DYNAMIC ANALYSES OF HIGH-SPEED RAILWAY BRIDGE STRUCTURE UNDER MOVING VEHICLES

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ABSTRACT. *The suitability of the structure above a high-speed railway bridge beam is analyzed based on the dynamics of moving railroad vehicles. To verify the accuracy of numerical simulation and to establish a practical numerical model, a theory predicting the impact of a single or multiple moving railroad cars on a simple-supported beam with damping will be developed and then implemented to analyze real case-study data. The results indicate that the vibrating acceleration induced and beam-end rotation angle need to be superpositioned with several mode shapes to obtain the approximate results. When the vehicular loading frequency approaches the bridge natural cycle, an excited resonance of the bridge occurs; the vehicle speed that causes the resonance can be accurately calculated using numerical analyses. Although the assumed conditions used in the numerical analyses may cause some analytical errors, these errors are quite limited and thus the numerical method is rather accurate for practical applications.*

AMS Classification: 37M05

Keywords: Moving Loads, Simulation of Finite Element Method, Resonance speed of motor vehicle.

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1. INTRODUCTION

The high-speed railroad bridge structure being constantly impacted by the moving high-speed railroad cars causing the bridge to deform abnormally leading to rail twist and possible derail of the moving vehicle. Additionally, the excessive bridge vibration may also reduce the contact between the moving vehicle and the rail that leads to derail problem. Therefore, a reliable method is proposed in this study for analyzing the dynamics of bridges under the influence of moving vehicles. The suitability of bridge upper structure can thus be determined based on the dynamics of moving railroad cars.

Taiwan is limited in land space but is heavily populated such that a high-speed transportation railroad in the west corridor to ease the traffic problem. In order to alleviate the environmental impact and traffic blockage of areas adjacent to the railroad route, the total length of planned over-pass bridges constitutes 75 % of the total railroad length. How to assure that these bridges are stable under the moving of high-speed vehicles to provide a safe and comfort service to the passengers is an important topic. The bridging vibrating problem caused by the passing high-speed railroad cars has been studied since the 19th century. During the earlier stage, the analytical capability was quite limited; only simplified models were to complete the analysis. The wide use of computers after 1960 has enabled the implementation of complicated cars and bridge models in computer data simulation. Current dynamic simulation of moving railroad cars concentrates on simulating moving loaded cars and moving mass.

Simulating moving loaded weight was first studied by Timoshenko [1] by considering a single loading passing simple support bridges without damping at a uniform speed to study the dynamic response. The result shows that the car speed is related to the natural frequency of the bridge; the bridge response will be exaggerated indefinitely with time. Gandi and Ramondenc [2] studied the dynamic response of a series of simulated moving cars passing simple supported composite bridges. They discovered that the bridge showed obvious enlargement effect under the moving high-speed railroad cars. Matsura [3] observed that the bridge resonance was related to not only the natural bridge frequency but also the railroad car length. Yau and Yang [4] based on theoretical analyses of the impact caused by moving equally-spaced cars on simple-supported bridges to propose the conditions for the occurrence and elimination of bridge resonance. Simulating the mass movement involves mutual non-linear inter-reaction between the moving vehicle and bridge structure; thus it cannot be solved theoretically. The analyses are mostly carried with numerical simulation that requires the use of simplifying models of various degrees for simulating the passing railroad cars. For example, Akin and Mofid [5] only took the inertial effect of the moving mass into consideration; Chu, Garg and Wang [6] considered the characteristic feature of the car suspension devise to simulate cars with mass of single degree of freedom.

2. Theoretical Solution for Simple-Supported Beam with Damping

Theoretical solutions for simple-supported beam with damping subject to a single or multiple moving loadings are derived to form the bases for comparing the numerical simulation results in order to verify the accuracy of numerical simulation analyses.

2.1. Simple-Support Beam with Damping Subject to a single moving load.

The dynamic movement of a simple-supported beam with damping subject a single moving load can be defined by the following equation:

$$m\frac{\partial^2 y}{\partial t^2} + c\frac{\partial y}{\partial t} + EI\frac{\partial^4 y}{\partial x^4} = F_0 H(t)\delta(x - \nu t)\ ,\ 0 < t < \frac{L}{\nu} \tag{1}$$

Where :

EI : Stiffness of Beam $(N/m^2 \cdot m^4)$;
c : damping (s/m);
ν : Vehicle speed (m/s);
L : Length of Beam (m);
F_0 : Loading (N);
$y(x,t)$: Vertical movement of Beam (m);
$H(\bullet)$: Heaviside function;
$\delta(\bullet)$: Delta function;
m : Unit mass of the simple-support beam (kg/m);

Assuming that

$$y(x,t) = \sum_{n=1}^{\infty} W_n(x)\ \eta_n(t) \tag{2}$$

Where :

$W_n(x)$ is the nth natural vibration modeshape that conforms to the boundary condition of a simple-supported beam;

$$W_n(x) = \sqrt{\frac{2}{mL}} \sin \frac{n\pi}{L} x;$$

Substituting Equation (2) into Equation (1), Equation (3) is obtained:

(3)

$$m \sum_{n=1}^{\infty} W_n(x)\ \ddot{\eta}_n(t) + c \sum_{n=1}^{\infty} W_n(x)\ \dot{\eta}_n(t) + EI \sum_{n=1}^{\infty} W''''{}_n(x)\ \eta_n(t) = F_0 H(t) \delta(x - \nu t)$$

