

**EXISTENCE OF MASSLESS AND NON-EXISTENCE OF
MASSIVE SCALAR WAVES WITH GRAVITATIONAL WAVES
FOR GENERALIZED PERES SPACE-TIME**

V.R. PATIL¹, D.D. PAWAR², AND A.G. DESHMUKH³

¹DEPARTMENT OF MATHEMATICS, ARTS, SCIENCE AND COMMERCE COLLEGE
CHIKHALDARA DIST AMRAVATI-444807, INDIA

E-MAIL: VRPATIL2007@REDIFFMAIL.COM

²GOVT. VIDARBHA INSTITUTE OF SCIENCE AND HUMANITIES
AMRAVATI-444604, INDIA

E-MAIL: DYPAWAR@YAHOO.COM

³EX. READER AND HEAD, DEPT. OF MATHEMATICS, G.V.I.S.H, AMRAVATI
JOINT DIRECTOR, (H.E.), NAGPUR DIVISION, NAGPUR-440022, INDIA

E-MAIL: DRAGD2003@YAHOO.CO.IN

ABSTRACT. This paper deals with the existence of massless scalar field with plane gravitational waves, plane gravitational waves coupled with electromagnetic waves and non-existence of massive scalar field with plane gravitational waves, plane gravitational waves coupled with electromagnetic waves for generalized Peres space-time.

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1. INTRODUCTION

Takeno [8] has rigorously studied plane gravitational waves mathematically and shown the co-existence of gravitational waves with electromagnetic waves. Pawar

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et al. [5] has established the co-existence of plane gravitational waves with electromagnetic waves in six dimensional space-time, Deshmukh et al. [1] have studied the non-existence of massive scalar wave with gravitational waves and coupled with gravitational and electromagnetic waves in four dimensional space time for $Z = (z - t)$ and $Z = (\frac{t}{z})$ -type waves on the line of Takeno [8]. For our investigation, we have consider the Einstein-Maxwell field equations in general relativity as,

$$(1) \quad R_{ij} = (-8\pi) \left[T_{ij} - \frac{1}{2} g_{ij} T \right], \quad (i, j = 1, 2, 3, 4),$$

where R_{ij}, T_{ij} are Ricci tensor and energy momentum tensor of scalar field respectively. Furthermore Lal and Pandey [2] have worked on Peres space-time [6] and shown that the plane waves g_{ij} 's of Peres space-time is the solution of weakened field equations. Takeno [9] has investigated the generalized Peres space-time represented by the metric

$$(2) \quad ds^2 = -Adx^2 - Bdy^2 - (1 - E) dz^2 - 2Edzdt + (1 + E) dt^2,$$

where $A = A(x, Z), B = B(x, Z)$ and $E = E(x, y, Z)$ in which $Z = (z - t)$.

Lal and Pradhan [7] have taken space-time (2) and concluded that, the g_{ij} 's of generalized Peres space-time (2) is the Plane wave-like solution of weakened field equations. It is not difficult to verify that the metric (2) admits, a parallel null vector and also a gravitational null field, that condition which characterize Peres space-time of Takeno [9]. Accordingly the space-time represented by the metric (2) is called a generalized Peres space-time. In the present work, we have investigated the co-existence of massless scalar field with plane gravitational waves and together with electromagnetic waves. Further, we have linked up the existence of massless scalar field with gravitational waves and along with electromagnetic waves to the eigen values of energy momentum tensor. Moreover, we have obtained the non-existence of massive scalar field with plane gravitational waves and coupled with electromagnetic waves. Further the non-existence of massive scalar field with plane gravitational waves, coupled with electromagnetic waves to the eigen values of energy momentum tensor. It has been observed that the waves do not exist which are related to the non-zero equal eigen values of energy momentum tensor.

2. EXISTENCE OF MASSLESS SCALAR FIELD WITH GRAVITATIONAL WAVES AND COUPLED TO ELECTROMAGNETIC WAVES

Case (I): Scalar field coupled with gravitational field.

