THE DIFFERENTIAL TRANSFORM METHOD FOR THE NUMERICAL TREATMENT OF DIFFERENTIAL-ALGEBRAIC EQUATIONS WITH INDEX 2

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Abstract. We will consider the numerical solution of the Hessenberg linear index-2 differential algebraic equations (DAE's) using the differential transform method. We will first present a method to reduce the index of the DAE to index-1. This results in a problem easier to solve. Numerical examples in addition to the comparison with the work of others will also be presented to show the efficiency of the method.

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1. Introduction

The problem under consideration has the general form

$$
\begin{aligned}
A(t,z)\; z' &= f(t,z) \\
0 &= g(t,z)
\end{aligned}
\tag{1}
$$

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where $f : I \times D \longrightarrow R^n$, $g : I \times D \longrightarrow R^m$ and $A : I \times D \longrightarrow L(R^{n+m}, R^n)$ with rank $A(t, z) = n,\ I \subseteq R$ an open interval and $D \subseteq R^{n+m}$ open and nonempty, g is continuously differentiable, $A,\ f$ and g' Lipschitz-continuous with respect z. We will concentrate on a special case of (1) that is semiexplicit differential equation(DAE) of the form

$$\begin{aligned} x'(t) &= f(x(t), y(t)) \\ 0 &= g(x(t), y(t)) \end{aligned} \tag{2}$$

with initial condition $x(0) = x_0$. Here $f : D_1 \times D_2 \longrightarrow R^n;\ g : D_1 \times D_2 \longrightarrow R^m$ with D_1 and D_2 are open nonempty subsets of R^n and R^m respectively. Also, g is continuously differentiable, f and g' are Lipschitz-continuous and

$$rank(g'(x(t), y(t))) = m.$$

Any solution (x, y) of (2) must satisfy the constraint manifold

$$M = \{(x, y) \in D_1 \times D_2;\ g(x(t), y(t)) = 0\}.$$

Both types of DAEs mentioned above were considered for example by [15] where some theoretical aspects of impasse points were discussed. The index-1 condition at $(x,\ y) \in g^{-1}(0)$ is satisfied if and only if $g_y(x, y) \in L(R^m)$ is nonsingular. This means the constraint equation $g(x(t), y(t)) = 0$ is uniquely solvable for $y = y(x)$ in some neighborhood of $x_0 \in D_1$ and $y_0 \in D_2$. Hence the DAE (2) is locally transferable to state form

$$x' = f(x, y(x))$$

and hence can be changed to an ordinary differential equation. This is guaranteed by the Implicit Function Theorem.

Definition: The DAE given in (2) is said to be of index-1 if and only if $g_y(x, y) \in L(R^m)$ is nonsingular while it said to be of index-2 if and only if $g_x(x, y)f_y(x, y)$ is nonsingular.

We will be concerned here with the index-2 DAE. We will first transform the problem to index-1 DAE and then use differential transforms. This is done by first transforming the DAE to algebraic format to produce some kind of recursion formula which is then used to produce the solution up to an arbitrary order using

Mathematica. This approach proved to be simple and effective and the rate of convergence is usually high.

Recently many authors showed interest in solving differential-algebraic systems which are common in applied sciences and engineering. Traditionally, the methods used to solve these systems differentiate the algebraic equations to transform the problem into a system of ordinary differential equations, then solving the resulting system of ODE's. Such approach proved to be inefficient since chances are that the numerical solution of the resulting system is significantly different than the original problem, see Brenan et. al.[6].

