcal{F}_0] = 0, \tag{16}$$

$$\mathrm{Var}[I_1(t,T)|\mathcal{F}_0] = [(1-\alpha)^2\sigma_S^2 + \sigma_L^2 - 2(1-\alpha)\rho_{SL}\sigma_S\sigma_L](T-t), \tag{17}$$

$$\mathrm{Var}[I_2(t,T)|\mathcal{F}_0] = [\sigma_K^2 + \sigma_L^2 - 2\rho_{KL}\sigma_K\sigma_L](T-t), \tag{18}$$

$$\mathrm{Cov}[I_1(t,T), I_2(t,T)|\mathcal{F}_0] = [(1-\alpha)\sigma_S(\rho_{SK}\sigma_K - \rho_{SL}\sigma_L) - \rho_{KL}\sigma_K\sigma_L + \sigma_L^2](T-t). \tag{19}$$

The following proposition presents analytic expressions for the first two moments and the covariance of the exponentiated random processes $\exp\{I_1(t,T)\}$ and $\exp\{I_2(t,T)\}$ at fixed times $t_1$ and $t_2$.

**Proposition 3.2:** For random processes $\{I_i(t,T)\}_{i=1,2}$ the following equalities hold for $0 \le t_1 \le t_2 < T$ (formulas 20-23):

$$\mathbb{E}[\exp\{I_i(t_1,T)\}|\mathcal{F}_0] = \exp\left\{\frac{1}{2}\mathrm{Var}[I_i(t_1,T)|\mathcal{F}_0]\right\}, \tag{20}$$

$$\mathrm{Var}[\exp\{I_i(t_1,T)\}|\mathcal{F}_0] = (\exp\{\mathrm{Var}[I_i(t_1,T)|\mathcal{F}_0]\} - 1)\exp\{\mathrm{Var}[I_i(t_1,T)|\mathcal{F}_0]\}, \tag{21}$$

$$\mathrm{Cov}[\exp\{I_i(t_1,T)\}, \exp\{I_i(t_2,T)\}|\mathcal{F}_0] = \exp\left\{\frac{1}{2}(\mathrm{Var}[I_i(t_1,T)|\mathcal{F}_0] + \mathrm{Var}[I_i(t_2,T)|\mathcal{F}_0])\right\}(\exp\{\mathrm{Var}[I_i(t_2,T)|\mathcal{F}_0]\} - 1), \tag{22}$$

$$\mathrm{Cov}\big[\exp\{I_i(t_1,T)\}, \exp\{I_j(t_2,T)\}|\mathcal{F}_0\big] = \exp\left\{\frac{1}{2}\big(\mathrm{Var}[I_i(t_1,T)|\mathcal{F}_0] + \mathrm{Var}\big[I_j(t_2,T)|\mathcal{F}_0\big]\big)\right\}\big(\exp\big\{\mathrm{Cov}\big[I_i(t_2,T), I_j(t_2,T)|\mathcal{F}_0\big]\big\} - 1\big), \tag{23}$$

$$j = 1,2; j \ne i.$$

***Proof:*** Expressions above follow from the fact that $\{I_i(t,T)\}_{i=1,2}$ are normally distributed at each fixed time t and therefore $\big\{\exp\{I_i(t,T)\}\big\}_{i=1,2}$ are lognormally distributed.

In the context of assessing the adequacy of RW, we define the conditional expectation criterion as follows.

**Definition 3.3:** The conditional expectation of RW given $\mathcal{F}_0$ is called adequate (sufficient) if, under the admissible parameter values in formula (1), the following condition holds:

$$0 < \mu^s \leq \mathbb{E}[\mathrm{RW}(\pi, \theta)|\mathcal{F}_0] < \infty, \tag{24}$$

where $\mu^s$ is the threshold level of RW adequacy (sufficiency).

The optimization problem consists of minimizing the variance of RW in formula (13) given a specific level of its mean ($\mu$):

$$\begin{cases} \min_{\pi^1,\theta^1} \mathrm{Var}[\mathrm{RW}(\pi^1, \theta^1)|\mathcal{F}_0], \\ \mathbb{E}[\mathrm{RW}(\pi^1, \theta^1)|\mathcal{F}_0] = \mu \geq 0, \end{cases} \tag{25}$$

where for $\pi^1$ and $\theta^1$ the admissibility condition is satisfied according to formula (1).

The Lagrangian function for the problem above is defined as follows (formula 26):

$$\mathrm{L}(\pi^1, \theta^1, \lambda, \omega_1, \omega_2, \omega_3, \omega_4) = \mathrm{Var}[\mathrm{RW}(\pi^1, \theta^1)|\mathcal{F}_0] - \lambda(\mathbb{E}[\mathrm{RW}(\pi^1, \theta^1)|\mathcal{F}_0)] - \mu) - \omega_1(\pi^1 - s_1^2) - \omega_2(\pi^1 - \pi^{\sup} + s_2^2) - \omega_3(\theta^1 - s_3^2) - \omega_4(\theta^1 - 1 + s_4^2), \tag{26}$$

where $\lambda, \omega_1, \omega_2, \omega_3, \omega_4$ are Lagrange multipliers and $s_1^2, s_2^2, s_3^2, s_4^2$ are non-negative slack variables.

**Proposition 3.4:** The corresponding analytical solution of formula (26) for the case $s_i^2 \neq 0$ ($i = 1,4$) (i.e., for $0 < \pi^1 < \pi^{\sup}$ and $0 < \theta^1 < 1$) is given by:

$$\theta^{1,*} = \frac{\gamma_2\gamma_4 - \gamma_3\gamma_5}{(\gamma_1 - \gamma_3)\gamma_5 + (\gamma_2 - \gamma_3)\gamma_4}, \tag{27}$$

$$\pi^{1,*} = \pi^{1,*}(\mu) = \frac{\mu[(\gamma_1 - \gamma_3)\gamma_5 + (\gamma_2 - \gamma_3)\gamma_4]}{\gamma_1{\gamma_5}^2 - 2\gamma_3\gamma_4\gamma_5 + \gamma_2{\gamma_4}^2}, \tag{28}$$

where formulas 29-33:

$$\gamma_1 = \mathrm{Var}\left[\sum_{\tau=0}^{T-1}\left\{\left(\sum_{k=1}^{N} A(x_0 + \tau, \tau; k)\right)\exp\{I_1(\tau, T)\}\right\}|\mathcal{F}_0\right], \tag{29}$$

