

CHAOS CONTROL IN GENESIO SYSTEM USING ACTIVE BACKSTEPPING DESIGN

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ABSTRACT. In this paper, an active backstepping design is proposed for controlling chaotic Genesio system to a steady state as well as tracking of any desired trajectory to be achieved in a systematic way. This method is also applied to achieve chaos synchronization of two identical Genesio system with each other. The proposed method, which is based on Lyapunov stability theorem, combines active control and backstepping methods. Numerical simulations verify the effectiveness and feasibility of the method.

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1. INTRODUCTION

Chaos is a very interesting nonlinear phenomenon which has been intensively studied during the last four decades. It is very common and has been detected in a large number of dynamic systems of various physical nature. However, chaos is usually undesirable in practice, and it restricts the operating range of many electronic and mechanic devices. Recently, controlling chaos has attracted a great deal of attention in science and engineering.

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Nowadays, in order to achieve the control, stabilization and synchronization of chaotic systems, different techniques and methods ([5]-[18]) have been proposed. For instance, OGY method [11], differential geometric method [6], inverse optimal control [12], feedback and nonfeedback control ([4],[14],[16], [2]), adaptive control ([13], [9]).

In recent years, backstepping design ([10],[17],[19]) and active control ([15], [8],[3], [1]) have been widely recognized as two powerful design methods to control chaotic systems. Backstepping design can guarantee global stability, tracking and transient performance for a broad class of strict feedback nonlinear systems. The active control technique gives the flexibility to construct a control law so that it can be used widely to control various nonlinear systems including chaotic systems. Consequently, the main aim here is to use a combination of these two approaches, i.e., an active backstepping method.

Active backstepping design ([20], [18]) is a systematic design approach and consists in a recursive procedure that skillfully interlaces the choice of a Lyapunov function with the design of active control. Besides, the control scheme can also be used to track any desired trajectories in a systematic way and to achieve chaos synchronization.

In this paper, we propose a novel active controller to suppress chaos in Genesio system with certain parameters, based on the backstepping design. The effectiveness and feasibility of the proposed control technique are numerically verified.

2. SYSTEM DESCRIPTION

The Genesio system proposed by Genesio and Tesi [7] is one of the paradigms of chaos since it captures many features of chaotic systems. It includes a simple square part and three simple ordinary differential equations that depend on three positive real parameters. The dynamic equations of the system is given by

$$\begin{aligned}
 \dot{x} &= y \\
 \dot{y} &= z \\
 \dot{z} &= -ax - by - cz + x^2
 \end{aligned}
 \tag{2.1}$$

where x , y and z are the state variables and a , b and c are real positive constants.

With respect to the Genesio system (2.1) we have the following observations:

Lemma 2.1. *For $a > 0$, $b > 0$ and $c > 0$ the system (2.1) has two equilibria $O(0, 0, 0)$ and $A(a, 0, 0)$.*

Proof. Let

$$(2.2) \quad \begin{aligned} y &= 0 \\ z &= 0 \\ -ax - by - cz + x^2 &= 0 \end{aligned}$$

Then the system has two equilibria, which are, respectively described as follows:

$$O(0, 0, 0) \quad \text{and} \quad A(a, 0, 0)$$

□

The Jacobian matrix of system (2.1)

$$(2.3) \quad J = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a + 2x & -b & -c \end{pmatrix}$$

To gain its eigenvalues, we let

$$|A - \lambda I| = 0$$

i.e.,

$$(2.4) \quad \lambda^3 + c\lambda^2 + b\lambda + (a - 2x) = 0$$

Let $A = c$, $B = b$ and $C = a - 2x$. According to Routh-Hurwitz conditions, with respect to (2.4), the real parts of the roots are all negative if and only if $A > 0$, $C > 0$ and $BC - A > 0$, i.e.,

$$(2.5) \quad c > 0, \quad a - 2x > 0, \quad bc - (a - 2x) > 0$$

Lemma 2.2. *For system (2.1), we have the following results:*

- (i) *if $a > 0$ and $bc > a$, then $O(0, 0, 0)$ is a stable focus*
- (ii) *if $a > 0$ and $bc < a$, then $O(0, 0, 0)$ is a saddle-focus; it is unstable*
- (iii) *if $a > 0$ and $bc + a > 0$, $A(a, 0, 0)$ is a saddle-focus; it is unstable.*