Equation (2) is multiplied by $W_r(x)$ and then integrated with respect to x from 0 to L; the orthogonal characteristics of $W_r(x)$ are used to yield Equation (4);

$$\ddot{\eta}_n(t) + 2\xi W_n(x)\ \dot{\eta}_n(t) + \omega_n^2\ \eta_n(t) = F_n(t) \tag{4}$$

Where :

$$\begin{aligned}
\xi &= \frac{c}{2m\omega_n}\ \text{, damping ratio;} \\
\omega_n^2 &= \frac{EIn^4\pi^4}{mL^4}\ \text{, the } n^{th} \text{ natural frequency of simple-supported beam;} \\
F_n(t) &= \int_0^L F_0 H(t)\delta(x-\nu t)W_n(x)\,dx \\
&= \int_0^L F_0 H(t)\delta(x-\nu\ t)\sin\frac{n\pi x}{L}\sqrt{\frac{2}{mL}}\,dx \\
&= F_0 \sin\frac{n\pi\nu t}{L}\sqrt{\frac{2}{mL}} \times \left[\ H(t) - H(t-\frac{L}{\nu})\ \right]
\end{aligned}$$

Equation (4) is a typical 2^{nd} order vibration differential equation; the solution can be directly solved using the Duhamel integration with the initial condition assumed to be 0, thus :

(5)

$$y(x,t) = \left(\sum_{n=1}^{\infty}\left[\frac{1}{\sqrt{1-\xi^2}\ \omega_n}\int_0^t F_n(\tau)\exp^{-\xi_n(t-\tau)}\sin\sqrt{1-\xi^2}\ \omega_n(t-\tau)\,d\tau\right]\times W_n(x)\right)$$

Equation (5) is integrated to obtain Equation (6), yields

$$y(x,t) = \frac{F_0}{\sqrt{1-\xi^2}}\frac{2}{\pi^2}\sqrt{\frac{L^2}{mEI}}\sum_{n=1}^{\infty}\frac{1}{n^2}\sin\frac{n\pi x}{L}[X_1 - X_2] \tag{6}$$

Where :

$$X_1 = \frac{A\Gamma^t_{10} - B\Gamma^t_{20}}{A^2 - B^2} \ , \ X_2 = \frac{A\Gamma^t_{1L/\nu} - B\Gamma^t_{2L/\nu}}{A^2 - B^2}$$

$$A = \frac{\alpha^2 + \beta^2 + \gamma^2}{\gamma^2} \ , \ B = \frac{2\alpha\beta}{\gamma^2}$$

$$\alpha = \frac{n\pi\nu}{L} \ , \ \beta = \sqrt{1-\xi^2}\ \omega_n \ , \ \gamma = \xi\omega_n$$

$$\Gamma_1(t) = [\frac{1}{\gamma}\exp^{-\gamma(t-\tau)}\sin\alpha\tau\sin\beta(t-\tau) - \frac{\alpha}{\gamma^2}\exp^{-\gamma(t-\tau)}\cos\alpha\tau\sin\beta(t-\tau) + \frac{\beta}{\gamma^2}\exp^{-\gamma(t-\tau)}\cos\beta(t-\tau)\sin\alpha\tau]$$

$$\Gamma_2(t) = [\frac{1}{\gamma}\exp^{-\gamma(t-\tau)}\cos\alpha\tau\cos\beta(t-\tau) - \frac{\alpha}{\gamma^2}\exp^{-\gamma(t-\tau)}\sin\alpha\tau\cos\beta(t-\tau) + \frac{\beta}{\gamma^2}\exp^{-\gamma(t-\tau)}\sin\beta(t-\tau)\cos\alpha\tau]$$

$$\Gamma^b_{ac} = \Gamma_a(b) - \Gamma_a(c)$$

2.2. **Simple-Support Beam with Damping Subject to multiple moving loads:** The solution derived in the above section can be super-positioned to obtain the dynamic response of a simple-supported beam subject to multiple moving loads. Assuming that $y_i(x,t)$ represents the response of a simple-support beam subject to the ith moving load, the total vertical movement of the simple-support beam subject to n moving loads can be expressed as:

$$u(x,t) = \sum_{i=1}^{n} y_i(x, t-t_i)P_i \tag{7}$$

Where :

t_i : the time of the ith moving load entering the simple-support beam $t_i = \frac{\sum L_i}{\nu}$, L_i is the interval between the ith and the $(i+1)^{th}$ loads, L_1 is 0;

P_i :Value of the ith moving load.

3. Numerical Solution for Bridges under Moving Loads

A simple-supported bridge in practical application is installed with partition beam at the ends for anchoring the shearing or pre-stress cables; its cross-sectional area is thus not uniform. Hence, the finite element analysis is usually implemented for simulating the actual situation. To a specific node of the bridge, the moving load is equivalent to an instantaneously changing loading. Therefore, the moving loading can be used to simulate the bridge dynamic response to the moving loads. When applying the finite element analysis, the movement of each node is assumed to be unknown and the movement of the section between two adjacent nodes can be simulated with appropriate movement functions. If the load is applied to a node element, appropriate external forces are simulated to be applied at both ends of the element. When considering the load P being applied to a single element, shown in Fig.1, the load applied at point i leads to leading value of P at node i and zero load at node j. When the load is applied at node j, the loading values are P at node j and 0 at node i. Likewise, when the loading is shifted from i to j, the loading weight is reduced from P to 0 at node i and from 0 to P at node j. If the element is not long, a linear simulation can be applied to simulate the process of variation under external loading.

When considering multiple loadings, the time history of each moving loading is super-positioned; the loading time history of each node in the model can thus be determined with known distance between two adjacent loadings, speed and magnitude. Major procedures of the finite element method to analyze the structural response to moving loadings are summarized as follows:

(1) Establishing the Structural Analytical Model;

(2) Establishing the loading time history for each node according to distance between and speed of the applied loadings as well as the node location in the structural model nodes;

(3) Carrying out the time history analysis.