$$\gamma_2 = \mathrm{Var}\left[\sum_{\tau=0}^{T-1}\left\{\left(\sum_{k=1}^{N} B(x_0 + \tau, \tau; k)\right)\exp\{I_2(\tau, T)\}\right\}|\mathcal{F}_0\right], \tag{30}$$

$$\gamma_3 = \mathrm{Cov}\left[\sum_{\tau=0}^{T-1}\left\{\left(\sum_{k=1}^{N} A(x_0 + \tau, \tau; k)\right)\exp\{I_1(\tau, T)\}\right\}, \sum_{\tau=0}^{T-1}\left\{\left(\sum_{k=1}^{N} B(x_0 + \tau, \tau; k)\right)\exp\{I_2(\tau, T)\}\right\}|\mathcal{F}_0\right], \tag{31}$$

$$\gamma_4 = \sum_{\tau=0}^{T-1}\left\{\left(\sum_{k=1}^{N} A(x_0 + \tau, \tau; k)\right)\mathbb{E}[\exp\{I_1(\tau, T)\}|\mathcal{F}_0]\right\}, \tag{32}$$

$$\gamma_5 = \sum_{\tau=0}^{T-1}\left\{\left(\sum_{k=1}^{N} B(x_0 + \tau, \tau; k)\right)\mathbb{E}[\exp\{I_2(\tau, T)\}|\mathcal{F}_0]\right\}. \tag{33}$$

***Proof:*** Analytical expressions can be obtained by setting the partial derivatives of the Lagrangian function with respect to $\pi^1$, $\theta^1$, and $\lambda$ to zero under the condition that $\omega_i = 0$ ($i = 1.4$).

**Definition 3.5:** Let $(\pi_\mu^{1,\mathrm{opt}}, \theta_\mu^{1,\mathrm{opt}})$ denote the optimal solution to formula (25) for $\mu > 0$.

**Proposition 3.6:** The set of all points $(\pi^1, \theta^1)$ satisfying $\pi^1 = 0$ and $\theta^1 \in [0,1]$ constitutes the optimal solutions to problem formula (25) when $\mu = 0$. At these points, the global minimum variance of RW is attained.

**Definition 3.7:** The set of all optimal solutions to problem (25) for $\mu > 0$ forms the efficient frontier curve – mean-variance couples that minimize the risk for a specific level of expectation.

The preceding propositions and definitions are consistent with conventional results in optimization theory and variance minimization, as discussed, for instance, in Alonso-García and Devolder [16], Devolder and Melis [17].

We define the following criteria to assess the compliance of the optimal solution with the adequacy condition.

**Definition 3.8:** We shall say that the optimal solution to problem formula (25) satisfies the adequacy (sufficiency) condition if the condition in formula (24) is met for it.

After defining the efficient frontier, the selection of a specific combination $\{\pi, \theta\}$ on this curve depends on the cohort's preferences regarding risk and is accomplished by maximizing the mean-variance utility function (formula 34):

$$\max_{\pi^2,\theta^2} \mathrm{U}[\mathrm{RW}(\pi^2, \theta^2)|\mathcal{F}_0] = \max_{\pi^2,\theta^2} \left\{ \mathbb{E}[\mathrm{RW}(\pi^2, \theta^2)|\mathcal{F}_0] - \frac{\varphi}{2} \mathrm{Var}[\mathrm{RW}(\pi^2, \theta^2)|\mathcal{F}_0] \right\}, \tag{34}$$

where for $\pi^2$ and $\theta^2$ the admissibility condition is satisfied according to formula (1), and $\varphi$ is the risk aversion coefficient for the cohort.

The Lagrangian function for the problem above is defined as follows (formula 35):

$$\mathrm{L}(\pi^2, \theta^2, \omega_1, \omega_2, \omega_3, \omega_4) = \mathrm{U}[\mathrm{RW}(\pi^2, \theta^2)|\mathcal{F}_0] - \omega_1(\pi^2 - s_1^2) - \omega_2(\pi^2 - \pi^{\sup} + s_2^2) - \omega_3(\theta^2 - s_3^2) - \omega_4(\theta^2 - 1 + s_4^2), \tag{35}$$

where $\omega_1, \omega_2, \omega_3, \omega_4$ are Lagrange multipliers and $s_1^2, s_2^2, s_3^2, s_4^2$ are non-negative slack variables.

**Proposition 3.9:** The Lagrangian function in formula (35) in case of $\omega_i = 0$ $(i = 1,2,3,4)$ has a critical point of $(\pi^{2,*}, \theta^{2,*})$, where:

$$\theta^{2,*} = \theta^{1,*}, \tag{36}$$

$$\pi^{2,*} = \frac{1}{\varphi} \cdot \frac{(\gamma_1 - \gamma_3)\gamma_5 + (\gamma_2 - \gamma_3)\gamma_4}{\gamma_1\gamma_2 - {\gamma_3}^2}, \tag{37}$$

where expressions for $\gamma_i$ $(i = 1,2, \dots,5)$ are given in equations formulas (29)-(33).

***Proof:*** Analytical expressions can be obtained by setting the partial derivatives of the Lagrangian function with respect to $\pi^2$ and $\theta^2$ to zero, under the condition that $\omega_i = 0$ $(i = 1.4)$.

**Remark 3.10:** From Proposition 3.9, it follows that the level of risk aversion φ does not affect the proportion $\theta^{2,*}$, but there is an inverse relationship between φ and the pension contribution rate $\pi^{2,*}$.

**Proposition 3.11:** There exists a $\mu^* \in \mathbb{R}^+$ such that (formula 38):

$$\mu^* = \pi^{2,*}[\gamma_4\theta^{1,*} + \gamma_5(1-\theta^{1,*})] = \frac{1}{\varphi} \cdot \frac{\gamma_1{\gamma_5}^2 - 2\gamma_3\gamma_4\gamma_5 + \gamma_2{\gamma_4}^2}{\gamma_1\gamma_2 - {\gamma_3}^2}, \quad (38)$$

for which the following equality holds (formula 39):

$$\pi^{2,*} = \pi^{1,*}(\mu^*), \quad (39)$$

where $\pi^{1,*}(\mu^*)$ and $\pi^{2,*}$ are defined from formula (28) and formula (37) respectively.