A system of DAE's is characterized by its index, which is simply as mentioned above the number of differentiations required to convert it into a system of ordinary differential equations. We will define and consider the index-2 DAE's. Note that generally, the higher the index of the problem, the harder the method of solution. To simplify we will consider a special method to reduce the index by one which will result in an index-1 DAE, see Babolian and Hosseini[5]. This makes it easier to solve numerically. More on such index-1 DAE can be found in Attili[1, 2] where the systems considered have some types of singularities. Other methods that are concerned with the numerical solution of DAEs of index-1 are as follows, for example, Arclength parameterization was considered by Ren and Spence[16], Differential transforms for DAE of index-1 was considered by Ayaz[4], Series and Pade' approximations were considered by Celik and Bayram[8, 9] and Attili et. al.[3]. The methods of reducing and modified reducing index have be generalized and numerically implemented by Hosseini[14], the parameterization method for DAE arising in optimal control problems was done by Gorbunov and Lutoshkin[13] while fractional step methods were considered by Vijalapura et al.[19]. For index 2 DAEs we mention for example Celik[10] where Pade' approximation was used to solve the DAE. Runge-Kutta methods were suggested and used by Schropp[17] and Sun and Jiang[18].

The outline of the paper will be as follows. In section two, we will discuss the differential algebraic equation in general how we can change index 2 to index 1 DAEs. This results in a DAE that is easier to solve by the differential transform method. In the third section, we will give some details of the differential transform method in addition to a table which will be used in solving our problems. The last section will contain some numerical details and examples to show the efficiency of the method.

2. Index One and Two DAE's

Consider again the problem given by (2) of the form

$$\begin{aligned} x'(t) &= f(x(t), y(t)) \\ 0 &= g(x(t), y(t)) \end{aligned} \tag{3}$$

which can be written in the form $F(t, x, x\prime) = 0$ with some given initial condition $x(0) = x_0$. We will be concerned here with index-2 DAE; that is, where $g_x f_y$ is nonsingular .Assume that

$$\begin{aligned} f(x(t), y(t)) &= Ax + By + q(t) \\ g(x(t), y(t)) &= Cx + r(t) \end{aligned} \tag{4}$$

In this case the system (2) becomes

$$\begin{aligned} (a) \quad x^{'}(t) &= Ax(t) + By(t) + q \\ (b) \quad 0 &= Cx(t) + r \end{aligned} \tag{5}$$

where $f : D_1 \times D_2 \to \mathbb{R}^n$; $g = D_1 \times D_2 \to \mathbb{R}^m$ with D_1 and D_2 are open none empty sets of $\mathbb{R}^n$ and $\mathbb{R}^m$ respectively and g is continuously differentiable. Also $A_{n\times n}$. $B_{n\times 1}$, $C_{1\times n}$, $q\ _{n\times 1}$ are matrices with $n \geq 2$. We will consider the case when $n = 3$ in more details. The index 2 condition is equivalent to saying that the scalar $\det(CB(t)) \neq 0$ for all $t \in D_1$. Solving the first part of (5) for y, we obtain

$$y = (CB)^{-1}C[x^{'} - Ax - q].$$

Substituting back into $(5 - a)$, we get

$$[I - B(CB)^{-1}C][x^{'} - Ax - q] = 0.$$

The system (5) with $n = 3$ becomes

$$\begin{aligned} (a) \quad [I - B(CB)^{-1}C][x^{'} - Ax - q] &= 0 \\ (b) \quad Cx + r &= 0 \end{aligned} \tag{6}$$

with $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix}, \quad C = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \end{bmatrix}.$

Theorem: The Hessenberg index-2 system given by (6) with $n = 3$ is equivalent to index-1 DAE system of the form

$$E_1 x' + E_2 x = E_3$$

where

$$E_1 = \begin{bmatrix} D \\ 0 \end{bmatrix}, \quad E_2 = \begin{bmatrix} & -DA & \\ c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \end{bmatrix}, \quad x = \begin{bmatrix} Dq \\ -r \end{bmatrix}$$

with

$$\begin{aligned} D &= \begin{bmatrix} b_{11}b_{22} - b_{12}b_{21} & b_{11}b_{32} - b_{12}b_{31} & b_{21}b_{32} - b_{22}b_{31} \end{bmatrix} \\ &= [B_3,\ B_2,\ B_1]. \end{aligned}$$