Proof. (i) and (ii) The Jacobian matrix of system (2.1) at $O(0,0,0)$ is

$$(2.6) \quad J = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a & -b & -c \end{pmatrix}$$

whose characteristics polynomial is

$$(2.7) \quad \lambda^3 + c\lambda^2 + b\lambda + a = 0$$

Equations in (2.5) implies that the real parts of the roots of (2.7) are all negative if and only if

$$(2.8) \quad c > 0, \quad a > 0 \text{ and } bc - a > 0$$

(iii) The Jacobian matrix at the equilibrium point $A(a,0,0)$ is

$$(2.9) \quad J = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a & -b & -c \end{pmatrix}$$

and whose characteristic polynomial is

$$(2.10) \quad \lambda^3 + c\lambda^2 + b\lambda - a = 0$$

According to Routh-Hurwitz conditions , the real parts of the roots of (2.10) are all negative if and only if

$$(2.11) \quad c > 0, \quad -a > 0 \text{ and } bc + a > 0$$

Since $a > 0$, the second condition in (2.11) can not hold, that is one of the eigenvalues has a positive real part. So, the equilibrium point $A(a,0,0)$ is unstable. $\square$

For $a = 6$, $b = 2.92$ and $c = 1.2$, the Genesio system has a chaotic attractor as shown in Figure 1. It has two unstable equilibria $O(0,0,0)$ and $A(6,0,0)$ by Lemma 2.2

3. ACTIVE BACKSTEPPING CONTROL OF GENESIO SYSTEM

In this section we will use backstepping design method to design an active controller for stabilizing the chaotic behavior of the Genesio system.

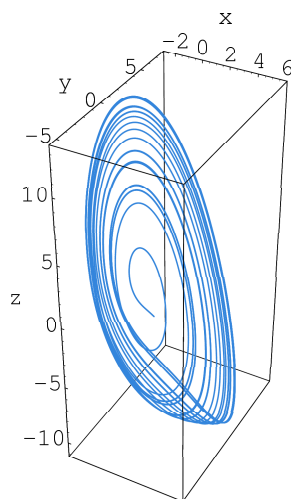


FIGURE 1. Chaotic attractor of Genesio system

Let the target unstable periodic orbit of the Genesio system be $w = (\tilde{x}, \tilde{y}, \tilde{z})^T$, which is itself a solution of the system (2.1) it satisfies

$$\begin{aligned}
 \dot{\tilde{x}} &= \tilde{y} \\
 \dot{\tilde{y}} &= \tilde{z} \\
 \dot{\tilde{z}} &= -a\tilde{x} - b\tilde{y} - c\tilde{z}\tilde{x}^2
 \end{aligned}
 \tag{3.1}$$

Thus, an important problem of controlling chaos in system (2.1) under investigation here is to select an appropriate control function $u(t) = (u_1, u_2, u_3)^T$ such that the trajectory of the controlled system

$$\begin{aligned}
 \dot{x} &= y + u_1(t) \\
 \dot{y} &= z + u_2(t) \\
 \dot{z} &= -ax - by - cz + x^2 + u_3(t)
 \end{aligned}
 \tag{3.2}$$

asymptotically approaches the target periodic orbit w , in the sense that

$$\lim_{t \rightarrow \infty} \|x - w\| = 0$$

where $\|\cdot\|$ is the Euclidean norm.

The subtraction of (3.1) from (3.2) now gives the following error system

$$\begin{aligned} \dot{e}_1 &= e_2 + u_1(t) \\ \dot{e}_2 &= e_3 + u_2(t) \\ \dot{e}_3 &= -ae_1 - be_2 - ce_3 + 2\tilde{x}e_1 + e_1^2 + u_3(t) \end{aligned} \quad (3.3)$$

where $e_1 = x - \tilde{x}$, $e_2 = y - \tilde{y}$, $e_3 = z - \tilde{z}$ are the error states. As long as the control input can stabilize the system, the error vector $e = (e_1, e_2, e_3)^T$ converges to zero as time t goes to ∞ . This implies that the trajectory of the controlled system (3.2) asymptotically approaches to the target unstable periodic orbit w .