The scalar field is described by the tensor,

$$(3) \quad T_{ij} = \frac{1}{4\pi} \left[V_i V_j - \frac{1}{2} g_{ij} V_s V^s \right], \quad (i, j = 1, 2, 3, 4),$$

where V is the scalar function of Z and $V_i = \frac{\partial V}{\partial x^i}$.

$$(4) \quad V_s V^s = V_s g^{sp} V_p = 0.$$

Using equation(3) and (4) we obtained,

$$(5) \quad T_{ij} = \frac{1}{4\pi} [V_i V_j] \Rightarrow T = 0.$$

Also, the non-vanishing components V_3 and V_4 are assumes the form

$$(6) \quad V_3 V_3 = -V_3 V_4 = V_4 V_4 = \bar{V}^2.$$

Using equation (3),(4) and (5), the field equation (1) yield,

$$(7) \quad R_{ij} = (-2) V_i V_j.$$

For the line element (2), we have

$$(8) \quad R_{33} = -R_{34} = R_{44} = \epsilon,$$

where

$$\epsilon = \frac{\alpha}{A} + \frac{\beta}{B},$$

in which

$$\alpha = \frac{1}{2} \left[\bar{\bar{A}} + E_{xx} - \frac{1}{2A} (A^2 - A_x E_x) \right],$$

$$\beta = \frac{1}{2} \left[\bar{\bar{B}} + E_{yy} - \frac{1}{2B} (B^2 - B_y E_y) \right].$$

From equation (6) and (7), we obtain

$$(9) \quad R_{33} = -R_{34} = R_{44} = (-2) \bar{V}^2.$$

Above equation (9) is compatible with equation (8) and in consequence of that we established the existence of massless scalar field with gravitational waves. Now, we

relate the existence of massless scalar field with eigen value problem of the energy momentum tensor by using eigen equation,(Takeno[8])

$$(10) \quad \det(T_{ij} - \lambda g_{ij}) = 0$$

where λ is the scalar curvature of Einstein space.

Solving the above equation, we get

$$(11) \quad \lambda^4 \det(g_{ij}) = 0.$$

Since $\det(g_{ij}) = -AB \neq 0$, this implies

$$(12) \quad \lambda^4 = 0.$$

From this, it is clear that all the four eigen values are zero. Hence the principle directions are not unique. Thus the co-existence of gravitational waves with massless scalar waves are related to eigen values of T_{ij} are all zero.

Case (II): Scalar field coupled with gravitational and electromagnetic field.

The scalar field is described by the tensor,

$$(13) \quad T_{ij} = E_{ij} + \frac{1}{4\pi} \left[V_i V_j - \frac{1}{2} g_{ij} V_s V^s \right], \quad (s = 1, 2, 3, 4),$$

where V is the scalar function of Z and $V_i = \frac{\partial V}{\partial x^i}$ and

$$(14) \quad E_{ij} = \frac{1}{4} g_{ij} F_{kl} F^{kl} - F_{il} F_{jm} g^{lm},$$

is the energy momentum tensor of an electromagnetic field (Landu and Lifshitz [4]).

And F_{ij} is the electromagnetic Maxwell skew tensor given by

$$(15) \quad F_{ij} = K_{j;i} - K_{i;j} = K_{j,i} - K_{i,j}, \quad K_i = K_i(Z),$$

semicolon (;) means covariant derivative with respect to g_{ij} and comma (,) stand for partial derivative with respect to x^i .

From equation (14) and (15), we obtain

$$E_{33} = -E_{34} = E_{44} = \frac{B\sigma^2 + A\rho^2}{AB}.$$

Using (4), the gravitational field (13) and the field equation (1), reduces to

$$(16) \quad T_{ij} = E_{ij} + \frac{1}{4\pi} [V_i V_j] \text{ and}$$

$$(17) \quad R_{ij} = (-8\pi) \left[E_{ij} + \frac{1}{4\pi} V_i V_j - \frac{1}{2} g_{ij} T \right], \quad (i, j = 1, 2, 3, 4).$$

