Journal of Interdisciplinary Mathematics
ISSN: 0972-0502 (Print), ISSN: 2169-012X (Online)
DOI : 10.47974/JIM-2406

An exact solution model to nonlinear equations for circuit design and analysis

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Abstract

In the realm of electrical circuit analysis, this paper presents a groundbreaking approach that revolutionizes the handling of nonlinear differential equations. Our method not only provides exact solutions but does so with unparalleled efficiency, optimizing computational resources and minimizing storage requirements. One of its remarkable features is its robustness, as it exhibits insensitivity to initial conditions. At the heart of our innovation lies a hybrid technique that seamlessly integrates trial and exponential functions. This method transcends the limitations of traditional approaches accommodating nonlinear differential equations of any order. To demonstrate its real-world applicability and performance, we apply our method to a novel communication circuit. The results affirm its prowess by delivering precise solutions without resorting to conventional numerical methods. By harnessing the

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power of this method, engineers and researchers gain an invaluable tool for circuit analysis, paving the way for enhanced accuracy and efficiency in the realm of electrical engineering.

Subject Classification (2010): *34A34, 65Z05.*

Keywords: *Circuit analysis, Differential equations, Exact solution, Nonlinear, System equations.*

1. Introduction

1.1 *Nonlinear systems*

Nonlinear differential equations (NDEs) have a wide range of applications in physics and engineering. Some of these circuits are of great importance to ensure security in the communication system some of which are known as oscillators or modulation units. Such circuit designs are represented by nonlinear differential equations which make analysis very difficult. Nonlinear differential equations use numerical methods to obtain approximate solutions. However, it is well known that there is no fixed solution available for a general nonlinear circuit representation [1-4]. This method is time-consuming and requires a large storage capacity for evaluation. Several researchers have introduced alternative methods to overcome these problems. Stochastic solvers (SSs) are the latest research topics introduced to solve nonlinear differential equations [5, 6,7,8]. An author introduced a numerical inverse Laplace transform method to solve accurately solve nonlinear differential equations as in [9]. Alternatively, to Laplacian methods, the authors of [30] successfully applied Modified Sumudu Adomian Decomposition method for solving nonlinear partial decomposition method.

The local method uses the Runge-Kutta numerical analysis to solve the nonlinear differential equations [10, 11]. The global method uses a suitable basis set to produce a solution [12]. The global method becomes more difficult to process when the complexity of the nonlinear equation increases.

Researchers try many other methods than the classical numerical methods to produce exact solutions to nonlinear equations [13]. Some of which produces an exact solution to a specific type of nonlinear equation [14]. A research paper introduced a generalized bifurcation method to find an exact solution with great success [15]. The paper [16] investigated to produce an exact solution to nonlinear equations using the simplest equation method. The author of the book [17] introduces a new method for the separation of variables to reduce complexity in the calculation procedure. Some authors used hybrid methods to find an exact solution to nonlinear type equations [18, 19, 20].

It is known that nonlinear equations play a great deal of importance in circuit design and analysis in engineering. As it can be understood from the intense interest of the researchers, it is of great importance to produce exact solutions to the nonlinear equations in which they are represented to increase the performance of the circuits in the field of engineering. In today's technology, the circuits (oscillator, modulator) used in communication systems are at the

forefront. This is motivated by the application of the non-linear communication circuits and the search for an exact solution is the objective of this work.

1.2 Trial equation methods

In papers [21] and [22], the trial equation methods are introduced to the solution of the constant-coefficient higher-order nonlinear differential equation,

$$M\ (u, u', u'', \dots) = 0 \tag{1}$$

The trial equation methods simply assume that the solution $u = F(u)$ where $F(u)$are polynomial functions and,

$$u' = \sum_{i=0}^{m} a_i u^i \tag{2}$$

After these assumptions, the nonlinear Equation (1) is reduced to a system of simple linear equations. After solving this linear system, the coefficient a_i, i=1,…, m can be found. The prime object is to solve the Equation (2) to solve for u such that it can be substituted in Equation (1) to solve F(u). After these introductory papers, some extended versions of the initial trial equation methods have been introduced. Further details are covered in papers [23, 24], and [25]. Paper [25] generalized all defined trial equation methods and named as "generalized trial equation method". The generalized trial equation method is successfully applied to some nonlinear differential equations.