Step 1.

Let $z_1 = e_1$ then we can obtain its derivative

$$\dot{z}_1 = \dot{e}_1 = e_2 + u_1 \quad (3.4)$$

where $e_2 = \alpha_1(z_1)$ is regarded as a virtual controller.

For the design of $\alpha_1(z_1)$ to stabilize z_1 -subsystem (3.4), we can choose the following Lyapunov function

$$V_1(z_1) = \frac{1}{2}z_1^2 \quad (3.5)$$

The derivative of V_1 is

$$\dot{V}_1 = z_1 \dot{z}_1 = z_1(\alpha_1 + u_1) \quad (3.6)$$

If we choose $\alpha_1 = -pz_1$, ($p > 0$) and

$$u_1 = 0 \quad (3.7)$$

then $\dot{V}_1$ is negative definite, i.e.,

$$\dot{V}_1 = -pz_1^2 < 0, \quad (p > 0)$$

This implies that the z_1 -subsystem (3.4) is asymptotically stable. Since the virtual control function $\alpha_1(z_1)$ is estimative, the error between e_2 and $\alpha_1(z_1)$ is

$$z_2 = e_2 - \alpha_1 \quad (3.8)$$

Then we obtain the following (z_1, z_2) -subsystem

$$\begin{aligned} \dot{z}_1 &= -pz_1 + z_2 \\ \dot{z}_2 &= -p^2z_1 + pz_2 + e_3 + u_2(t) \end{aligned} \quad (3.9)$$

where $e_3 = \alpha_2(z_1, z_2)$ is regarded as a virtual controller.

Step 2.

In order to stabilize the (z_1, z_2) -subsystem (3.9), we can choose a Lyapunov function V_2 as follows

$$V_2 = V_1 + \frac{1}{2}z_2^2 \quad (3.10)$$

Its derivative is given by

$$\begin{aligned} \dot{V}_2 &= z_1\dot{z}_1 + z_2\dot{z}_2 \\ &= -pz_1^2 - z_2^2 + z_2((1-p^2)z_1 + (1+p)z_2 + \alpha_2 + u_2) \end{aligned} \quad (3.11)$$

If $\alpha_2(z_1, z_2) = 0$ and

$$u_2 = (p^2 - 1)z_1 - (1 + p)z_2 \quad (3.12)$$

then

$$\dot{V}_2 = -pz_1^2 - z_2^2 < 0 \quad (3.13)$$

is negative definite, therefore (z_1, z_2) -subsystem (3.9) is asymptotically stable.

Similarly, we assume that

$$\bar{z}_3 = e_3 - \alpha_2 \quad (3.14)$$

then we can drive the following (z_1, z_2, z_3) -subsystem:

$$\begin{aligned} \dot{z}_1 &= -pz_1 + z_2 \\ \dot{z}_2 &= -z_1 - z_2 + z_3 \\ \dot{z}_3 &= (-a + bp + 2\tilde{x})z_1 - bz_2 - cz_3 + z_1^2 + u_3 \end{aligned} \quad (3.15)$$

Step 3.

We can choose a Lyapunov function V_3 as follows

$$V_3 = V_2 + \frac{1}{2}\bar{z}_3^2 \quad (3.16)$$

Its derivative is given by

$$\begin{aligned} \dot{V}_3 &= z_1 \dot{z}_1 + z_2 \dot{z}_2 + z_3 \dot{z}_3 \\ (3.17) \quad &= -pz_1 - z_2 - cz_3^2 + z_3((1-b)z_2 + (-a + bp + 2\tilde{x})z_1 + z_1^2 + u_3) \end{aligned}$$

If

$$(3.18) \quad u_3 = (a - bp - 2\tilde{x})z_1 + (b - 1)z_2 - z_1^2$$

then

$$\dot{V}_3 = -pz_1^2 - z_2^2 - cz_3^2 < 0$$

is negative definite, therefore (z_1, z_2, z_3) -system (3.15) is asymptotically stable. Since $\dot{V}_3$ is negative definite, it follows that in the (z_1, z_2, z_3) coordinates the equilibrium $O(0, 0, 0)$ is global asymptotically stable. In view of $z_1 = e_1$, $z_2 = e_2 - \alpha_1(z_1)$ and $z_3 = e_3 - \alpha_2(z_1, z_2)$, this implies that e_1 , e_2 and e_3 go to zero asymptotically. In other words, the trajectory of the controlled system (3.15) asymptotically approaches the target unstable periodic orbit $w = (\tilde{x}, \tilde{y}, \tilde{z})^T$.