In addition, we shall consider a critical point $(\pi^{2,*}, \theta^{2,*})$ on a boundary segment, where $\pi^{2,*} = \pi^{\sup}$ and $0 < \theta^{2,*} < 1$. The characteristics and implications associated with this point will be discussed in the sensitivity analysis of Section 3.

**Proposition 3.12:** The Lagrangian function in formula (35) for both $\omega_2 \neq 0$ and $\omega_1 = \omega_3 = \omega_4 = 0$ has a critical point of $(\pi^{2,*}, \theta^{2,*})$, where formulas 40, 41:

$$\theta^{2,*} = \frac{(\pi^{\sup}\varphi)^{-1}(\gamma_4 - \gamma_5) + \gamma_2 - \gamma_3}{\gamma_1 + \gamma_2 - 2\gamma_3}, \quad (40)$$

$$\pi^{2,*} = \pi^{\sup}, \quad (41)$$

where expressions for $\gamma_i$ $(i = 1.5)$ are given in equations formulas (29)-(33).

***Proof:*** Analytical expressions can be obtained by setting the partial derivatives of the Lagrangian function with respect to $\pi^2$, $\theta^2$, and $\omega_2$ to zero under the conditions $\omega_2 \neq 0$ and $\omega_1 = \omega_3 = \omega_4 = 0$.

**Remark 3.13:** Given that $\gamma_1 + \gamma_2 - 2\gamma_3 > 0$, we observe an inverse relationship between φ and $\theta^{2,*}$ when $\gamma_4 - \gamma_5 > 0$. Conversely, when $\gamma_4 - \gamma_5 < 0$, there is a direct relationship between φ and $\theta^{2,*}$.

**Definition 3.14:** Let $(\pi^{2,\mathrm{opt}}, \theta^{2,\mathrm{opt}})$ denote the optimal solution to problem formula (34). The optimal solution will also be referred to as the optimal choice for the cohort (considering the risk preferences).

As before, the preceding propositions and definitions align with conventional results in portfolio optimization.

We define the following criteria to assess the compliance of the optimal choice with the adequacy condition.

**Definition 3.15:** The optimal choice for the cohort satisfies the adequacy (sufficiency) condition if the condition in formula (24) is met.

## 3. Results and Discussion

This section presents a numerical illustration utilizing results from Section 2 and data on Kazakhstan's demographic and economic development. Additionally, the scenario analysis explores the feasibility of achieving optimal solutions that satisfy the adequacy condition of RW – a critical measure of the pension system's effectiveness in providing individuals with retirement benefits relative to their earnings [22].

As a reference cohort, we will select a group of men and women who commence their employment at the age of $x_0 = 22$ in 2024 and make pension contributions at a rate of $\pi$ ($0 \leq \pi \leq \pi^{sup} = 25\%$) until reaching the normal retirement age of 63. This allows for a theoretical career span of $T = 41$ years, aligning with the assumptions established in Alonso-García and Devolder [16] to ensure comparability with this study. Also, a career spanning from age 22 until the normal pension age conforms to the baseline assumptions of OECD cohort-based pension models, which are specifically tailored to evaluate future entitlements under today's parameters and rules.

We use country-specific projections of cohort-based mortality rates from the United Nations population database, which account for mortality improvements over time [23].

Table 1 defines the primary assumptions for the financial, non-financial, and wage processes.

**Table 1**

**Parameters of $S_0(t), S(t), K_\tau(t)$ and $L_{x,k}(t)$ processes for the base scenario**

| Financial assets | Notional (non-financial) asset | Wage | Interactions between processes |
|---|---|---|---|
| $\alpha = 90\%$<br>$r = 5.0\%$<br>$\mu_S = 7.5\%$<br>$\sigma_S = 18.0\%$ | $\mu_K = 5.5\%$<br>$\sigma_K = 5.0\%$ | $\mu_L = 5.0\%$<br>$\sigma_L = 4.0\%$ | $\rho_{SK} = 0.0$<br>$\rho_{SL} = 0.0$<br>$\rho_{KL} = 0.5$ |

*Source:* compiled by the authors.

The rationale for establishing the above values for the financial market is as follows. As of the beginning of 2024, total pension assets amounted to approximately 39 billion US dollars, with about 90% allocated to various bonds (predominantly government and quasi-government bonds) and cash deposits, and only around 9% invested in equities, mostly in foreign markets. The relatively lower allocation to equities is due to the NBRK's highly conservative approach to managing pension assets and investing funds in reliable financial instruments to ensure their long-term safety, as specified in its investment policy statement. In this context, the 2023 MCGPI index ranked Kazakhstan's pension system with the lowest score for the share of investment in growth assets (including equities and property) among the 47 retirement income systems

worldwide. Notably, twenty-one of these 47 systems have a proportion of growth assets above 45%.

Therefore, the bond and cash components of a portfolio are considered proxies for the risk-free asset, with the assumption that their share will remain unchanged in the future, establishing $\alpha = 90\%$. A long-term real risk-free rate of 1.5% and a long-term target inflation rate of 3.5% in the country are assumed. The remaining portion of the portfolio is regarded as a risky asset, representing a stock market index. The drift $\mu_S$ and volatility $\sigma_S$ parameters of the risky asset are set at 7.5% and 18.0%, respectively, to reflect the anticipated performance of a broad global stock market index, such as the MSCI ACWI. The analysis emphasizes the global stock market because the domestic market alone might lack sufficient capacity to attract and absorb new investments, potentially offering lower comparable returns. International diversification across developed and emerging markets aims to achieve more stable and predictable investment outcomes. Given that exchange rate fluctuations are expected to have minimal impact over an extended investment horizon compared to overall equity market performance, fixed exchange rates are assumed for simplicity.

The drift parameter of the wage process is established at to $\mu_L = 5.0\%$, indicating an anticipated long-term real growth rate of average wages of 1.5%, while its volatility is determined at $\sigma_L = 4.0\%$, derived from the historical dynamics of economy-wide average wages. In reality, the notional rate may be associated with numerous economic variables, including GDP growth rates, average earnings, and contributions [24]. This study asserts that the instantaneous mean of the non-financial process is equal to the sum of the productivity growth rate, which closely aligns with the average wage growth rate, and the growth rate of the labour force. These two requirements are broadly regarded as essential components of the rate of return for a PAYG system [25]. Assuming an average long-term growth rate of the labour force between 0.5% and 0.7%, derived from population forecasts and labour force participation rates, and utilising the previously established long-term growth rate of average earnings at 5.0%, we designate $\mu_K$ as 5.5% with a volatility of $\sigma_K = 5.0\%$.