4. **Results of case-study analyses and Discussion**

The results of analyses on two case studies are discussed in the following paragraphs. Case study 1: A bridge with the following characteristics and loading conditions is used: Young's Modulus (E) = 2.82106 t/m^2, Moment of Inertia (I) = 9 m^4, Mass (M) =30000 kg/m, Bridge Length (L) =30 m, Vehicle Speed (V)=200 km/hr, damping ratio (ξ) =0.025, and Loading Value (F_0) =12 t. Case Study 2: To investigate the accuracy of this model for conducting numerical analysis, the high-speed cars operated on Japan Sinkansen (SKS) railroad are analyzed. The actual bridge information is as follows: Young's Modulus (E) = 2.82106 t/m^2, Moment of Inertia (I) = 7.84 m^4, Mass (M) =41740 kg/m, Bridge Length (L) =30 m (Effective Span = 27.8 m), Vehicle speed (V) =210 km/hr and damping ratio (ξ) =0.025.

4.1. **Results and Discussion.**

$Figs.2 \sim 4$ show the results for Case Study 1 analysis on the dynamic response of a simple-supported beam subject to a single moving load passing. In these figures, the results of various super-positioned mode shapes are compared to reveal that the displacement reaction of the simple beam is basically controlled by the first mode shape. As the vibrating acceleration and the beam-end angle of rotation are concerned, at least several super-positioned mode shapes are needed to obtain approximate results.

$Figs.5 \sim 7$ show the Case Study 2 analytical results; they indicate the beam midpoint maximum acceleration as well as the angles of rotation for the left-most and right-most ends with respect to the car speed for multiple passing cars at 210 km/hr. The results show that the bridge beam dynamic response increases with greater car speed. When the car speed reaches a specially defined " Critical Speed ", the dynamic response of the simple-supported beam will suddenly increase.

When a single car with a fixed length passes over the bridge at a constant speed, its rollers apply a periodic pounding impact on the bridge beam. If the loading frequency approaches the natural frequency of the bridge, a vigorous resonance of the bridge will be induced thus causing the critical speed.

$Fig.8 \sim 11$ shows the first secondary critical speed calculated using the formula $V = \frac{fw_1 L_b}{n}$ where n = 1,2,3 $\cdots$ for a 25-meter single SKS car passing over a simple-supported beam with length L_b. The results reveal that the predicated critical speed is very close to the resonant speed shown in $Fig.8 \sim 11$.

Table 1 lists the predicated critical speed for the French TGV and the Germany ICE car sets passing over simple-supported beams under similar conditions. The analytical results for a car passing a 30-meter simple-supported beam at 210 km/hr shown in $Figs.12 \sim 15$ reveal that both the analytical solution and the finite element solution yield similar displacement and vibrating acceleration at the mid-point of the bridge.

5. Conclusions

In conclusion, the numerical analysis lead to the conclusion that more than three super-positions are need to obtain suitable approximate results. The conditions assumed in the finite element analysis have poor accuracy leading to discrepancies between the analytical results and numerical analysis results on the beam-end angle of rotation. However, the difference is small with its maximum value less than 3 %. Hence, the numerical model proposed in this paper can yield accurate results for practical applications. Additionally, this numerical analysis method will allow one to calculate the accurate resonating car speed; it can be applied for predicting the dynamic impact caused by the moving rollers on the bridge super-structure and the resulting resonance. The results will be valuable references for future design and evaluation of high-speed railroad bridges.

Acknowledgement

In memory of the outstanding Structural Engineer of Taiwan, Lee, I-Shan, P.E., passed away on Feb., 2006. He contributed his life to structural design and build High Speed Railroad for Taiwan.

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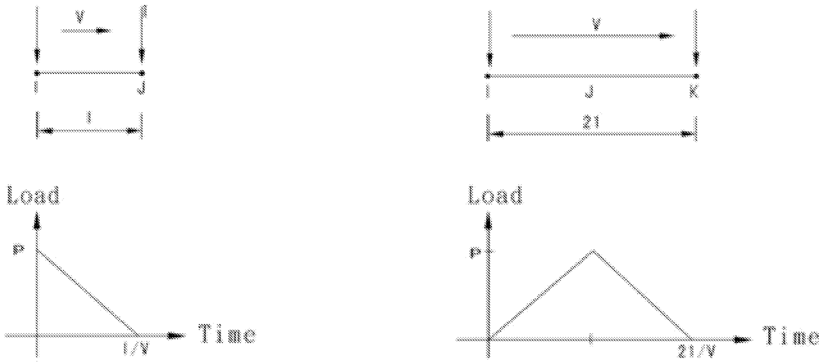
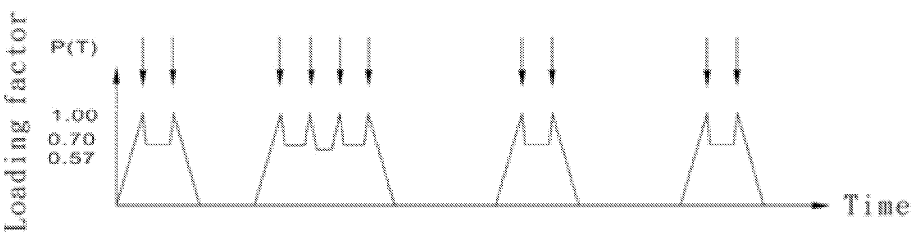
Table 1. The predicated critical speed for TGV and ICE car sets passing over 30 m simple-supported bridge