3.1. Numerical results. (Controlling chaotic behavior to steady states) For the numerical simulation, we assume that the initial states of the controlled system (3.2) are $(x(0), y(0), z(0)) = (3, -4, 2)$. The parameters are chosen as $a = 6$, $b = 2.92$, $c = 1.2$ and $p = 0.5$. Figure 2 (a)-(b) show that the Genesio system can be stabilized with control functions (3.7), (3.12) and (3.18) to the steady states $O(0, 0, 0)$ and $A(6, 0, 0)$.

4. TRACKING ANY DESIRED TRAJECTORY

This section is to find a control law $u(t)$ so that a scalar output $x(t)$ can track any desired trajectory $r(t)$. We add control input $u(t) = (u_1, u_2, u_3)^T$ to the Genesio system (2.1). Thus the controlled system is

$$\begin{aligned} \dot{x} &= y + u_1(t) \\ (4.1) \quad \dot{y} &= z + u_2(t) \\ \dot{z} &= -ax - by - cz + x^2 + u_3(t) \end{aligned}$$

Step 1

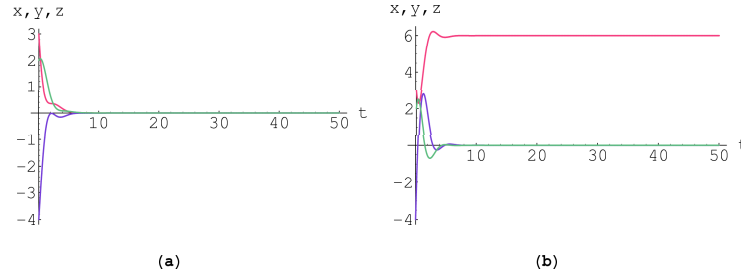


FIGURE 2. Controlling Genesio system using active backstepping design; (a) stabilization to steady state $O(0,0,0)$, (b) stabilization to $A(6,0,0)$. The control is activated at $t = 7$. Red, blue and green denote the x, y and z trajectories, respectively.

Assume that $x_1 = x - r$, then we can obtain its derivative

$$(4.2) \quad \dot{x}_1 = \dot{x} - \dot{r} = y - \dot{r}$$

where $y = \alpha_1(x_1)$ is regarded as a virtual controller

For the design of $\alpha_1(x_1)$ to stabilize x_1 -subsystem (4.2), we can choose the following Lyapunov function

$$(4.3) \quad V_1 = \frac{1}{2}x_1^2$$

The derivative of V_1 is

$$(4.4) \quad \dot{V}_1 = x_1\dot{x}_1 = x_1(\alpha_1 + u_1 - \dot{r})$$

If we choose $\alpha_1(x_1) = -px_1 + \dot{r}$, ($p > 0$) and

$$(4.5) \quad u_1 = 0$$

such that $\dot{V}_1 = -px_1^2$ is negative definite. This implies that the x_1 -subsystem (4.2) is asymptotically stable. Since the virtual control function $\alpha_1(x_1)$ is estimative, define an error variable

$$(4.6) \quad x_2 = y - \alpha_1$$

We can obtain the following (x_1, x_2) -subsystem

$$(4.7) \quad \begin{aligned} \dot{x}_1 &= -px_1 + x_2 \\ \dot{x}_2 &= -p^2x_1 + px_2 + z - \ddot{r} + u_2 \end{aligned}$$

where $z = \alpha_2(x_1, x_2)$ is regarded as a virtual controller.