Historically, over the entire period of the DC system's operation in the country from 1998 to 2023, including the years with private pension funds, the correlation between the performance of these instruments and average wage growth is low, at 0.19, and statistically insignificant at the 5% level. Specifically, for the main stock market index in Kazakhstan (KASE index), which has data available only from 2007 onwards, and the MSCI ACWI index (returns are measured in U.S. dollars), the correlations with average wage growth are negative at -0.22 and -0.16, respectively, and also statistically insignificant. Therefore, we assume that the $\rho_{SL}$ value will remain fixed at 0.0 in the future. As the notional rate is closely linked to average wages, it is assumed that $\rho_{KL}$ is moderately positive at 0.5 and that $\rho_{SK}$ equals $\rho_{SL}$. Subsequently, alternative scenarios are considered in which some of these assumptions are relaxed, and other possible options are explored.

We'll analyze population employment and income data next. According to the Bureau of National Statistics, Kazakhstan had 9.1 million workers in 2023. About 6.9 million people (76%) were employed, while 2.2 million (24%) were self-employed. The baseline scenario assumes this distribution structure throughout the cohort's career. We examine 16 profession profiles (sectors) for employed people, their age-specific monthly earnings, and sector-wise employment shares. Using the historical level of 7.5, the employed population's projected yearly pension contribution frequency is 63%.

In analysing self-employed workers as a 17th career profile, the focus is solely on individual entrepreneurs, given their significant prevalence within this group. These individuals typically do not receive a wage in the traditional sense, as this term is generally associated with employment where an employer pays an employee. As per legislation, individual entrepreneurs retain the freedom to determine the income used for calculation of pension contributions, which may not align with their actual business earnings. Consequently, there is a potential incentive for them to set this income at the minimum standard (the minimum monthly wage) stipulated by legislation in Kazakhstan to reduce mandatory social contributions. As a result, it is hypothesized that the income path of such workers, utilized for determining pension contributions, remains tied to the minimum wage over their entire career. The estimated annual frequency of pension contributions within this group is set at the historical level of 4.8, resulting in a lower contribution density of 40% compared to that of employed workers.

As a result, for the base case, the average density of contributions across all workers is estimated to be 57%. Further, we assume a conservative estimate of the risk aversion coefficient at a level of 3.0, indicating that the cohort, overall, leans towards preferring safety over risk. This aligns with the findings of the Adilkhanova [26], which analysed data from approximately 12,000 households in Kazakhstan.

The value of $\mu^s$ is selected as 7.3 so that, with a real discount rate of 1%, the pension wealth would provide, on average, the representative cohort member with an annual pension income equivalent to 40 percent of the economy-wide average wage over the expected pay-out period. This threshold level aligns with the minimum recommended level specified by the ILO Convention on Social Security.

Figure 1(a) illustrates the efficient frontier for the base case, representing the set of optimal risk-return combinations for RW. The global minimum of the RW variance is attained at a point situated at the origin, whereas the portion of the efficient frontier curve corresponding to $\mu < 7.53$ is generated by optimal solutions derived from formula (27) and (28), where $0 < \pi_{\mu}^{1,opt} < \pi^{sup}$ and $0 < \theta_{\mu}^{1,opt} < 1$. Meanwhile, points along the curve with expected values not less than 7.3 are determined by optimal solutions that satisfy the adequacy condition (the green part of the curve).

The optimal choice for the cohort is the pair $(\pi^{2,opt}, \theta^{2,opt}) = (25.0\%, 50.8\%)$ obtained from expressions formulas (40) and (41). This

solution corresponds to the point located at the intersection of the efficient frontier with the indifference curve, reflecting the cohort's risk preference (Figure 1(b)).

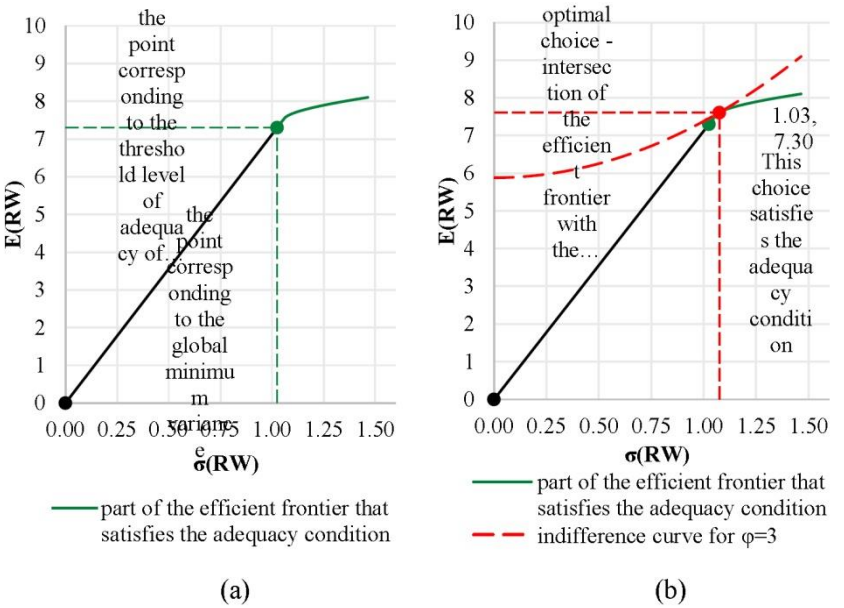



**Figure 1**
**(a) The efficient frontier and (b) the optimal choice for the cohort on this frontier under the base case**

*Source:* compiled by the authors.

It is important to emphasize that in the base scenario, the cohort can tolerate a pension contribution rate exceeding the legislated 15%, suggesting that there is room for increasing contributions beyond the current level. In essence, the implementation of initial parameters and assumptions under the base case leads to the formation of sufficient pension capital because the corresponding expected value of $RW$ (7.6) is higher than $\mu^s$. However, we note that raising the contribution rate should be approached with caution, as it carries potential negative repercussions, including a diminished standard of living for workers, reduced labour force participation, and increased incentives for income underreporting, which could distort employment behaviours.