Type of vehicle	length of Single car (m)	Resonance speed (km/hr)		
		n=1	n=2	n=3
French TGV	18.7	312.1	156.1	104.0
Germany ICE	26.4	220.3	146.9	110.2



(a) Single passing cars



(b) Multiple passing cars

FIGURE 1. The load P being applied to a single element

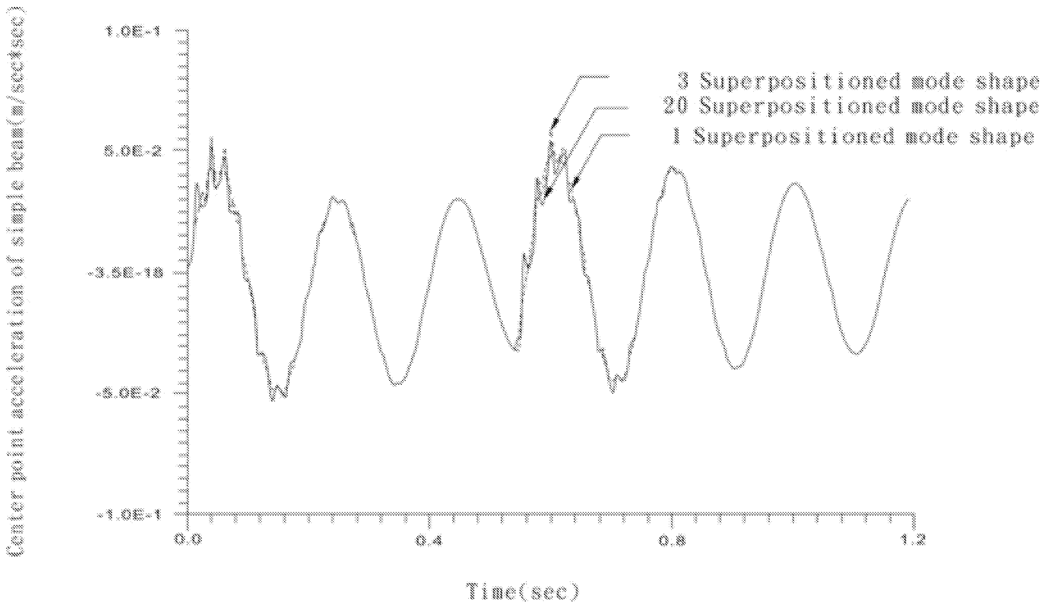


FIGURE 2. Center point displacement for Case Study 1 on the dynamic response of a simple-supported beam

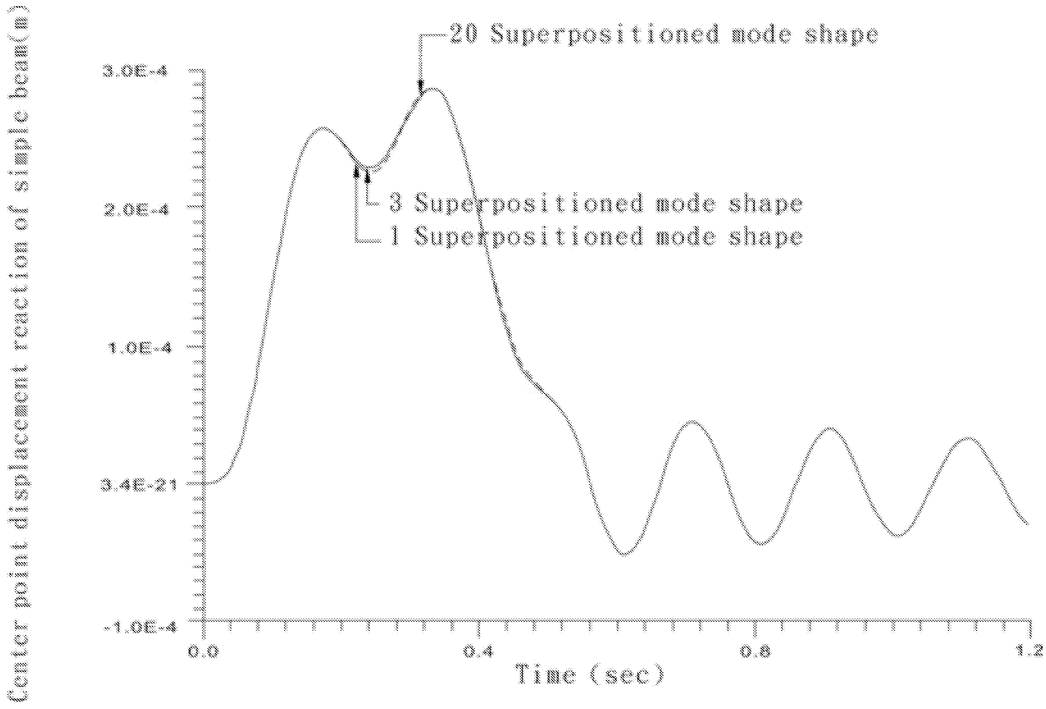


FIGURE 3. Center point acceleration for Case Study 1 on the dynamic response of a simple-supported beam