Proof: Expanding (6) and if we look at $(I - B(CB)^{-1}C)$, then with

$$\begin{aligned} \det(CB) &= -(b_{12}c_{11} + b_{22}c_{12} + b_{32}c_{13})(b_{11}c_{21} + b_{21}c_{22} + b_{31}c_{23}) \\ &\quad +(b_{11}c_{11} + b_{21}c_{12} + b_{31}c_{13})(b_{12}c_{21} + b_{22}c_{22} + b_{32}c_{23}) \\ &= \begin{vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{vmatrix} . \begin{vmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{vmatrix} + \begin{vmatrix} b_{11} & b_{12} \\ b_{31} & b_{32} \end{vmatrix} . \begin{vmatrix} c_{11} & c_{13} \\ c_{21} & c_{23} \end{vmatrix} \\ &\quad + \begin{vmatrix} b_{21} & b_{22} \\ b_{31} & b_{32} \end{vmatrix} . \begin{vmatrix} c_{12} & c_{13} \\ c_{22} & c_{23} \end{vmatrix} \\ &= |C_3| . |B_3| + |C_2| . |B_2| + |C_1| . |B_1| \neq 0 \end{aligned}$$

by assumption, we are using the notation C_i and B_i to mean the matrices C and B with the $i-th$ row, $i-th$ column eliminated respectively. With this notation, we will have

$$E = \left(I - B(CB)^{-1}C\right) = \frac{1}{\det(CB)} \begin{bmatrix} c_3b_3 & -c_3b_2 & c_3b_2 \\ -c_2b_3 & c_2b_2 & -c_2b_1 \\ c_1b_3 & -c_1b_2 & c_1b_1 \end{bmatrix}.$$

It is clear that E has 3 linearly dependent rows or columns since each is a scalar multiple of the other. Which means that E is such that $Rank(E) = 1$. In addition,

$$\begin{aligned}\det\begin{bmatrix} D \\ C \end{bmatrix} &= \det\begin{bmatrix} b_{11}b_{22}-b_{12}b_{21} & b_{11}b_{32}-b_{12}b_{31} & b_{21}b_{32}-b_{22}b_{31} \\ c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \end{bmatrix} \\ &= \det(BC) \neq 0.\end{aligned}$$

This means that the system

$$\begin{aligned} &(a) \quad & D.[x^{'} - Ax - q] &= 0 \\ &(b) \quad & Cx + r &= 0 \end{aligned} \tag{7}$$

is of full rank, a system of 3 equations in three unknowns, completing the proof.

One can generalize this result for $n > 3$ as follows:

Theorem: Consider the system given in (6) when $n > 3$, if there exist k such that $1 \leq k \leq n$, with $c_k(t) \neq 0$ for $t \in I$ where I is some interval, then the $k-th$ row of the matrix $I - B(CB)^{-1}C$ is linearly dependent with respect to other rows.

A similar result can be found in Attili et. al.[3] and for the proof one can refer to it there. As a result if we define

$$\bar{E} = \sum_{i=0}^{n} C_i B_i \left[I - B(CB)^{-1}C\right]$$

then we obtain the matrix D by eliminating the $k-th$ row (k as given in the Theorem) to transform the system from an index-2 overdetermined DAE system to an index-1 system with $n-$equations in $n-$unknowns that has the form given in (7).