We will proceed further with analysing the sensitivity of results to variations in input parameters. First, we will examine the optimal choice for the cohort, as well as the feasibility of meeting the adequacy condition, across different levels of risk aversion and correlations, while keeping other parameters unchanged. This scenario closely parallels the study conducted in Alonso-García and Devolder [16], allowing for a comparative analysis of optimal allocation in

funding. The authors [16] define the ratio between the market rate (notional rate) over the rate of increase in wages as the financial (demographic) process. Then it is assumed that the financial and demographic processes are correlated when they interact during the same period of time $t-1$ to $t$, but are uncorrelated otherwise. In contrast, in this study, potential interactions between stochastic processes $S(t), K_{\tau}(t)$ and $L_{x,k}(t)$ are assessed analytically through pairwise correlations in formula (2).

For the purposes of the analysis, the initial step involves determining the potential variations in pairwise correlations between stochastic processes. As previously noted, the assumption remains that the $\rho_{SL}$ value will continue into the future at a fixed level of 0.0, as indicated in Table 1, consistent with the statistical insignificance of the relationship between the financial market and wage dynamics during the historical period. Given that the rules and parameters of the newly implemented NDC system are not yet fully legislated, the sensitivity analysis will deviate from the base assumption to explore varying levels of pairwise correlations $\rho_{SK}$ and $\rho_{KL}$, ranging from -0.5 to +0.5 in increments of 0.25.

Regarding the level of risk aversion, its impact will be evaluated across an extended range from 1 to 8. Table 2 demonstrates how variations in correlations and risk aversion levels influence the optimal choices for the cohort. Henceforth, results satisfying the adequacy condition outlined in formula (24) are marked in green, whereas results not meeting this criterion are marked in red.

At higher levels of risk aversion ($\varphi \geq 5$), there is a reduction in the optimal pension contribution rate compared to the base scenario, while the optimal funded proportion remains fixed, assuming all else being equal. Expression's formulas (36) and (37) primarily describe the dynamics of these optimal solutions. This suggests that individuals with a more conservative approach opt to allocate a smaller portion of their income for retirement, aiming to minimize risks, even if it entails having a smaller pension capital in the future. These findings are consistent with Remark 3.10.

Conversely, as the level of risk aversion decreases, the optimal pension contribution rate increases until reaching the upper constraint boundary of 25%, at which point the optimal share in funding declines. Expression's formulas (40) and (41) describe such optimal solutions. The latter result contrasts with the findings of J. Alonso-García and P. Devolder [16], which observed a negative correlation between risk aversion and funding allocation: as risk aversion diminishes, the weighted mean allocation to riskier funding increases, and vice versa.

The variance in results arises from differing assumptions on expected financial returns and hypothetical returns. In the numerical examples presented by Alonso-García and Devolder [16], the presumption of a positive expected value for the disparity between funded and demographic returns was crucial for substantiating the admissibility of optimal allocations in contexts where financial and demographic processes operated independently. This assumption aligns with the hypothesis that, in a dynamically efficient economy, the nominal rate would be lower than the rate of return on financial assets [24].

**Table 2**

**Ex-ante optimal choice for the cohort ($\pi^{2,opt}, \theta^{2,opt}$), percentage, and feasibility of meeting the adequacy condition: comparison for different levels of correlations ($\rho_{SK}, \rho_{KL}$) and risk aversion φ**

| Correlations ($\rho_{SK}, \rho_{KL}$) | Risk aversion levels, φ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| (-0.5, -0.5) | (25, 83) | (25, 97) | (25, 100) | (25, 100) | (24, 100) | (20, 100) | (17, 100) | (15, 100) |
| (-0.5, -0.25) | (25, 72) | (25, 87) | (25, 91) | (25, 94) | (24, 95) | (20, 95) | (17, 95) | (15, 95) |
| (-0.5, 0.0) | (25, 60) | (25, 74) | (25, 79) | (25, 82) | (25, 83) | (21, 83) | (18, 83) | (16, 83) |
| (-0.5, 0.25) | (25, 46) | (25, 61) | (25, 66) | (25, 68) | (25, 69) | (25, 70) | (21, 70) | (19, 70) |
| (-0.5, 0.5) | (25, 31) | (25, 46) | (25, 51) | (25, 53) | (25, 54) | (25, 55) | (25, 56) | (25, 57) |
| (-0.25, -0.5) | (25, 86) | (25, 100) | (25, 100) | (25, 100) | (24, 100) | (20, 100) | (17, 100) | (15, 100) |
| (-0.25, -0.25) | (25, 74) | (25, 91) | (25, 96) | (25, 99) | (24, 100) | (20, 100) | (17, 100) | (15, 100) |
| (-0.25, 0.0) | (25, 61) | (25, 77) | (25, 83) | (25, 85) | (25, 87) | (21, 87) | (18, 87) | (16, 87) |
| (-0.25, 0.25) | (25, 45) | (25, 62) | (25, 68) | (25, 70) | (25, 72) | (23, 73) | (20, 73) | (18, 73) |
| (-0.25, 0.5) | (25, 29) | (25, 45) | (25, 51) | (25, 53) | (25, 55) | (25, 56) | (25, 57) | (23, 57) |
| (0.0, -0.5) | (25, 91) | (25, 100) | (25, 100) | (25, 100) | (24, 100) | (20, 100) | (17, 100) | (15, 100) |
| (0.0, -0.25) | (25, 77) | (25, 95) | (25, 100) | (25, 100) | (24, 100) | (20, 100) | (17, 100) | (15, 100) |
| (0.0, 0.0) | (25, 62) | (25, 81) | (25, 87) | (25, 90) | (25, 92) | (20, 92) | (18, 92) | (15, 92) |
| (0.0, 0.25) | (25, 45) | (25, 64) | (25, 70) | (25, 73) | (25, 75) | (22, 76) | (19, 76) | (17, 76) |
| (0.0, 0.5) | (25, 26) | (25, 44) | (25, 51) | (25, 54) | (25, 56) | (25, 57) | (24, 58) | (21, 58) |
| (0.25, -0.5) | (25, 96) | (25, 100) | (25, 100) | (25, 100) | (24, 100) | (20, 100) | (17, 100) | (15, 100) |
| (0.25, -0.25) | (25, 81) | (25, 100) | (25, 100) | (25, 100) | (24, 100) | (20, 100) | (17, 100) | (15, 100) |
| (0.25, 0.0) | (25, 64) | (25, 86) | (25, 93) | (25, 96) | (24, 98) | (20, 98) | (17, 98) | (15, 98) |
| (0.25, 0.25) | (25, 44) | (25, 66) | (25, 73) | (25, 77) | (25, 79) | (21, 79) | (18, 79) | (16, 79) |
| (0.25, 0.5) | (25, 21) | (25, 43) | (25, 51) | (25, 55) | (25, 57) | (25, 58) | (22, 59) | (19, 59) |
| (0.5, -0.5) | (25, 100) | (25, 100) | (25, 100) | (25, 100) | (24, 100) | (20, 100) | (17, 100) | (15, 100) |
| (0.5, -0.25) | (25, 86) | (25, 100) | (25, 100) | (25, 100) | (24, 100) | (20, 100) | (17, 100) | (15, 100) |
| (0.5, 0.0) | (25, 67) | (25, 92) | (25, 100) | (25, 100) | (24, 100) | (20, 100) | (17, 100) | (15, 100) |
| (0.5, 0.25) | (25, 43) | (25, 69) | (25, 78) | (25, 83) | (25, 85) | (21, 85) | (18, 85) | (15, 85) |
| (0.5, 0.5) | (25, 15) | (25, 42) | (25, 51) | (25, 56) | (25, 58) | (24, 60) | (21, 60) | (18, 60) |