From equation(16), we get

$$(18) \quad T = T_i^i = 0.$$

Using equation (18), equation (17) yield,

$$(19) \quad R_{ij} = (-2) [4\pi E_{ij} + V_i V_j], \quad (i, j = 1, 2, 3, 4).$$

Using equation(6), equation (19) implies

$$R_{33} = (-2) [4\pi E_{33} + \bar{V}^2],$$

$$R_{34} = (-2) [4\pi E_{34} + \bar{V}^2],$$

$$R_{44} = (-2) [4\pi E_{44} + \bar{V}^2].$$

$$(20) \quad R_{33} = -R_{34} = R_{44} = (-2) \left[4\pi \left(\frac{B\sigma^2 + A\rho^2}{AB} \right) + \bar{V}^2 \right].$$

Equation(20) is compatible with (8) and hence it follows the existence of massless scalar waves coupled with gravitational and electromagnetic waves. Also, we have linked up the eigen value of energy momentum tensor with existence of massless scalar field by using characteristic equation,

$$(21) \quad \det (T_{ij} - \lambda g_{ij}) = 0,$$

where T_{ij} are given in equation(16).

Solving the above equation, we get

$$(22) \quad \lambda^4 \det(g_{ij}) = 0.$$

Since $\det (g_{ij}) = -AB \neq 0$, this implies

$$(23) \quad \lambda^4 = 0.$$

From this, it is clear that all the four eigen values are zero (*i.e.* $\lambda = 0$). Hence the principle directions are not unique. Thus the existence of gravitational waves with massless scalar field are related to eigen values of T_{ij} are all zero.

3. NON-EXISTENCE OF MASSIVE SCALAR FIELD WITH GRAVITATIONAL WAVES AND COUPLED WITH ELECTROMAGNETIC WAVES

Case (i): Massive scalar field with gravitational waves.

Since $V_s V^s = 0$, the energy momentum tensor for massive scalar field is

$$(24) \quad T_{ij} = \frac{1}{4\pi} \left[V_i V_j + \frac{1}{2} g_{ij} m^2 V^2 \right], \quad (i, j = 1, 2, 3, 4).$$

$$\Rightarrow T = T^i_i = \frac{1}{2\pi} m^2 V^2,$$

where $V_i = \frac{\partial V}{\partial x^i}$, V is the scalar function of Z .

Also, the non-vanishing components V_3 and V_4 assumes the form as in equation (6).

Using equation (3), (4) and (5), the field equation (1) assumes the form,

$$(25) \quad R_{ij} = (-2) \left[V_i V_j - \frac{1}{2} g_{ij} m^2 V^2 \right].$$

Using equation (6) and (25), we obtain

$$R_{33} = (-2) \left[\bar{V}^2 + \frac{1}{2} (1 - E) m^2 V^2 \right],$$

$$R_{34} = R_{43} = (-2) \left[-\bar{V}^2 + \frac{1}{2} (E) m^2 V^2 \right],$$

$$(26) \quad R_{44} = (-2) \left[\bar{V}^2 - \frac{1}{2} (1 + E) m^2 V^2 \right].$$

The equation (26) is compatible with (8) only when $m^2 = 0$, hence it expresses the fact that massive scalar plane wave do not exist with gravitational waves. In this case it is worthy to note the analogous result of Ray and Rao [7] with respect to axially symmetric massive scalar field. Now, we have linked up the non-existence of massive scalar field with the eigen value problem of matter tensor as follows:

$$(27) \quad \det(T_{ij} - \lambda g_{ij}) = 0$$

where

$$T_{33} = \frac{1}{4\pi} \left[\bar{V}^2 - \frac{1}{2} (1 - E) m^2 V^2 \right],$$

$$T_{34} = T_{43} = \frac{1}{4\pi} \left[-\bar{V}^2 - \frac{1}{2} E m^2 V^2 \right],$$

$$T_{44} = \frac{1}{4\pi} \left[\bar{V}^2 + \frac{1}{2} (1 + E) m^2 V^2 \right].$$

Solving the above equation, we get

$$(28) \quad \left[\lambda - \frac{1}{8\pi} m^2 V^2 \right]^4 \det(g_{ij}) = 0.$$

Since $\det(g_{ij}) = -AB \neq 0$, this implies

$$(29) \quad \lambda = \frac{1}{8\pi} m^2 V^2.$$

We have obtained four equal non-zero eigen values equal to $\frac{m^2 V^2}{8\pi}$. Thus the non-existence of gravitational waves with massive scalar field is related non-zero eigen values of T_{ij} .

