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Applications of fuzzy sets in decision making

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Abstract

In this paper, the applications of fuzzy sets to decision making are considered. The Bellman-Zadeh approach is discussed, and decision making in the set of alternatives is studied. This paper is continuation of paper [26] in References.

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1. Introduction

The problem of achieving a fuzzy defined goal, formulated by Bellman and Zadeh, is based on the assumption that the decision goal and the set of alternatives are considered as equivalent fuzzy sets of a universal set, containing alternatives. This assumption allows the decision for the considered problem to be found relatively simply.

Topics discussed in this paper are considered in [1]-[32].

For the basic concepts and results, used in this paper, see, for example, [26].

Let X be such a universal set, containing alternatives, i.e., a set of all possible choices of the person making the decision. *Fuzzy goal* in X is a fuzzy set in X, denoted by G. In this case, the fuzzy membership function $m_G: X \to [0,1]$ is considered.

Example 1. If $X = \mathbf{R}$, then a fuzzy set in X, for example, is the set

$\{x$ is significantly larger than 10$\}$.

The fuzzy goals are assumed to be completely described by the membership functions. The greater the degree of membership of the alternative x to the considered fuzzy set G, that is, the greater the value of $m_G(x)$, the greater the degree of achievement of this goal when choosing the alternative x as a decision.

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The *fuzzy constraints* are also described by fuzzy sets. In Example 1, the fuzzy relation can be described, for example, as follows: x should be approximately equal to 35.

To solve a problem for achieving a fuzzy goal means to achieve this goal (to one degree or another) and to satisfy the constraints.

The Bellman-Zadeh approach for solving the above problem consists in the following. Let the alternative x ensure achievement of the goal with degree $m_G(x)$ and let x satisfy the constraints with degree $m_C(x)$. Then it is assumed that the degree of membership of this alternative to the decision for the problem is equal to the minimum of the values of $m_G(x)$ and $m_C(x)$. A *fuzzy decision* for the problem of achieving a fuzzy goal is the intersection of the fuzzy sets of the goal and of the constraints, that is, the membership function of the decision is

$$m_D(x) = \min\{m_G(x), m_C(x)\}.$$

If we have multiple goals $G_1, \dots, G_n$ and constraints $C_1, \dots, C_l$, the fuzzy decision is described by a membership function as follows

$$m_D(x) = \min\{m_{G_1}(x), \dots, m_{G_n}(x); m_{C_1}(x), \dots, m_{C_l}(x)\}.$$

If the goals and the constraints differ in their degree of importance, and if the corresponding coefficients of relative importance $g_1, \dots, g_n$ of the goals and coefficients $c_1, \dots, c_l$ of importance of the constraints are given, then $m_D(x)$ can be defined as

$$m_D(x) = \min\{g_1 m_{G_1}(x), \dots, g_n m_{G_n}(x); c_1 m_{C_1}(x), \dots, c_l m_{C_l}(x)\}.$$

Example 2. Let $X = \{1,2, \dots,10\}$ be the considered universal set, and let the following data be given:

a) a fuzzy goal $G =$ "x is close to 5",

b) the first fuzzy constraint $C_1 =$ "x is close to 4",

c) the second fuzzy constraint $C_2 =$ "x is close to 6".

The membership functions of G, C_1, and C_2 are presented in the following Table 1.

Table 1, Example 2.

x	1	2	3	4	5	6	7	8	9	10
$m_G(x)$	0	0.1	0.4	0.8	1.0	0.7	0.4	0.2	0.1	0
$m_{C_1}(x)$	0.3	0.6	0.9	1.0	0.9	0.7	0.5	0.3	0.2	0
$m_{C_2}(x)$	0.2	0.4	0.6	0.7	0.9	1.0	0.8	0.6	0.6	0.2

Then about the decision D for the considered problem, we obtain the following membership function $m_D(x)$ (see Table 2).

Table 2, Example 2.

x	1	2	3	4	5	6	7	8	9	10
$m_D(x)$	0	0.1	0.4	0.7	**0.9**	0.7	0.4	0.2	0.1	0

Since the largest value of $m_D(x)$ is 0.9 according to the above table, then the decision D is interpreted as follows: x should be close to 5. This decision

can be considered as a non-strictly formulated instruction, whose achievement guarantees the achievement of a fuzzily set goal. However, the question which alternative should be chosen remains open.

There are various ways for solving this uncertainty. The most common way, proposed by Zadeh, is to choose an alternative with the maximum membership degree to the considered fuzzy set, i.e., the alternative, determined by the following condition

$$\max_{x \in X} m_D(x) = \max_{X} \min\{m_G(x), m_C(x)\}.$$

Such alternatives are called *maximizing alternatives*.

In many cases, a decision under fuzzy conditions is made not at one stage but at several stages – these are *multi-stage decision making processes*. Such an approach is mainly applied to the so-called dynamical systems.

2. Relations of fuzzy preference in the considered set of alternatives

The decision process, that is, the choice of the best (the most rational) alternative of the universal set X of alternatives, can be made under various degrees of information, available to the decision maker.