*Note:* Solutions that satisfy the adequacy condition are marked in green, while those that do not are marked in red.
*Source:* compiled by the authors.

In contrast, in this study, it is the negative expected value of the difference between the funded and unfunded components ($\gamma_4 - \gamma_5 < 0$) that defines the positive relationship between the risk aversion level and proportion in funding, as indicated by Remark 3.13. Indeed, under the base case, the long-run mean level of the notional rate ($\mu_K = 5.5\%$ from Table 1), based on both wage growth and labour force growth, exceeds the mean level of the market rate ($90\%r + 10\%\mu_S = 5.25\%$), based on assumed weights of financial assets. Additionally, the expected value of the unfunded component is amplified by the survivorship dividend, which can be viewed as an extra return beyond the notional rate.

As mentioned earlier, the relatively lower yield of the funded component is due to the NBRK's conservative approach to managing pension assets and

predominantly investing funds in reliable financial instruments to ensure their long-term safety. Nevertheless, the sensitivity analysis will later be complemented by exploring additional scenarios for funded returns and risks, considering enhanced portfolio diversification. The latter is based on the premise that the existence of unfunded pensions serves as an important risk hedge through which diversification into equities can take place. We will now examine the impact of pairwise correlations $(\rho_{SK}, \rho_{KL})$ on the optimal choice for the cohort and the feasibility of meeting the adequacy condition.

The expression $I_2(t, T)$ represents the difference between the increments of two Wiener processes, linked through $\rho_{KL}$. Therefore, higher positive values of $\rho_{KL}$ are likely to reduce the potential volatility of the non-funded part. Indeed, in this case, $I_2(t, T)$ would fluctuate less because changes in one process are partially offset by changes in the other. Consequently, with reduced volatility, the optimal allocation to the non-funded part should rise, as it now becomes less risky relative to the funded part, assuming all other factors remain constant. Furthermore, when combined with a relatively higher expected notional return, this would result in an increased expected value of the RW, consequently enhancing the likelihood of meeting the adequacy condition.

Table 2 illustrates these findings clearly. As the processes $\Delta W_K(t, T)$ and $\Delta W_L(t, T)$ move more closely in tandem, we observe a gradual decline in the optimal funded proportion, holding all else constant. Moreover, the adequacy condition is shown to be satisfied mostly for nonnegative values of $\rho_{KL}$ when $\varphi = 2$. For higher levels of risk aversion, the adequacy condition is only met when there is a stronger positive relationship, specifically with $\rho_{KL} = 0.5$.

Regarding the correlation between processes $W_S(t)$ and $W_K(t)$, denoted by $\rho_{SK}$, Table 2 generally demonstrates that greater diversification benefits are observed as this correlation moves towards negative levels. This observation aligns with the principles of Modern Portfolio Theory (MPT), which emphasize that an investor can achieve these benefits by reducing the correlation between the returns of the assets selected for the portfolio. However, the study Alonso-García and Devolder [16] does not strongly support this result for the ex-ante case, where the optimal allocation is not significantly affected by the correlation between the demographic and the financial processes. On the other hand, for the ex-post case, the J. Alonso-García and P. Devolder demonstrate that there are few diversification benefits when these processes are positively correlated, thus reinforcing the above notion.

As shown in Table 2, higher diversification contributes to achieving a sufficient level of pension capital. For instance, across scenarios where $\varphi = 2$ and $\rho_{KL} = -0.25$, the adequacy condition is met only when the optimal solution corresponds to $\rho_{SK} = -0.5$, indicating the highest possible diversification. This is because higher negative correlations of $\rho_{SK}$ make allocating a larger share to the unfunded component more attractive. This, combined with a relatively higher expected notional return, results in an increase in the expected pension capital.

In essence, the above implications highlight the importance of having the notional return closely tied to the average wage index while being inversely related to the performance of financial assets in order to ensure diversification benefits and sufficient growth in pension capital over time. Although the idea of a negative correlation between financial returns and economy-wide measures (notional returns) may seem counterintuitive, empirical research provides evidence supporting this relationship. For instance, an analysis of historical datasets for 19 developed and 15 developing countries showed a negative correlation between the growth of real per capita GDP (which can feature in NDC rules) and real returns on equities, with coefficients of -0.39 and -0.41, respectively, as there are sound reasons for this result [27-29]. However, in countries where pension assets can be invested freely in any market and are not restricted to the home market (as in the case of this study), these correlations could change significantly [30].

Regarding Kazakhstan, the correlation between per capita economic growth and domestic stock index returns is remarkably similar to the above figures, with a value of -0.42 in nominal terms and -0.31 in real terms. Additionally, there is a statistically significant moderate correlation (0.63) between per capita GDP growth rates and average wage growth over the period of 1998-2023. Therefore, given a thorough analysis of these relationships with relevant economic justifications, the GDP per capita indicator potentially could be another option to consider as a feature in the NDC formula, particularly if policymakers consider increasing the exposure of the investment portfolio to domestic equities in the future.

Next, the impact of diversifying pension assets within the DC system will be explored. To achieve this, a comparative analysis will be conducted. In addition to the current investment strategy under the base case (Base), two alternative, less conservative strategies will be considered. These alternatives involve increasing the allocation to the risky asset $1-\alpha$ while maintaining all other parameters consistent with the base case. Specifically, Strategy A, with an $1-\alpha$ of 30%, and Strategy B, with an $1-\alpha$ of 45%, will be examined for levels of risk aversion ranging from 1 to 8. The results of this analysis are presented in Table 3.