Finally, the generalized trial equation method was also a pioneer in the development of a method for solving nonlinear systems of equations. Recently, Ozyapici and Bilgehan firstly introduced a new method for the solution of a non-linear system of differential equations in the paper [26]. The method can be stated as:

$$\begin{aligned} x'_1 &= F_1(x_1, x_2, \dots, x_n) \\ x'_2 &= F_2(x_1, x_2, \dots, x_n) \\ &\dots \\ x'_n &= F_n(x_1, x_2, \dots, x_n). \end{aligned} \tag{3}$$

Consequently, the solutions x_i , $i = 1, \dots, n$ are defined

$$x_i = \Psi_i(u),\ i = 1, \dots, n \tag{4}$$

where $u' = g(u)$. The selection of $\Psi_i(u)$, $i = 1, \dots, n$ may cause the function $g(u)$ to be nonlinear. The nonlinear processing is possible if Equation (3) is transformed into a linear representation. It is important to note that the generalized system of trial equation method can effectively be applied to nonlinear biological systems in the paper [26]. After the promising result of the paper [26], the method has also been successfully applied to electrical circuits [27].

The paper is organized as follows. In Section 2, the derivation process of the exact solution to nonlinear circuit application is introduced. Section 3 purpose-built circuit design is proposed. Presents the introduced exact solution method to the electrical circuit. In Section 4, the circuit has been analyzed with a

newly introduced hybrid model. Section 5, concluded with the findings of the newly introduced method.

2. A Hybrid Method Derivation of The Exact Solution to Nonlinear Circuits

The exact solution to the nonlinear differential equation has always been of great interest in applications. There are many alternative methods applied by researchers to solve a specific problem. This research work aims to produce a general solution to a system of nonlinear differential equations. Therefore, the flexible characteristics of the Trial equation and the exponential function are combined.

The nonlinear system of equations is considered to be general-purpose to apply in electronics circuit design as oscillator and communication circuit to be used for secure data transmission. Consider the following nonlinear differential equations as:

$$\begin{aligned} \frac{dx}{dt} &= -\beta \mathrm{X} + \alpha \mathrm{YZ} \\ \frac{dy}{dt} &= -\mu \mathrm{Y} + \gamma XZ + \eta \mathrm{X} \\ \frac{dz}{dt} &= -\lambda \mathrm{Z} + \delta \mathrm{XY} \end{aligned} \tag{5}$$

The system equations in (5) is an important model having a wide range of applications, especially in electrical and electronic engineering. Additionally, the system is very similar to the very famous Lorenz system of three nonlinear ordinary differential equations introduced by American mathematician Edward Lorenz. The Lorenz system is described as:

$$\begin{aligned} \frac{dx}{dt} &= \alpha(\mathrm{Y} - \mathrm{X}) \\ \frac{dy}{dt} &= X(\mu - Z) - \mathrm{Y} \\ \frac{dz}{dt} &= \mathrm{XY} - \lambda \mathrm{Z} \end{aligned} \tag{6}$$

The Lorenz Equation (6) also arises in simplified models for especially lasers, dynamos, DC motors, chemical reactions, and importantly electric circuits. It is observed from the research that it is very important to solve Equation (5). The solution of this model will also be a guide in solving other similar models as well as the Lorenz system. Since model (5) is three dimensional and complicated system, it is not easy to construct a trial equation method. So, an alternative approach should be considered for the formation of the trial equation methods. It is important to realize that the methods of the exponential function are suggested to be combined with the trial equation using the dependent variable z(t). In this method, the solution z(t) is assumed as:

$$z(t) = \frac{a_0 + a_1 \exp(a_2 t + a_3) + a_2 \exp(-a_2 t - a_3)}{a_4 + a_5 \exp(a_2 t + a_3) + a_6 \exp(-a_2 t - a_3)} \tag{7}$$

where $a_i = 0, \ldots 6$, are constants to be determined. For more details for the exp-function method, we refer to the paper [28] and the papers therein.

For the solution of model (5), we firstly combine the trial equation method with an exp-function method for deriving the exact solutions. Hence in this

paper, a new approach is being developed by combining the system of trial equation and exp-function method.

Our starting point for solving Equation (5) is:

$$X(u) = a_0 + a_1 \exp(p * u). \tag{8}$$

$$Y(u) = b_0 + b_1 \exp(p * u), \tag{9}$$

and

$$Z(u) = c_0 + c_1 \exp(p * u). \tag{10}$$

Let us now consider $X(u)$, $Y(u)$ and $Z(u)$ such that u'can be stated as:

$$u' = g(u) = d_0 + d_1 exp(p * u) + d_2 exp(-p * u) \tag{11}$$

The solution to Equation (11) determines the unknown coefficients stated in (8), (9), (10), and (11).