One of the ways of presenting information about a real situation is by the utility function (see, for example, [26]). However, such a way of describing the information is not always possible. A more universal way of describing the information is through the preference relations in the set of alternatives.

Let, based on information, received by the decision maker, a (non-fuzzy) strict preference relation R be defined in the set of feasible alternatives X, i.e., for each pair of alternatives $x, y \in X$, only one of the three statements listed below is made:

1. x is no worse than y, i.e., $x \geq y$ or, alternatively, $(x, y) \in R$;
2. y is no worse than x, i.e., $y \geq x$ or, alternatively, $(y, x) \in R$;
3. x, y are not comparable to each other, i.e., $(x, y) \notin R$ and $(y, x) \notin R$.

The above data helps to shrink the group of the reasonable options, including in this choice only the alternatives, which are not dominated by any other alternative in the set X.

Alternative x is said to be *strictly better* than the alternative y if $x \geq y$ and $y \ngeq x$, i.e., if $(x, y) \in R$ but $(y, x) \notin R$. The set of all ordered pairs (x, y) with this property is said to be a *strict preference relation* R^s in the considered set X.

To describe a fuzzy preference relation R in the considered set X, we can use the membership function $m_R(x, y)$, which for every pair of alternatives $(x, y) \in X$ is defined as the degree of fulfilment of the preference $x \geq y$.

Similarly to the "exact" case, *fuzzy relation of strict preference* can be defined as

$$m_R^s(x, y) = \begin{cases} m_R(x, y) - m_R(y, x), & \text{if } m_R(x, y) \geq m_R(y, x), \\ 0, & \text{otherwise.} \end{cases}$$

The totality of all alternatives, each of which cannot be dominated by any other alternative of the considered set X, is called the *fuzzy subset of non-dominable alternatives* and it is denoted $R^{n.d.}$. The membership function $m_R^{n.d.}(x)$ of the set of non-dominable alternatives is defined as

$$m_R^{n.d.}(x) = 1 - \sup_{y \in X} m_R^s(y, x), x \in X.$$

The set $X^{n.d.}$ of all alternatives, which satisfy the condition

$$m_R^{n.d.}(x^*) = \sup_{x \in X} m_R^{n.d.}(x),$$

that is, the set of all alternatives whose membership function takes maximum value, is called the *set of maximal non-dominable alternatives* in the considered set $(X, m_R(x))$.

Example 3. Let $X = \{x_1, x_2, x_3, x_4\}$ be a set of finite number of elements with a fuzzy preference relation of the form (see Table 3).

Table 3, Example 3.

$m_R(x_i, x_j) =$

$i \setminus j$	x_1	x_2	x_3	x_4
x_1	1.0	0.2	0.3	0.1
x_2	0.5	1.0	0.2	0.6
x_3	0.1	0.6	1.0	0.3
x_4	0.6	0.1	0.5	1.0

From the definition of $m_R^s(x, y)$ we get the following Table 4.

Table 4, Example 3.

$m_R^s(x_i, x_j) =$

$i \setminus j$	x_1	x_2	x_3	x_4
x_1	0	0	0.2	0
x_2	0.3	0	0	0.5
x_3	0	0,4	0	0
x_4	0.5	0	0.2	0

hence we obtain the following Table 5.

Table 5, Example 3.

$m_R^{n.d.}(x) =$

x_1	x_2	x_3	x_4
0.5	0.6	**0.8**	0.5

From the fact that the highest degree of non-dominability 0.8 corresponds to the alternative x_3 it follows that the choice of x_3 is the best choice (the most reasonable choice).

Given a fuzzy set $A \subset X$ and a relation between the fuzzy sets $A, B \subset X \times Y$. Find the values of B in Y.

The decision is made using the so-called *Zadeh's compositional rule*.

Example 4: Let $A =$ {a relatively low-placed apartment} and let the price B of A is high. Find the price B.

Let the basic sets be defined as follows:

$X = \{\text{floor} = 1,2,3,4,5\}$,

$Y = \{\text{price} = 100000,105000,110000,115000,120000\}$,

let A in $X = \{$a relatively low-placed apartment$\}$ be defined by the fuzzy set

$$A = \{(1,1), (2,0.8), (3,0.3), (4,0.2), (5,0.1)\},$$

and let the relation R in $X \times Y = \{$if the apartment is low-placed then its price is high$\}$ be defined by the following Table 6.

Table 6, Example 4.

$X \setminus Y$	100000	105000	110000	115000	120000
1	0.2	0.4	0.6	0.8	1.0
2	0.2	0.4	0.6	0.8	0.8
3	0.8	0.8	0.8	0.8	0.8
4	1.0	1.0	1.0	1.0	1.0
5	1.0	1.0	1.0	1.0	1.0

Then B in Y is the fuzzy set

$B = \{(100000,0.2), (105000,0.4), (110000,0.6), (115000,0.8), (120000,1)\}$

and since $m_B = 1$ corresponds to the highest price 120000, then B can be interpreted as "relatively high price".

3. Conclusion

In this paper, the applications of fuzzy sets to decision making are considered. The Bellman-Zadeh approach is discussed, and decision making in the set of alternatives is studied. This paper is continuation of paper [26] in References.

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