**Table 3**

**Ex-ante optimal choice for the cohort ($\pi^{2,opt}, \theta^{2,opt}$), percentage, and feasibility of meeting the adequacy condition: comparison for alternative investment strategies within the DC system**

| | Risk aversion levels, φ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Investment strategy** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** |
| Base | (25, 26) | (25, 44) | (25, 51) | (25, 54) | (25, 56) | (25, 57) | (24, 58) | (21, 58) |
| Strategy A | (25, 25) | (25, 27) | (25, 27) | (25, 27) | (24, 27) | (20, 27) | (17, 27) | (15, 27) |
| Strategy B | (25, 18) | (25, 15) | (25, 15) | (25, 14) | (21, 14) | (17, 14) | (15, 14) | (13, 14) |

*Note:* Solutions that satisfy the adequacy condition are marked in green, while those that do not are marked in red.

*Source:* compiled by the authors.

As shown in Table 3, firstly, there is a tendency for the optimal allocation to funding to decline as it becomes riskier. Additionally, the optimal contribution rate also decreases, but at higher levels of risk aversion ($\varphi \geq 5$). For instance, compared to the Base scenario, with $\varphi = 5$, the optimal contribution rate decreases by 1.0 and 4.0 percentage points under Strategy A and Strategy B, respectively. Allocating a smaller portion of the wage for retirement leads to a smaller pension capital in the future, which, in turn, heightens the risk of not achieving adequate pension levels.

Secondly, an escalation in risk aversion results in an augmentation of the optimal funding allocation in both the Base and Strategy A scenarios, a pattern previously seen in Table 2. Conversely, under Strategy B, a contrasting tendency is seen; specifically, the ex-ante allocation to funding diminishes as the risk aversion coefficient declines from 18% to 14%. This discrepancy occurs due to the allocation of a greater proportion of money to the hazardous asset in Strategy B, aligning return assumptions with those of J. Alonso-García and P. Devolder [16] in a dynamically efficient economy, hence ensuring γ_4-γ_5>0. Remark 3.13 indicates that this alignment enhances the comparability of the outcomes from both studies.

Additional assumptions within the above investment strategies are introduced by adjusting the parameters related to the employment structure of the population and density of contributions. Specifically, for the proportion of individuals classified as self-employed, denoted as $\alpha(x_0 + t, t; 17)$, two scenarios are analyzed in addition to the base scenario value of 24%. The effects of halving this value (12%) and doubling it (48%) are examined. Furthermore, the impact of varying the density of contributions is assessed by evenly decreasing and increasing its current average level under the base case from 57% to 30% and 90%, respectively. A higher density of contributions reflects greater levels of formality and improved economic conditions, typically attributed to well-established pension systems in developed countries [30-32]. The results of these scenarios are summarized in Table 4 and Table 5.

**Table 4**

**Ex-ante optimal choice for the cohort ($\boldsymbol{\pi^{2,opt}}$, $\boldsymbol{\theta^{2,opt}}$), percentage, and feasibility of meeting the adequacy condition: comparison of alternative scenarios for employment structure**

| | | Risk aversion levels, φ | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Investment strategy** | **Proportion of self-employed** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** |
| Base | 12% | (25, 26) | (25, 45) | (25, 51) | (25, 54) | (25, 56) | (25, 57) | (24, 58) | (21, 58) |
| | 24% | (25, 26) | (25, 44) | (25, 51) | (25, 54) | (25, 56) | (25, 57) | (24, 58) | (21, 58) |
| | 48% | (25, 24) | (25, 44) | (25, 50) | (25, 54) | (25, 55) | (25, 57) | (25, 58) | (22, 58) |
| Strategy A | 12% | (25, 25) | (25, 27) | (25, 27) | (25, 27) | (23, 27) | (19, 27) | (17, 27) | (15, 27) |
| | 24% | (25, 25) | (25, 27) | (25, 27) | (25, 27) | (24, 27) | (20, 27) | (17, 27) | (15, 27) |
| | 48% | (25, 25) | (25, 27) | (25, 27) | (25, 27) | (25, 27) | (21, 27) | (18, 27) | (15, 27) |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Strategy B | 12% | (25, 18) | (25, 15) | (25, 15) | (25, 14) | (21, 14) | (17, 14) | (15, 14) | (13, 14) |
| | 24% | (25, 18) | (25, 15) | (25, 15) | (25, 14) | (21, 14) | (17, 14) | (15, 14) | (13, 14) |
| | 48% | (25, 18) | (25, 16) | (25, 15) | (25, 14) | (22, 14) | (18, 14) | (16, 14) | (14, 14) |

*Note:* Solutions that satisfy the adequacy condition are marked in green, while those that do not are marked in red.
*Source:* compiled by the authors.

**Table 5**
**Ex-ante optimal choice for the cohort ($\pi^{2,opt}, \theta^{2,opt}$), percentage, and feasibility of meeting the adequacy condition: comparison of alternative scenarios for density of contributions**

| Risk aversion levels, φ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Investment strategy** | **Density of contributions** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** |
| Base | 30% | (25, 0) | (25, 27) | (25, 39) | (25, 45) | (25, 49) | (25, 51) | (25, 53) | (25, 54) |
| | 57% | (25, 26) | (25, 44) | (25, 51) | (25, 54) | (25, 56) | (25, 57) | (24, 58) | (21, 58) |
| | 90% | (25, 39) | (25, 51) | (25, 55) | (25, 57) | (21, 58) | (18, 58) | (15, 58) | (13, 58) |
| Strategy A | 30% | (25, 23) | (25, 25) | (25, 26) | (25, 27) | (25, 27) | (25, 27) | (25, 27) | (25, 27) |
| | 57% | (25, 25) | (25, 27) | (25, 27) | (25, 27) | (24, 27) | (20, 27) | (17, 27) | (15, 27) |
| | 90% | (25, 26) | (25, 27) | (25, 27) | (19, 27) | (15, 27) | (12, 27) | (11, 27) | (9, 27) |
| Strategy B | 30% | (25, 22) | (25, 18) | (25, 16) | (25, 15) | (25, 15) | (25, 15) | (25, 14) | (25, 14) |
| | 57% | (25, 18) | (25, 15) | (25, 15) | (25, 14) | (21, 14) | (17, 14) | (15, 14) | (13, 14) |
| | 90% | (25, 16) | (25, 15) | (22, 14) | (17, 14) | (13, 14) | (11, 14) | (9, 14) | (8, 14) |

*Note:* Solutions that satisfy the adequacy condition are marked in green, while those that do not are marked in red.
*Source:* Compiled by the authors.