3. Differential Transforms

It was first introduced by Zhou[20], who solved linear and non-linear electrical circuit problems using differential transforms. Chen and Ho[11] developed this method for partial differential equations and Ayaz[4] applied it for a system of differential equations. The differential transform of the function $y(x)$ is defined as follows:

$$Y(k) = \frac{1}{k!}\left(\frac{\partial^k y(x)}{\partial x^k}\right) \tag{8}$$

where $y(x)$ is the original function and $Y(k)$ is the transformed one. Also $\left(\frac{\partial^k}{\partial x^k}\right)$ represents the $k-th$ derivative with respect to x. Then to obtain the approximation to a given function $y(x)$, we will use the differential inverse transform of $Y(K)$ which is define as

$$y(x) = \sum_{k=0}^{\infty} x^k Y(k) \tag{9}$$

So, combining (8) and (9), we get

$$y(x) = \sum_{k=0}^{\infty} \frac{1}{k!}[\frac{\partial^k y(x)}{\partial x^k}]x^k. \tag{10}$$

From the definitions it can be seen that the concept of differential transform is derived from Taylor series expansion. It is just a generalization. The differential transform clearly satisfies the linear conditions

Original function	Transformed function
$f(x) = g(x) + h(x)$	$F(k) = G(k) + H(k)$
$f(x) = \alpha g(x)$	$F(k) = \alpha G(k)$.

The basic mathematical operations of differential transform that are commonly used are obtained and given in the following table

Original function	Transformed function
$f(x) = \frac{dg(x)}{dx}$	$F(k) = (k+1)G(k+1)$
$f(x) = \frac{d^m g(x)}{dx^m}$	$F(k) = (k+1)(k+2)\ ...\ (k+m)G(k+m)$
$f(x) = g(x)h(x)$	$F(k) = \sum\limits_{i=0}^{k} G(i)H(k-i)$
$f(x) = x^m$	$F(k) = \delta(k-m), \quad \delta(k-m) = \begin{cases} 1 & , \quad if \quad k = m \\ 0 & , \quad if \quad k \neq m \end{cases}$
$f(x) = e^{\alpha x}$	$F(k) = \frac{\alpha^k}{k!}$
$f(x) = (1+x)^m$	$F(k) = \frac{m(m-1)\ ...\ (m-k+1)}{k!}$
$f(x) = \sin(\varpi x + \alpha)$	$F(k) = \frac{\varpi^k}{k!} \sin\left(\frac{\pi k}{2} + \alpha\right)$
$f(x) = \cos(\varpi x + \alpha)$	$F(k) = \frac{\varpi^k}{k!} \cos\left(\frac{\pi k}{2} + \alpha\right)$

We will describe how to use this approach to solve DAE problems in the next section.

4. Numerical Examples

We will first consider an example on how to transform an index-2 DAE to a full rank index-1 DAE, for that reason consider the following example:

Example 1. With $t \in [-1,\ 1]$, consider

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}' = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} \sin\lambda^2 t \\ \cos\lambda^2 t \\ \lambda^2 t \end{bmatrix} y + \begin{bmatrix} q_1(t) \\ q_2(t) \\ q_3(t) \end{bmatrix}$$

$$\sin(\lambda^2 t)x_1 + \cos(\lambda^2 t)x_2 + \lambda^2 t x_3 = -r(t)$$

with suitable $q_i(t)$ and $r(t)$ to satisfy the given system.

It is easy to see that $(CB)^{-1}$ is just a scalar and

$$(CB)^{-1} = \frac{1}{(\sin\lambda^2 t)^2 + (\cos\lambda^2 t)^2 + \lambda^4 t^2}.$$

This leads to

$$B(CB)^{-1}C = (CB)^{-1} \begin{bmatrix} (\sin\lambda^2 t)^2 & \sin\lambda^2 t\cos\lambda^2 t & \lambda^2 t\sin\lambda^2 t \\ \sin\lambda^2 t\cos\lambda^2 t & (\cos\lambda^2 t)^2 & \lambda^2 t\cos\lambda^2 t \\ \lambda^2 t\sin\lambda^2 t & \lambda^2 t\cos\lambda^2 t & \lambda^4 t^2 \end{bmatrix}.$$

Examining this matrix we see that the second and third rows are dependent as expected. In this case the matrix D and after some simplification and eliminating the third row is given as

$$D = \begin{bmatrix} (\cos\lambda^2 t)^2 + \lambda^4 t^2 & -\sin\lambda^2 t\cos\lambda^2 t & -\lambda^2 t\sin\lambda^2 t \\ -\sin\lambda^2 t\cos\lambda^2 t & (\sin\lambda^2 t)^2 + \lambda^4 t^2 & -\lambda^2 t\cos\lambda^2 t \end{bmatrix}$$

which will change the problem from index-2 problem to index-1 full rank problem given by (7).