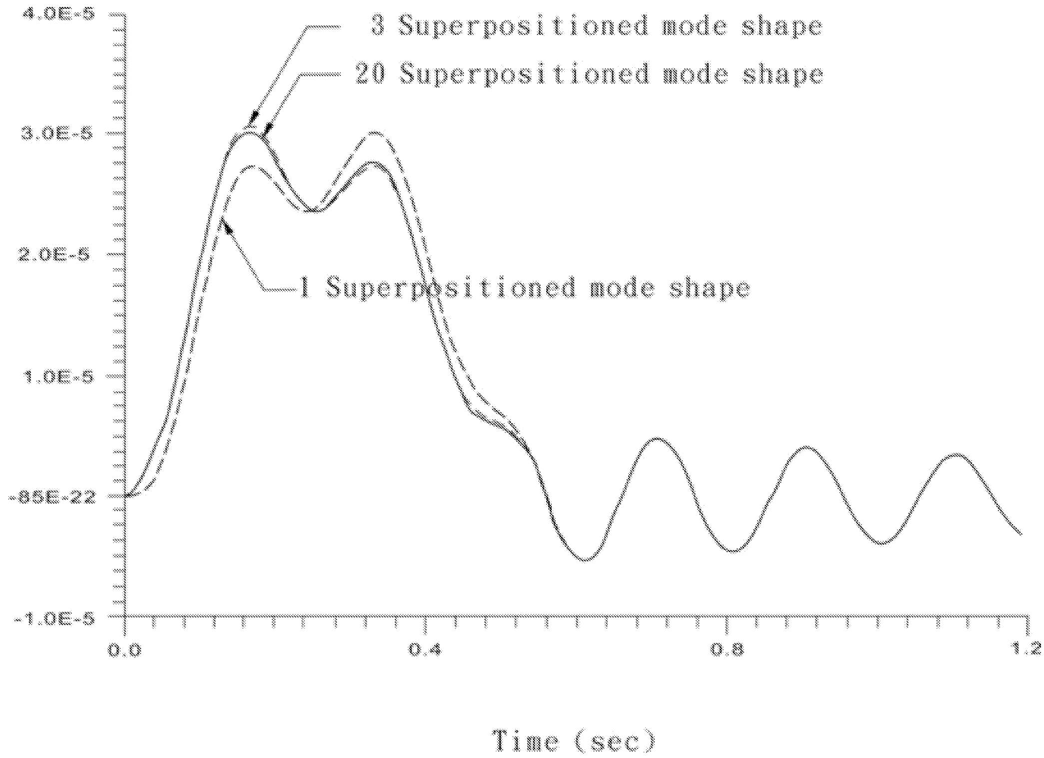


FIGURE 4. Left beam end rotation angle for Case Study 1 on the dynamic response of a simple-supported beam

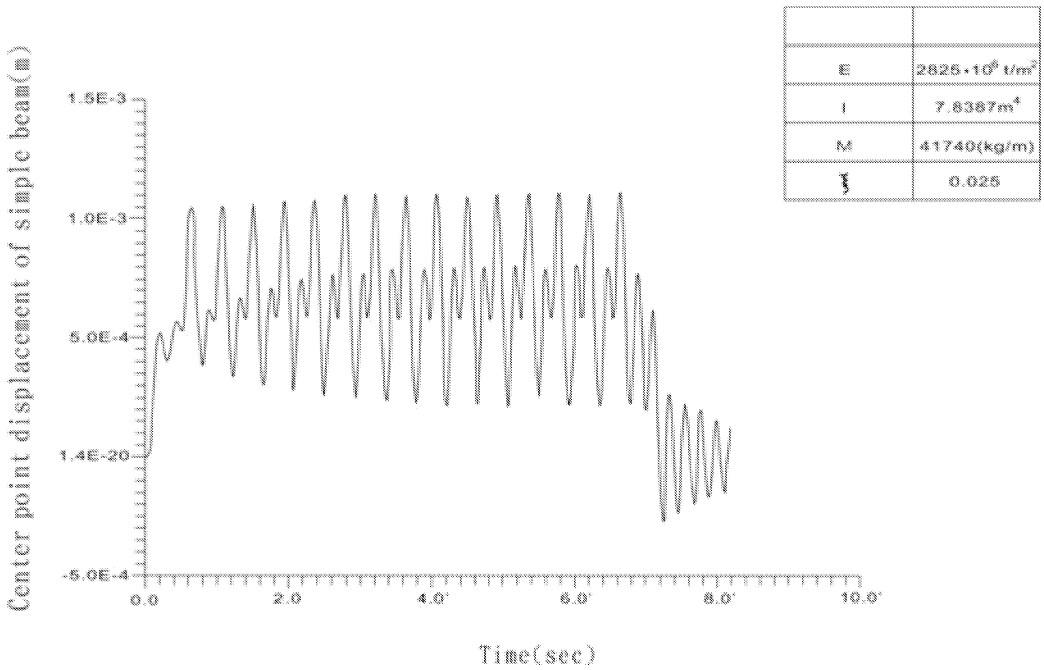


FIGURE 5. Center point displacement for Case Study 2 on the dynamic response of a simple-supported beam