Case (ii): Massive scalar field coupled with gravitational and electromagnetic waves. Since $V_s V^s = 0$, the energy momentum tensor for massive scalar field coupled with electromagnetic waves are given by

$$(30) \quad T_{ij} = \frac{1}{4\pi} \left[V_i V_j + \frac{1}{2} g_{ij} m^2 V^2 \right] + E_{ij}, \quad (i, j = 1, 2, 3, 4),$$

where V is the scalar function of Z and $V_i = \frac{\partial V}{\partial x^i}$ and

$$(31) \quad R_{ij} = (-2) \left[4\pi E_{ij} + V_i V_j - \frac{1}{2} g_{ij} m^2 V^2 \right], \quad (i, j = 1, 2, 3, 4).$$

Using equation (6), equation (31) implies

$$(32) \quad \begin{aligned} R_{33} &= (-2) \left[4\pi E_{33} + \bar{V}^2 + \frac{1}{2} (1 - E) m^2 V^2 \right], \\ R_{34} &= (-2) \left[4\pi E_{34} - \bar{V}^2 + \frac{1}{2} E m^2 V^2 \right], \\ R_{44} &= (-2) \left[4\pi E_{44} + \bar{V}^2 - \frac{1}{2} (1 + E) m^2 V^2 \right]. \end{aligned}$$

in which

$$E_{33} = -E_{34} = E_{44} = \frac{B\sigma^2 + A\rho^2}{AB}$$

Equation (32) is compatible with (8) only when $m^2 = 0$. Hence it follows the non-existence of massless scalar field coupled with gravitational and electromagnetic waves. Also, we associate the non-existence of massive scalar waves with the eigen value problem of matter tensor by using characteristic equation.

$$(33) \quad \det(T_{ij} - \lambda g_{ij}) = 0$$

where

$$\begin{aligned} T_{33} &= \frac{1}{4\pi} \left[\bar{V}^2 - \frac{1}{2} (1 - E) m^2 V^2 \right] + E_{33}, \\ T_{34} = T_{43} &= \frac{1}{4\pi} \left[-\bar{V}^2 - \frac{1}{2} E m^2 V^2 \right] + E_{34}, \\ T_{44} &= \frac{1}{4\pi} \left[\bar{V}^2 + \frac{1}{2} (1 + E) m^2 V^2 \right] + E_{44}. \end{aligned}$$

Solving the above equation we get,

$$(34) \quad \left[\lambda - \frac{1}{8\pi} m^2 V^2 \right]^4 \det(g_{ij}) = 0.$$

Since $\det(g_{ij}) = -AB \neq 0$, this implies

$$(35) \quad \lambda = \frac{1}{8\pi} m^2 V^2.$$

We have obtained four equal non-zero eigen values equal to $\frac{m^2 V^2}{8\pi}$. Thus the non-existence of gravitational waves with massive scalar waves is related non-zero eigen values of T_{ij} .

4. CONCLUSION

From the above results it is observed that, the existence of massless scalar field together with plane gravitational waves and electromagnetic waves are guaranteed by the four equal zero eigen values of energy momentum tensor for space-time. Moreover, the non-existence of massive scalar field coupled with plane gravitational waves and electromagnetic waves are assured the four equal non-zero eigen values of energy momentum tensor for space-time for generalized Peres space-time. If we choose $A = A(Z)$, $B = B(Z)$ and $E = E(Z)$, in which $Z = (z - t)$ and $C = 1$ then our results reduces to the Takeno's[8].

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