After substituting (8), (9), (10), and (11) into the nonlinear system of Equation (5) that yields the unknown parametric values. The parameter values are very much dependent on the following term.

$$exp(p * u), exp(2p * u) \text{ and constant.} \tag{12}$$

The unknown coefficients can be discovered by setting the terms in Equation (12) to zero. The system (5) has three nonlinear equations with nine algebraic equations that consist of the terms in (12), for the unknown parameters. The unknown parameters can be calculated as:

$$\begin{aligned}
&a_0 = \alpha_0, a_1 = \alpha_1, \\
&b_0 = -\eta\left(\alpha_0 + \frac{\lambda}{\sqrt{\gamma} * \sqrt{\delta}}\right), b_1 = -\frac{\eta * \alpha_1}{2\beta}, \\
&c_0 = -\frac{\mu(-\beta + \alpha_0\sqrt{\gamma}\sqrt{\delta} + \lambda)}{2\beta\gamma}, c_1 = \frac{-\mu\sqrt{\delta}\alpha_1}{2\beta\sqrt{\gamma}}, \\
&d_0 = \frac{2(\alpha_0\sqrt{\gamma}\sqrt{\delta} + \lambda)}{p}, d_1 = \frac{\sqrt{\gamma}\sqrt{\delta}\alpha_1}{p}, d_2 = \frac{-\beta\lambda + (\alpha_0\sqrt{\gamma}\sqrt{\delta} + \lambda)^2\sqrt{\delta}\alpha_1}{\sqrt{\gamma}\sqrt{\delta}\alpha_1 p}, \\
&\mu = \beta - \lambda, \alpha = \frac{4p^2\gamma}{\mu^2}.
\end{aligned} \tag{13}$$

The parameters in Equation (13) are substituted into Equation (8), (9), (10) and each equation in the system (5) is redefined accordingly.

Considering the parameter in (13), the first-order differential Equation (11) can explicitly be solved and produces

$$u(t) = g(u) = \frac{ln\left(-(\alpha_0\sqrt{\gamma}\sqrt{\delta} - \lambda - \sqrt{p.\lambda})tanh\left(\sqrt{\beta\lambda}t + \sqrt{\beta\lambda}\sqrt{\gamma}\sqrt{\delta}\alpha_1 p.C\right)\right)}{\sqrt{\gamma}\sqrt{\delta}\alpha_1 p} \tag{14}$$

The variable C denotes an integral constant and $tanh$ is a tangent hyperbolic function.

Finally, exact solutions of the nonlinear differential equations can respectively be obtained after using the hybrid method as:

$$X(t) = \frac{-\lambda - \sqrt{\beta\lambda}tanh\left(\sqrt{\beta\lambda}(t + \sqrt{\gamma}\sqrt{\delta}\alpha_1 p.C)\right)}{\sqrt{\gamma}\sqrt{\delta}} \tag{15}$$

$$(t) = \frac{-\eta\sqrt{\frac{\lambda}{\beta}}-tanh\left(\sqrt{\beta\lambda}\left(t+\sqrt{\gamma}\sqrt{\delta}\alpha_1 p.C\right)\right)}{2\sqrt{\gamma}\sqrt{\delta}} \tag{16}$$

$$Z(t) = \frac{\eta+\mu\sqrt{\frac{\lambda}{\beta}}-tanh\left(\sqrt{\beta\lambda}\left(t+\sqrt{\gamma}\sqrt{\delta}\alpha_1 p.C\right)\right)}{2\gamma} \tag{17}$$

where C is integral constant.

3. Purpose-Built Generalized Circuit Design

The systems that have been studied the most in the field of communication are Chua, Rossler oscillators, and Lorenz systems. The dynamic behavior of such systems is well known and it offers an advantage in the evaluation process. They offer a broadband spectrum and secure transmission in communication systems. A new purposely designed oscillating circuit has been introduced. The newly introduced mathematical method was applied to the circuit for signal analysis purposes. The block diagram to be designed is given in Fig 1.