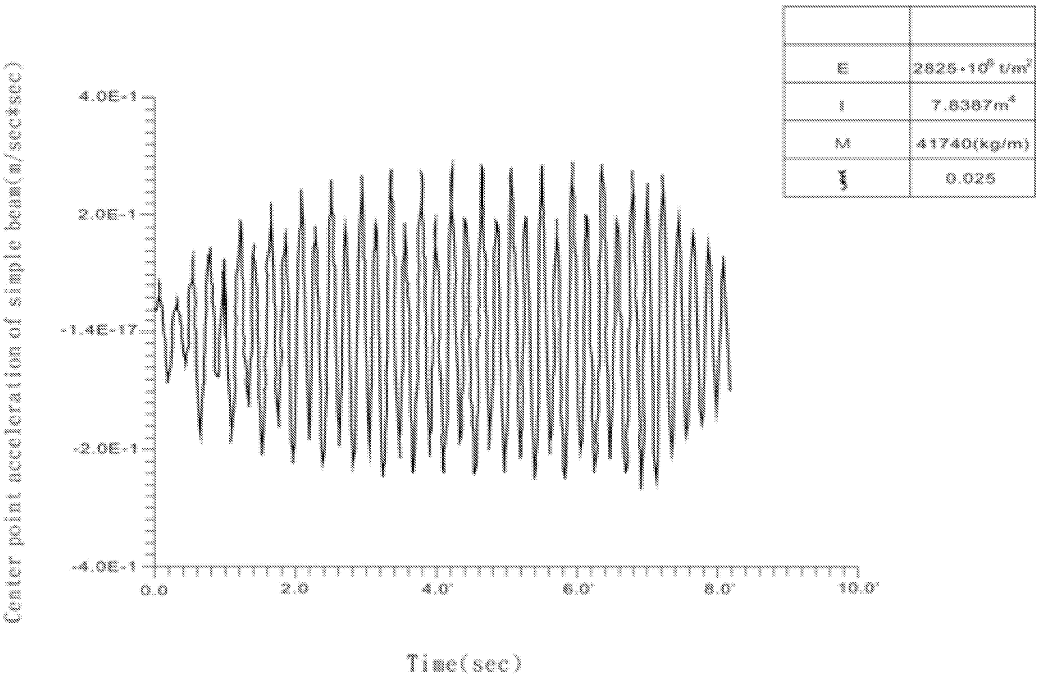


FIGURE 6. Center point acceleration for Case Study 2 on the dynamic response of a simple-supported beam

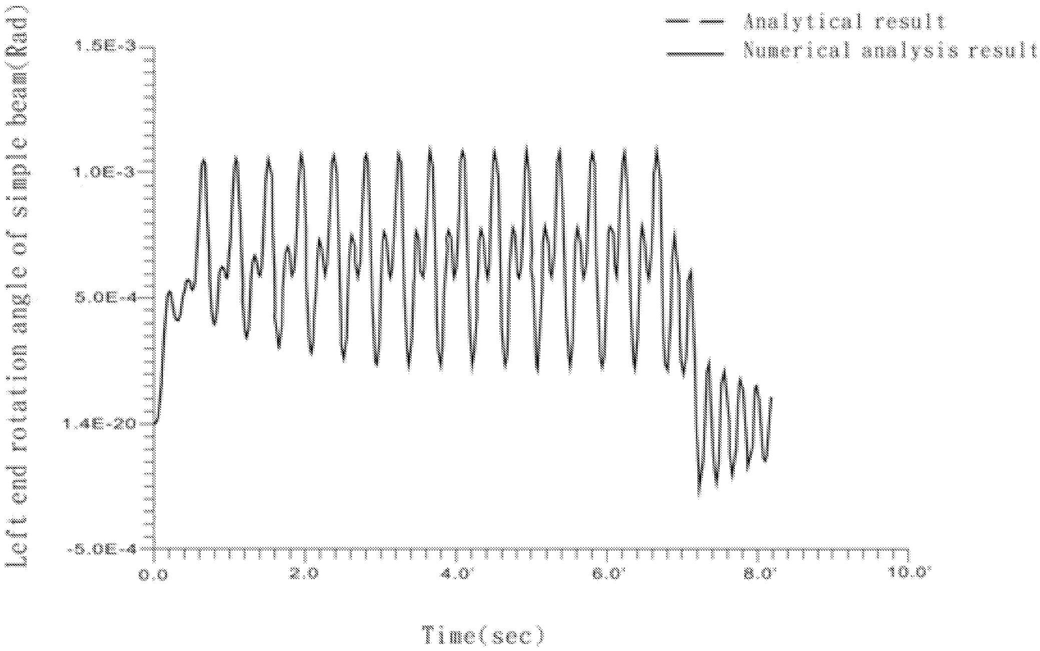


FIGURE 7. Left end rotation angle for Case Study 2 on the dynamic response of a simple-supported beam