Example 2. We will consider an example on how to use differential transforms to solve a differential algebraic equation. For that reason consider the system (see[9])

$$\begin{pmatrix} 1 & -t & t^2 \\ 0 & 1 & -t \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} v_{1'} \\ v_{2'} \\ v_{3'} \end{pmatrix} + \begin{pmatrix} 1 & -(1+t) & t^2+2t \\ 0 & -1 & t-1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \sin t \end{pmatrix}$$

subject to the initial condition

$$\begin{pmatrix} v_1(0) \\ v_2(0) \\ v_3(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

Using the differential transformation table given in the previous section, we obtain for the first equation the following

$$\begin{aligned} 0 &= (k+1)V_1(k+1) - \sum_{r=0}^{k} \delta(r-1)(k-r+1)V_2(k-r+1) \\ &+ \sum_{r=0}^{k} \delta(r-2)(k-r+1)V_3(k-r+1) + V_1(k) - \sum_{r=0}^{k} \delta(r-1)V_2(k-r) \\ &- V_2(k) + \sum_{r=0}^{k} \delta(r-2)V_3(k-r) + 2\sum_{r=0}^{k} \delta(r-1)V_3(k-r) \end{aligned}$$

Which when simplified we obtain

$$\begin{aligned} V_1(k+1) &= \frac{1}{k+1}[kV_2(k) - (k-1)V_3(k-1) - V_1(k) + V_2(k-1) \\ &\quad + V_2(k) - V_3(k-2) - 2V_3(k-1)] \end{aligned}$$

Similarly for the second equation,

$$V_2(k+1) = \frac{1}{k+1}\left[kV_3(k) + V_2(k) + V_3(k-1) + V_3(k)\right] \tag{11}$$

While for third equation, we use the Maclaurin's series expansion of $\sin t$ given as

$$\sin t = \sum_{i=0}^{\infty} \frac{(-1)^{i+2}}{(2i+1)!} t^{2i+1}$$

As a result we have

$$\begin{aligned} V_1(0) &= 1, \quad V_1(1) = 0, \quad V_1(2) = \frac{3}{2!}, \quad V_1(3) = \frac{2}{3!}, \quad V_1(4) = \frac{5}{4!}, \\ V_1(5) &= \frac{4}{5!}, \quad V_1(6) = \frac{7}{6!}, \quad V_1(7) = \frac{6}{7!}, \quad V_1(8) = \frac{9}{8!}, \quad V_1(9) = \frac{8}{9!}, ... \end{aligned}$$

and

$$v_1(t) = 1 + \frac{3}{2!}t^2 + \frac{2}{3!}t^3 + \frac{5}{4!}t^4 + \frac{4}{5!}t^5 + \frac{7}{6!}t^6 + \frac{6}{7!}t^7 + \frac{9}{8!}t^8 + \frac{8}{9!}t^9 + \ ...$$

and

$$v_2(t) = 1 + t + \frac{3}{2!}t^2 + \frac{1}{3!}t^3 - \frac{3}{4!}t^4 + \frac{1}{5!}t^5 + \frac{7}{6!}t^6 + \frac{1}{7!}t^7 - \frac{7}{8!}t^8 + \frac{1}{9!}t^9 + \ ...$$

Note that the exact solutions are $v_1(t) = e^{-t} + te^t$ and $v_2(t) = e^t + t\sin t$ and the obtained solutions are the first few terms of the expansion of these solutions.