A new three-dimensional nonlinear system of Equation (5) is proposed. The aim is to analyze the system and produce a solution to nonlinear differential equations representing the system at specific points. The system can be forced to operate under chaotic cases but the intention is to analyze the system under the equilibrium point to produce an exact solution to the equations at minimum computational complexity. Such analysis may be important when the circuit is operating in oscillating circuits or communication systems that require synchronization between the transmitting and the receiving unit. Figure 1 is a general representation of the system described in the system of Equation (5).

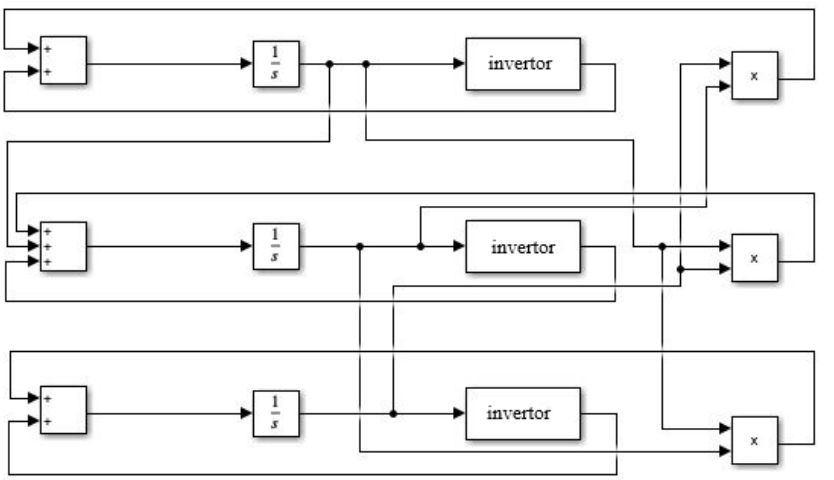

Figure 1
Block diagram of the system described in the system of Equation (5)

The block diagram in Figure 1 helps to represent the cause-and-effect relationship of the system. The functional operation of the units is identified by

the symbols. The operator 1/s indicates the integration operation and x represents a multiplier unit.

The block diagram of the system in Figure 1 can be derived by proper selection of electronic components as in Figure 2. The operational amplifiers marked as U1, U2, and U7 are summing the input signals to the system. The integrators are marked as U3, U4, and U8. The operational amplifiers marked as U5, U6, and U9 are used as inverters in the circuit. The analog multipliers are denoted as A1, A2, and A3.

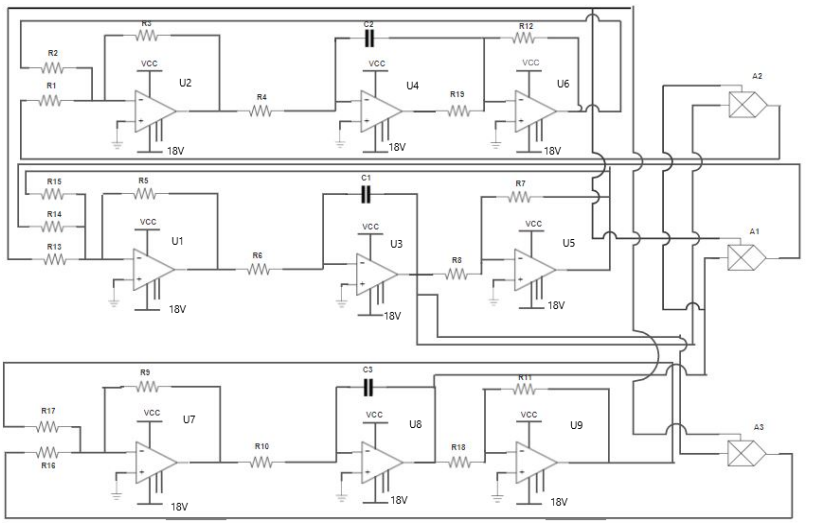

Figure 2

The circuit design of the system described in the system of Equation (5)

The circuit representation in Figure 1 can be analyzed by setting the differential equations at three points of interest. The general representation of the system described in the system of Equation (5) needs to be identified with the circuit components indicated in Figure 2. The variables α, β, γ, μ, η, δ and λ in the system of equation (5) needs to be defined with circuit components illustrated in Figure 2. The variables X, Y, Z are the state variables the circuit analysis in line with parameters used in the system of Equation (5).