Step 2

In order to stabilize the (x_1, x_2) -subsystem (4.7), we can choose a Lyapunov function V_2 as follows

$$(4.8) \quad V_2 = V_1 + \frac{1}{2}x_2^2$$

Its derivative is given by

$$(4.9) \quad \begin{aligned} \dot{V}_2 &= x_1\dot{x}_1 + x_2\dot{x}_2 \\ &= -px_1^2 - x_2 + x_2((1-p^2)x_1 + (1+p)x_2 - \ddot{r} + u_2 + \alpha_2) \end{aligned}$$

If $\alpha_2(x_1, x_2) = 0$ and

$$(4.10) \quad u_2 = (p^2 - 1)x_1 - (1 + p)x_2 + \ddot{r}$$

then

$$\dot{V}_2 = -px_1^2 - x_2^2 < 0$$

makes (x_1, x_2) -subsystem (4.7) is asymptotically stable. Similarly, assume that $x_3 = z - \alpha_2$ then we obtain the following subsystem

$$(4.11) \quad \begin{aligned} \dot{x}_1 &= -px_1 + x_2 \\ \dot{x}_2 &= -x_1 - x_2 \\ \dot{x}_3 &= (bp - a + 2r)x_1 - bx_2 - cx_3 + x_1^2 - ar - b\dot{r} + r^2 + u_3 \end{aligned}$$

In order to stabilize the (x_1, x_2, x_3) -system (4.11), we choose a Lyapunov function V_3 is given by

$$(4.12) \quad \begin{aligned} \dot{V}_3 &= x_1\dot{x}_1 + x_2\dot{x}_2 + x_3\dot{x}_3 \\ &= -px_1^2 - x_2 - cx_3^2 + x_3((bp - a + 2r)x_1 - bx_2 + x_1^2 - ar - b\dot{r} + r^2 + u_3) \end{aligned}$$

If we choose

$$(4.13) \quad u_3 = (a - bp - 2r)x_1 + bx_2 - x_1^2 + ar + b\dot{r} - r^2$$

then $\dot{V}_3 = -px_1^2 - x_2^2 - cx_3^2 < 0$, therefore (x_1, x_2, x_3) -system (4.11) is asymptotically stable. Since $\dot{V}_3$ is negative definite, it follows that in the (x_1, x_2, x_3) coordinates the equilibrium $O(0, 0, 0)$ is global asymptotically stable. In view of $x_1 = x - r$, $x_2 = y - \alpha_1(x_1)$ and $x_3 = z - \alpha_2(x_1, x_2)$, this implies that the output trajectory $x(t)$ of the controlled system (4.1) asymptotically approaches the target trajectory $r(t)$.

4.1. Numerical results. (Tracking a desired trajectory) In simulation, the parameters and the initial states of the controlled system (4.1) are chosen as $a = 6$, $b = 2.92$, $c = 1.2$, $p = 0.5$ and $(x(0), y(0), z(0)) = (3, -4, 2)$, respectively. Figure 3 shows the scalar output track the desired trajectory $r(t) = 3 \sin t$ with the control functions (4.5), (4.10) and (4.13).

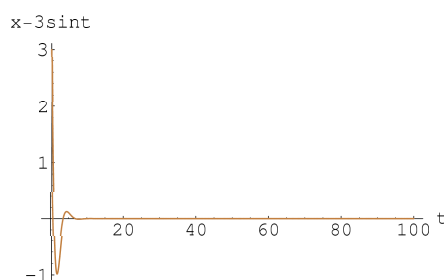


FIGURE 3. Track of $r(t) = 3 \sin t$

5. SYNCHRONIZATION OF GENESIO SYSTEM

We assume that there are two identical Genesio systems playing the roles of drive system and response system, respectively. Here, the drive system is described by

$$\begin{aligned}
 \dot{x}_1 &= y_1 \\
 \dot{y}_1 &= z_1 \\
 \dot{z}_1 &= -ax_1 - by_1 - cz_1 + x_1^2
 \end{aligned}
 \tag{5.1}$$

and the response system is given by

$$\begin{aligned}
 (5.2) \quad & \dot{x}_2 = y_2 + u_1(t) \\
 & \dot{y}_2 = z_2 + u_2(t) \\
 & \dot{z}_2 = -ax_2 - by_2 - cz_2 + x_2^2 + u_3(t)
 \end{aligned}$$