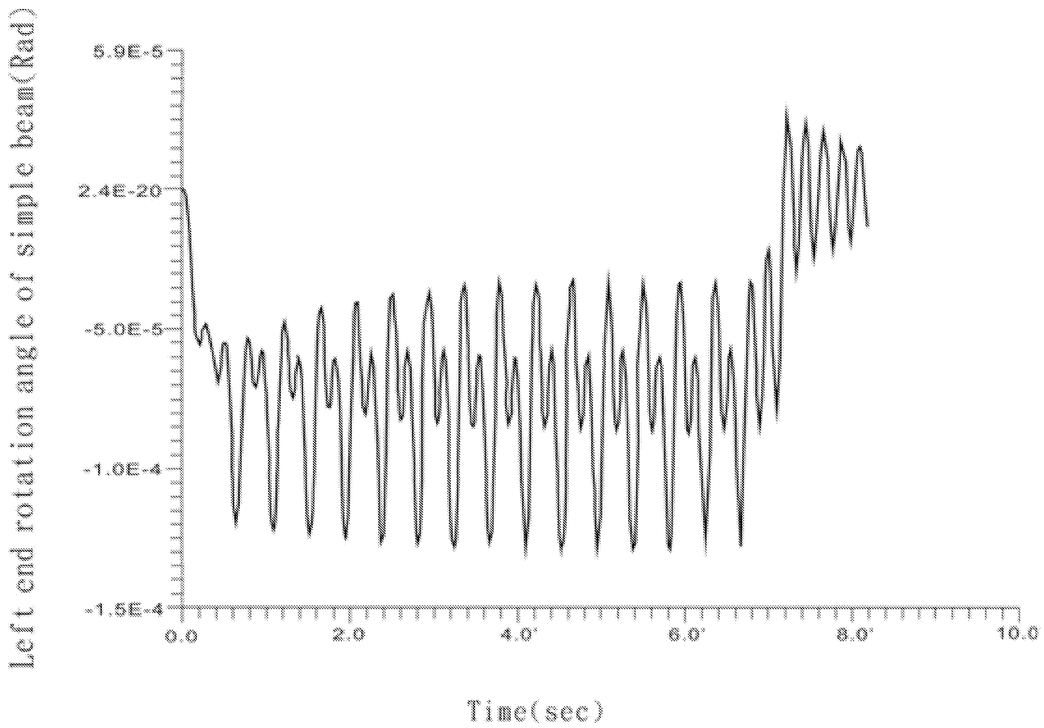


FIGURE 8. Right end rotation angle of simple beam (Rad)

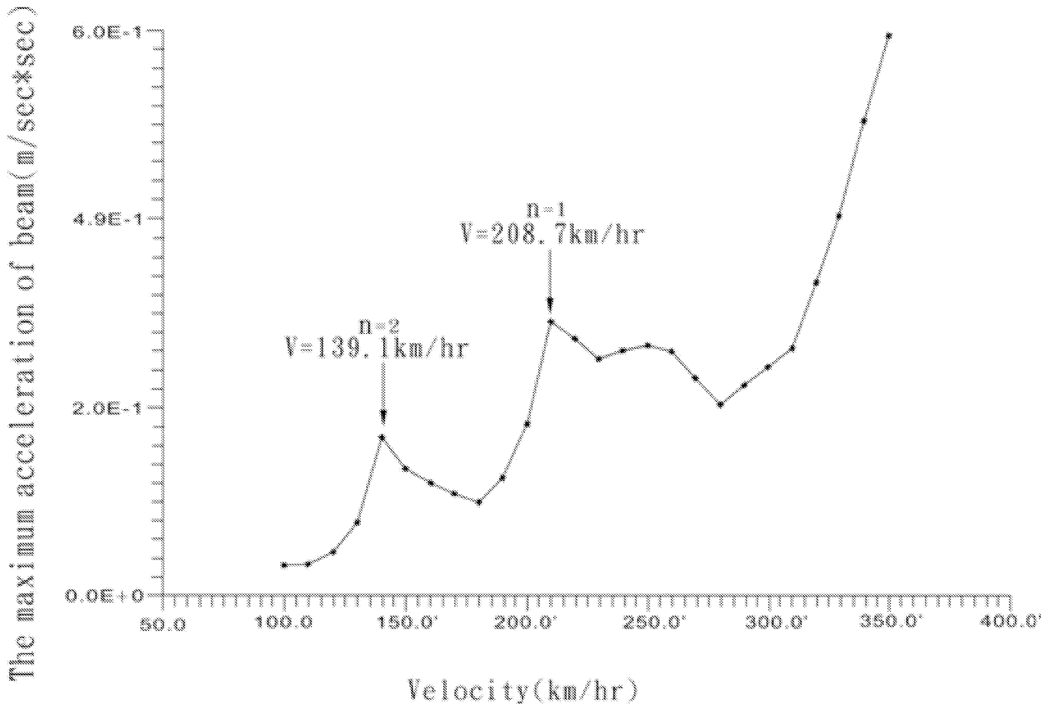


FIGURE 9. The Maximum acceleration of beam for various velocity of high speed train

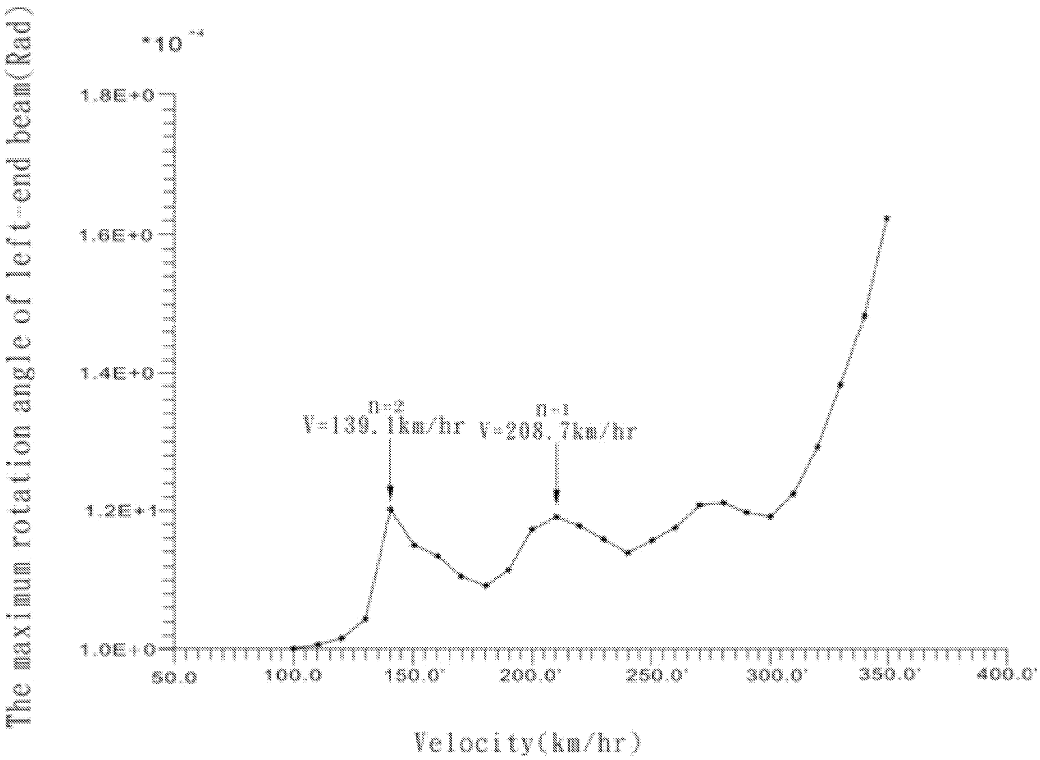


FIGURE 10. The Maximum rotation angle of left-end beam for various velocity of high speed train