The operational amplifier circuit analysis produces the following relationship.

$$\beta = \frac{R3}{R2R4C2}, \alpha = \frac{R3}{R1R4C2}, \mu = \frac{R5}{R15R6C1}, \gamma = \frac{R5}{R14R5C1},$$
$$\eta = \frac{R5}{R13R6C1}, \lambda = \frac{R9}{R17R10C3}, \delta = \frac{R9}{R16R10C3}$$

Substituting variable components in the system of Equation (5),

$$\begin{aligned}\frac{dx}{dt} &= -\frac{R3}{R2R4C2}x + \frac{R3}{R1R4C2}yz \\ \frac{dy}{dt} &= -\frac{R5}{R15R6C1}y + \frac{R5}{R14R5C1}xz + \frac{R5}{R13R6C1}x \\ \frac{dz}{dt} &= -\frac{R9}{R17R10C3}z + \frac{R9}{R16R10C3}xy\end{aligned} \tag{18}$$

4. Circuit Analysis

In this section, real numerical values of the circuit will be used for the analysis. The results in this section show that the nonlinear system defined in the system of Equation (5) with respective solutions of (15), (16), and (17) can be applied in real applications and it produces an exact solution to the system.

The circuit is a typical synchronization unit that can be used in a communication system. The accuracy of the result is very important to maintain the stability of the synchronized state for coupled circuits. The synchronization dynamics of various circuit systems are well defined [29].

Considering the operation of the practical op-amps, relevant electronic component values are selected as

R1 = 10 KΩ R2 = 36 KΩ R3 = 2.2 KΩ R4 = 20 KΩ R5 = R9 = 1.1 KΩ

R6 = 22 KΩ R10 = R13 = 27 KΩ R14 = 51 KΩ R15 = 77 KΩ R17 = 11 KΩ

C1 = C2 = C3 = 1μF

R7, R11, R12, R18, R19 =10 KΩ

The substitution of the selected component values in Equation (18) is evaluated as:

$$\begin{aligned}\frac{dx}{dt} &= -3.06x + 9.176328yz \\ \frac{dy}{dt} &= -0.64y + 0.98xz + 2x \\ &\text{and} \\ \frac{dz}{dt} &= -3.70z + 0.9475xy\end{aligned} \tag{19}$$

.

The general solutions of the nonlinear Equation (19) have been derived in Equation (8), (9), and (10) with a set of their necessary parameters (13). Remaining, we need to substitute the coefficients of (19) into corresponding solutions. In the nonlinear Equation (19) $\beta = -3.06, \gamma = 0.98, \mu = -2, \lambda = -3.710$, and $\delta = 0.9475$ are found in these real applications. Hence, the solutions

$$\begin{aligned}x(t) &= 3.8397 - 3.4918\tanh(3.3648t + 3.2423 * n * p * 0.98 * C) \\ y(t) &= 1.1411\tanh(3.3648t + 3.2423 * 0.98 * C) \\ z(t) &= -1.0204 + 1.1220\tanh(3.3648t + 3.2423 * 0.98 * C)\end{aligned} \tag{20}$$

are obtained where C denotes an integral constant and the numerical values are truncated to four digits after the decimal point.

For simplicity, n=p=C=1 has been assumed. Substitution of n, p, and c values into Equation (20) gives:

$$x(t) = 3.8397 - 3.4918\,tanh(3.3648t + 3.1774)$$
$$y(t) = 1.1411tanh(3.3648t + 3.1774) \quad (21)$$

and

$$z(t) = -1.0204 + 1.1220tanh(3.3648t + 3.1774)$$

5. Conclusion

In this study, we have unveiled a groundbreaking nonlinear analysis method that empowers engineers and researchers to conquer complex systems of equations with precision. Through the fusion of numerical and analytical techniques, we have not only proposed an exact solution but have also laid bare the inner workings of these nonlinear systems.

Our innovative approach, anchored in the synergy of trial and exponential equations, breathes life into mathematical setups that defy convention. The elusive solutions to nonlinear differential equations, even those laden with unknown circuit parameters or intricate variable interplay, succumb effortlessly to our method. The results obtained through our approach are nothing short of formidable. Robustness and reliability are its hallmark, offering a dependable compass for circuit design and analysis. What sets it apart is its simplicity compared to classical numerical methods, making it accessible to both novices and experts in the field. To underscore the practicality and accuracy of our method, we subjected it to rigorous testing with a real-world circuit. The results speak volumes, affirming its efficacy as an indispensable tool for authentic circuit design and analysis applications. In a world where complexity often begets confusion, our nonlinear analysis method shines as a beacon of clarity, promising a brighter and more efficient future for circuit engineers and analysts alike.

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Received May, 2025