where a, b, c are positive constants. We have introduced three functions u_1, u_2 and u_3 in (5.2). These functions are to be determined. Let us define the error states between the response system (5.2) that is to be controlled and the drive system (5.1) as $e_1 = x_2 - x_1$, $e_2 = y_1 - y_2$, $e_3 = z_2 - z_1$. Subtracting (5.1) from (5.2), we obtain the error system

$$\begin{aligned}
 (5.3) \quad & \dot{e}_1 = e_2 + u_1(t) \\
 & \dot{e}_2 = e_3 + u_2(t) \\
 & \dot{e}_3 = -ae_1 - be_2 - ce_3 + 2x_1e_1 + e_1^2 + u_3(t)
 \end{aligned}$$

Repeating the above design technique we obtain the control functions

$$\begin{aligned}
 (5.4) \quad & \dot{u}_1 = 0 \\
 & \dot{u}_2 = (p^2 - 1)z_1 - (1 + p)z_2 \\
 & \dot{u}_3 = (a - bp - 2x_1)z_1 + (b - 1)z_2 - z_1^2
 \end{aligned}$$

to make the zero solution of system (5.3) is asymptotically stable. Also, the output trajectory of the controlled system (5.2) asymptotically approaches the trajectory of the system (5.1).

5.1. Numerical results. (Synchronization of Genesio chaotic systems) The initial values of the drive and response systems (5.1) and (5.2) are $(x_1(0), y_1(0), z_1(0)) = (3, -4, 2)$ and $(x_2(0), y_2(0), z_2(0)) = (1, 2, 5)$, respectively, while the initial states of the error system (5.3) are $(e_1(0), e_2(0), e_3(0)) = (-2, 6, 3)$. The parameters are taken as $a = 6$, $b = 2.92$, $c = 1.2$ and $p = 0.5$. The dynamics of the errors for the drive system and response system are shown in Figure 4. Figure 5 (a)-(c) display the time response of states determined by drive system and the response system with the control functions (5.4).

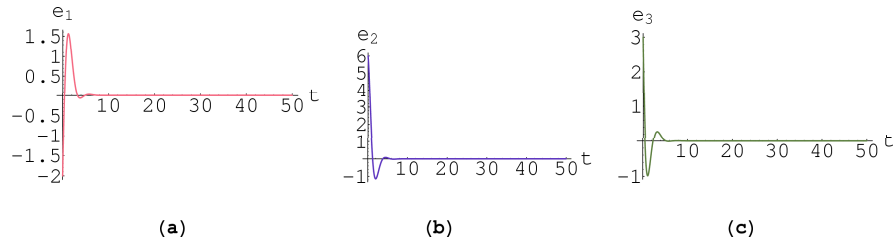
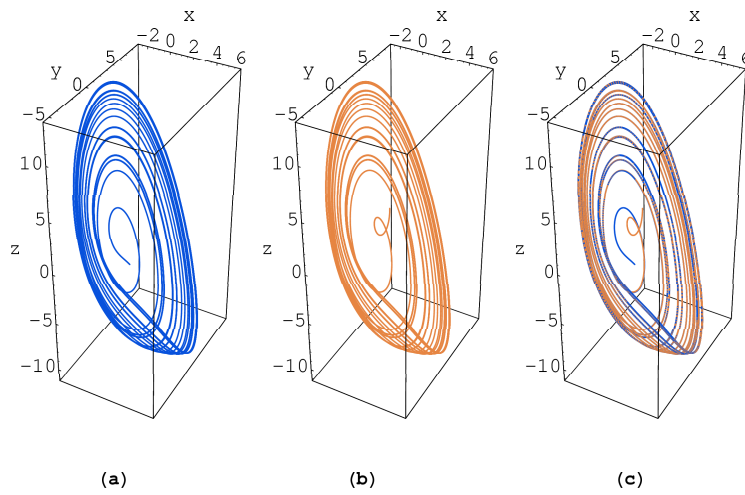
FIGURE 4. Synchronization error e_1, e_2 and e_3 with time

FIGURE 5. Synchronization of two identical Genesio systems with each other; (a) trajectory of the drive system, (b) trajectory of the response system, and (c) synchronization