In Table 4, we observe that changes in the employment structure have virtually no effect on the optimal choice for the cohort. The significant impact is seen in the feasibility of meeting the adequacy condition when the share of self-employed individuals is doubled to 48% under the Base scenario. In this case, solutions satisfy the adequacy condition only for risk aversion levels of 1 and 2. In contrast, in the other cases with a lower proportion of self-employed workers, the optimal choice for the cohort satisfies the adequacy condition for risk aversion levels up to 6. This discrepancy arises because an increased share of self-employed workers reduces the overall density of contributions for the cohort from 57% to 52%, due to their relatively lower frequency of contributions. Combined with lower expected funded returns under the Base scenario, this reduction leads to a notable decline in the overall pension wealth of the cohort.

Finally, Table 5 shows that a significant drop in the overall density of contributions from the current level of 57% to 30% (due to factors such as rising unemployment spells, increased informality, etc.) prevents attaining optimal solutions that fulfil the adequacy condition of relative pension wealth across all

investment strategies and risk aversion levels. This finding highlight, once again, the crucial importance of maintaining regular contributions throughout one's employment.

On the other hand, with a higher density of contributions (90%), the feasibility of meeting the adequacy condition remains unchanged compared to the base case. However, in this scenario, as risk aversion increases, the optimal pension contribution rate declines more rapidly, while the optimal allocation to funding generally remains aligned with the base case. Notably, this effect is more pronounced with more diversified investment portfolios. Specifically, under Strategies A and B, when $\varphi = 3$ and 4, the cohort can achieve adequate levels of relative wealth with optimal contribution rates that are lower by up to 6 and 8 percentage points, respectively, compared to the base case. This reduction in contribution rates can potentially ease financial burdens on individuals and businesses, leading to economic benefits [33-36].

Consequently, a strategic methodology for alleviating fiscal stresses while enhancing the pension system's capacity to provide sufficient retirement income in Kazakhstan may encompass a blend of initiatives. These measures encompass offering the potential for increased returns on defined contribution assets through improved diversification of the default investment portfolio, formalising participation for informal workers, including the self-employed, and promoting enhanced individual involvement with the pension system. Given adequate financial, political, and administrative capacity, this holistic approach merits thorough examination by the nation's policymakers and regulatory authorities to guarantee effective pension design [37-39].

The Kazakhstan case study may also provide insightful information with wide-ranging relevance, especially for countries dealing with comparable socioeconomic issues. These countries can draw meaningful parallels and adapt relevant strategies to enhance their own pension frameworks. More focused strategies, like tying NDC rules to per capita GDP growth, might offer customized solutions for nations with high levels of exposure to domestic stocks, while broad policy initiatives like diversifying pension assets and encouraging individual participation in the pension system are in line with best international practices.

Still, one important thing that our results show is that there isn't a single best way to set contribution rates and pension system mixes in every country. The optimal setups depend on many things, such as how risk-averse people are, how many people are contributing, and what the expected financial and nominal rates will be. This underscores the importance of a nuanced approach to policy development, one that considers the unique economic and demographic factors of each country.

Lastly, the numerical results are compared with practical implementations, aligning them with similar analyses conducted in Alonso-García and Devolder [16]. In practice, countries with well-developed DC and NDC systems, such as Sweden, Latvia, and Poland, have a total pension contribution rate ranging from 18.5% to 20%, with allocations to funding between 13.5% and 30%. With more

diversified investment portfolios (Strategies A and B), a higher overall density of contributions (90%), and reasonable risk aversion levels between 1 and 4, the ex-ante pension contribution rates vary from 17% to 25%, and the ex-ante allocations to funding range from 14% to 27%. These figures demonstrate close alignment with the practical implementations observed in the aforementioned countries.

Finally, there are several potential directions for future research that could address some of the limitations of the current study. Firstly, by allowing $\mu_S$, $\mu_K$, and $\mu_L$ to be stochastic, the model could be extended to enable a more general approach to determining expected financial return, notional return, and average wage growth. Second, the financial market in the current study includes only two types of financial assets. Expanding the model to incorporate additional instruments, such as a second risky asset representing a domestic stock market index or zero-coupon bonds, could better capture diverse investment opportunities and risks. Third, given that our analysis assumes a fixed allocation to financial assets, it would be interesting to explore a time-dependent approach to asset allocation, akin to life-cycle funds. Consequently, based on the work of Alonso-García and Devolder [16], it is essential to analyse the circumstances of diversification between funded and unfunded components over different concurrently existent cohorts, while considering liquidity restrictions.

## 4. Conclusions

This study enhances the comprehension of overall optimal pension system design by analysing Kazakhstan, a nation that has adopted a hybrid system featuring both DC and NDC elements. The study uses a mean-variance approach, modelling financial, non-financial, and wage dynamics while incorporating varying wage profiles and contribution densities in Kazakhstan to identify the optimal contribution rate and allocation between funded and unfunded components and to assess the feasibility of achieving adequate pension outcomes. The findings offer valuable structural insights into the design and performance of the pension system under varying economic and policy assumptions.

There is a clear inverse relationship between risk aversion and the optimal contribution rate, while the funded proportion generally remains unaffected. However, when the contribution rate reaches its upper constraint and becomes fixed, this changes: the effect of risk aversion on the funded share then depends on whether the economy is dynamically efficient.

The analysis underscores that closely linking notional returns to wage growth, while inversely aligning them with financial market performance, is essential for achieving diversification benefits and supporting the sustainable growth of pension capital. Overall, to ensure the long-term adequacy of Kazakhstan's pension system while easing fiscal pressures, a coordinated approach is needed. This includes broader diversification of DC assets, expansion of formal participation among workers, and stronger incentives to

encourage regular and consistent contributions, particularly from informal and self-employed individuals.

The limitations of the study include its fixed asset allocation strategies and single cohort analysis. Future research ought to investigate stochastic modelling of return parameters, a wider array of asset classes, dynamic investment strategies, and multi-cohort contexts that consider liquidity and intergenerational equity – thereby enhancing the policy significance of the proposed approach.

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