Example 3. Consider the index-2 differential algebraic equation (DAE) given by, see Celik[10]

$$\begin{aligned}
x_1' &= \left(\lambda - \frac{1}{2-t}\right) x_1 + (2-t)\, y + \left(\frac{3-t}{2-t}\right) e^t \\
x_2' &= \left(\frac{\lambda - 1}{2-t}\right) x_1 - x_2 + (\lambda - 1)\, y + 2e^t \\
0 &= (t+2)x_1 + (t^2-4)x_2 - (t^2+t-2)e^t
\end{aligned}$$

subject to the initial conditions:

$$x_1(0) = x_2(0) = 1.$$

Using Theorem 1 and as done in Example 1, it can be transformed into index–1 DAE that has the form

$$\begin{bmatrix} (\lambda-1) & \lambda(t-2) \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1' \\ x_2' \end{bmatrix} + \begin{bmatrix} \dfrac{\lambda-1}{2-t} & \lambda\,(t-2) \\ t+2 & t^2-4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$
$$= \begin{bmatrix} \dfrac{-2\lambda t^2 + (7\lambda+1)\,t - 5\lambda - 3}{2-t} e^t \\ \left(t^2+t-2\right) e^t \end{bmatrix}$$

Eliminating x_1 from the first equation to prepare it for the use of differential transforms we obtain,

$$\left(-t^2+4t-4\right) x_2' + \left(-10t^2+40t-40\right) x_2 = -11t^2e^t + 44te^t - 44e^t.$$

Using the transformation table given in the previous section leads to

$$\sum_{i=0}^{k} \delta(k-1)(k-i+1)X_2(k-i+1) + 10X_2(k) = \frac{11}{k!}$$

or

$$(k+1)X_2(k+1) + 10X_2(k) = \frac{11}{k!}.$$

Hence

$$X_2(k+1) = -\frac{10X_2(k)}{k+1} + \frac{11}{k!(k+1)}.$$

With $X_2(0) = 0$ we obtain

$$\begin{aligned} k &= 0, \quad X_2(1) = -10 + 11 = 1 \\ k &= 1, \quad X_2(2) = -\frac{10}{2} + \frac{11}{2} = \frac{1}{2} \end{aligned}$$

$$\begin{aligned} k &= 2, \quad X_2(3) = -\frac{10}{2.3} + \frac{11}{2!.3} = \frac{1}{3!} \\ k &= 3, \quad X_2(4) = -\frac{10}{3!.4} + \frac{11}{3!.4} = \frac{1}{4!} \end{aligned}$$

and so on leading to

$$\begin{aligned} x_2(t) &= \sum_{k=0}^{\infty} t^k X_2(k) \\ &= 1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \frac{t^4}{4!} + \frac{t^5}{5!} + \frac{t^6}{6!} + \frac{t^7}{7!} + ... \end{aligned}$$

which agrees very much with the exact solution $x_2(t) = e^t$. Using the algebraic part of the DAE, we obtain

$$x_1(t) = x_2(t) = 1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \frac{t^4}{4!} + \frac{t^5}{5!} + \frac{t^6}{6!} + \frac{t^7}{7!} + ...$$

To retrieve y and as mentioned before, we use the formula

$$y = \frac{1}{CB} C[x^{'} - Ax - q].$$

This gives

$$y(t) = \frac{-1}{2-t} \left(1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \frac{t^4}{4!} + \frac{t^5}{5!} + \frac{t^6}{6!} + \frac{t^7}{7!} + ... \right)$$

Again this agrees very much with the exact solution which is $y(t) = \frac{-e^t}{2-t}$. This competes very well with other methods presented in the literature.

5. Conclusion

In this paper, we considered how to transform an index-2 DAE to an index-1 DAE which makes it easier to solve. We applied the one dimensional differential transform to solve the resulting system. We have demonstrated that using the differential transform, DAE's can be easily transformed into algebraic equations which can be solved by simple manipulations. This approach proved to be effective in solving DAE's in general and those of index-2 in particular.

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