6. CONCLUSION

In this paper, we have proposed an effective control method for controlling chaotic Genesio system based on active backstepping design. The proposed technique enables stabilization of chaotic motion to a steady state as well as tracking of any desired trajectory to be achieved in a systematic way. Chaos synchronization of Genesio system is also achieved using that technique. The advantage of this method can

be summarized as follows: (i) this systematic design approach does not need to calculate the Lyapunov exponents and eigenvalues of the Jacobian matrix; hence it is simple and convenient; (ii) the design method can overcome the controller singularity problem resulting from the nonlinear term of quadratic type [10] and [19]; (iii) especially, it gives the flexibility to construct a control law so that the control strategy can be easily extended to other chaotic systems no matter whether they possess strange attractors or not; (iv) it extends the control capability for achieving a full range synchronization of all state variables in a proportional way; and (v) it guarantees the global stability of the closed-loop system.

REFERENCES

- [1] H.N. Agiza and M.T. Yassen, *Synchronization of Rossler and Chen chaotic dynamical systems using active control*, Phys. Lett. A 278 (2001), 191-197.
- [2] H.N. Agiza, *Controlling chaos for the dynamical system of coupled dynamos*, Chaos, Solitons and Fractals, 12 (2002).
- [3] E.W. Bai and K.E. Lonngren, *Sequential synchronization of two Lorenz systems using active control*, Chaos, Solitons, Fractals 11 (2000), 1041-1044.
- [4] G. Chen and X.N. Dong, *On feedback control of chaotic continuous time systems*, IEEE Trans Circ Syst. 40 (1993), 591-601.
- [5] G. Chen and X. Dong, *From Chaos to Order: Perspectives Methodologies and Applications*, World Scientific, Singapore, (1998).
- [6] C.C. Fuh and P.C. Tung, *Controlling chaos using differential geometric method*, Phys Rev Lett. 75 (1995), 2952-2955.
- [7] R. Genesio and A. Tesi, *Harmonic balance methods for the analysis of chaotic dynamics in nonlinear systems*, Automatica 28 (1992), 531-548.
- [8] M.C. Ho and Y.C. Hung, *Synchronization of two different two different systems by using generalized active control*, Phys.Lett.A 301 (2002), 424-428.
- [9] T.L. Liao and S.H. Lin, *Adaptive control and synchronization of Lorenz system*, J. Franklin Inst., 336 (1999).
- [10] J. Lu and S. Zhang, *Controlling Chen's chaotic attractor using backstepping design based on parameters identification*, Phys.Lett A, 286 (2001), 148-152.
- [11] E. Ott, C. Grebogi and J.A. Yorke, *Controlling chaos*, Phys Rev Lett. 64 (1990), 1196-1199.
- [12] E.N. Sanchez, J.P. Perez, M. Martinez and G. Chen, *Chaos stabilization: an inverse optimal control approach*, Latin Am. Appl. Res. 32 (2002).
- [13] M.T. Yassen, *Adaptive control and synchronization of a modified Chua's circuit system*, Appl. Math. Comp., 135 (2001).
- [14] M.T. Yassen, *Chaos control of Chen chaotic dynamical system*, Chaos, Solitons and Fractals, 15 (2003).

- [15] M.T. Yassen, *Chaos synchronization between two different chaotic systems using active control*, Chaos, Solitons and Fractals, 23 (2005), 131-140.
- [16] M.T. Yassen, *Cotrolling chaos and synchronization for new chaotic system using linear feedback*, Chaos, Solitons and Fractals, 26 (2005).
- [17] M.T. Yassen, *Chaos control of chaotic dynamical systems using backstepping design*, Chaos, Solitons and Fractals, 27 (2006).
- [18] M.T. Yassen, *Controlling, synchronization and tracking chaotic Liu system using active backstepping design*, Phys Lett A, 360 (2007).
- [19] Y. Yu and S. Zhang, *Controlling uncertain L system using backstepping design*, Chaos, Solitons and Fractals, 15 (2003), 897-902.
- [20] H. Zhang, X.K. Ma and B.L. Xue, *Synchronization of chaotic systems with parametric uncertainty using active sliding mode control*, Chaos, Solitons and Fractals, 21 (2004), 1249-1257.