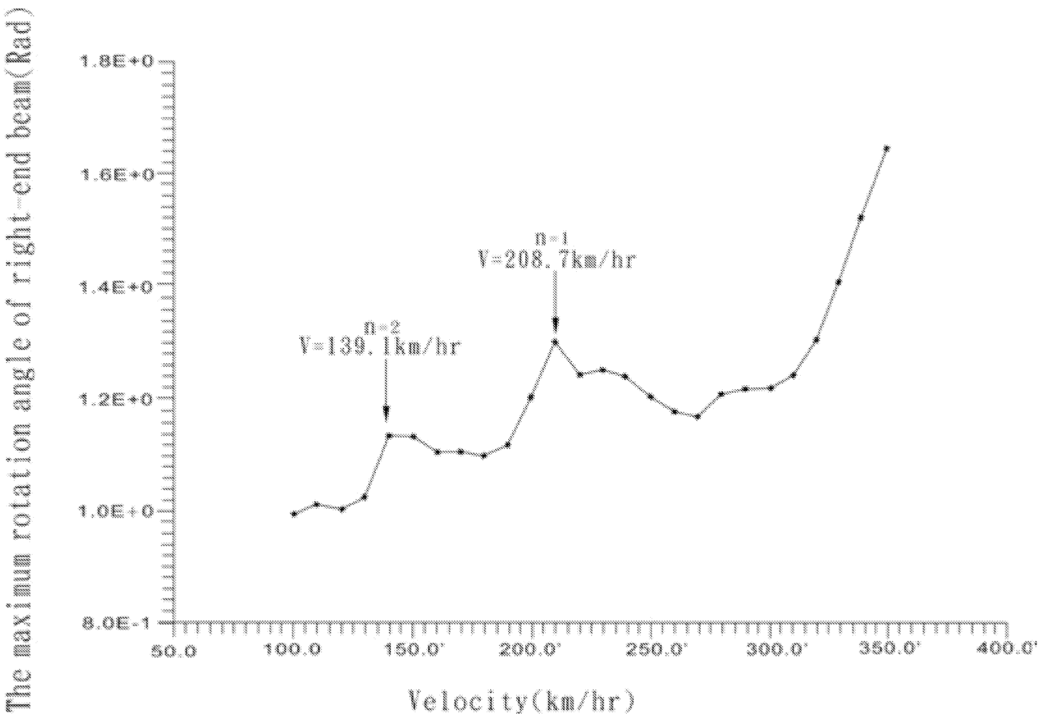


FIGURE 11. The maximum rotation angle of right end beam for 25-meter single SKS car passing over a simple-supported beam

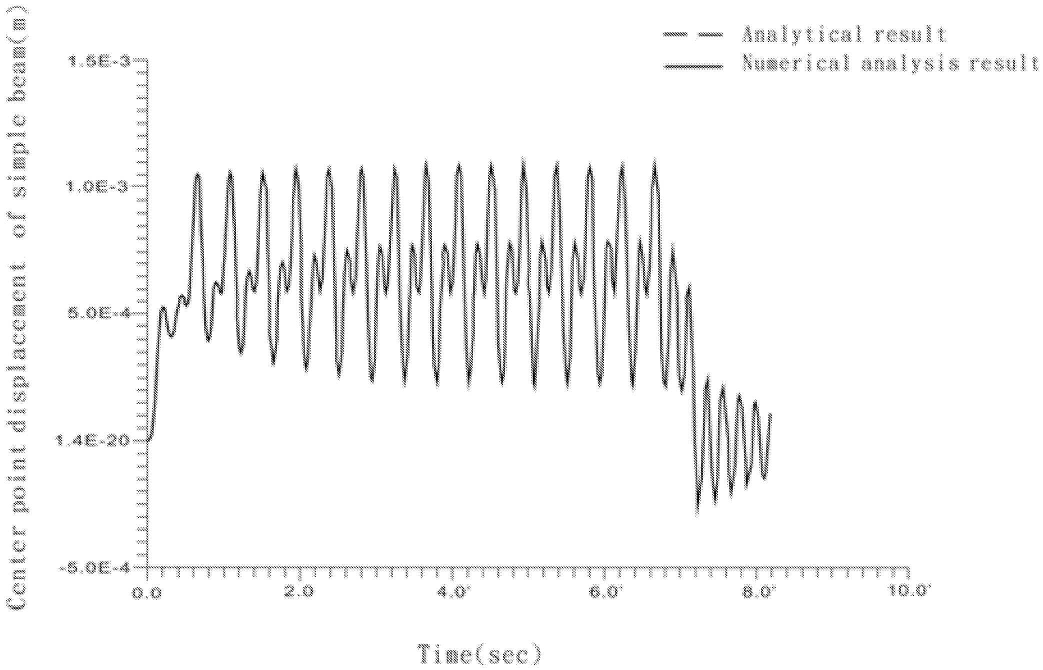


FIGURE 12. The center point displacement for 30-meter single car passing over a simple-supported beam with 210 km/hr

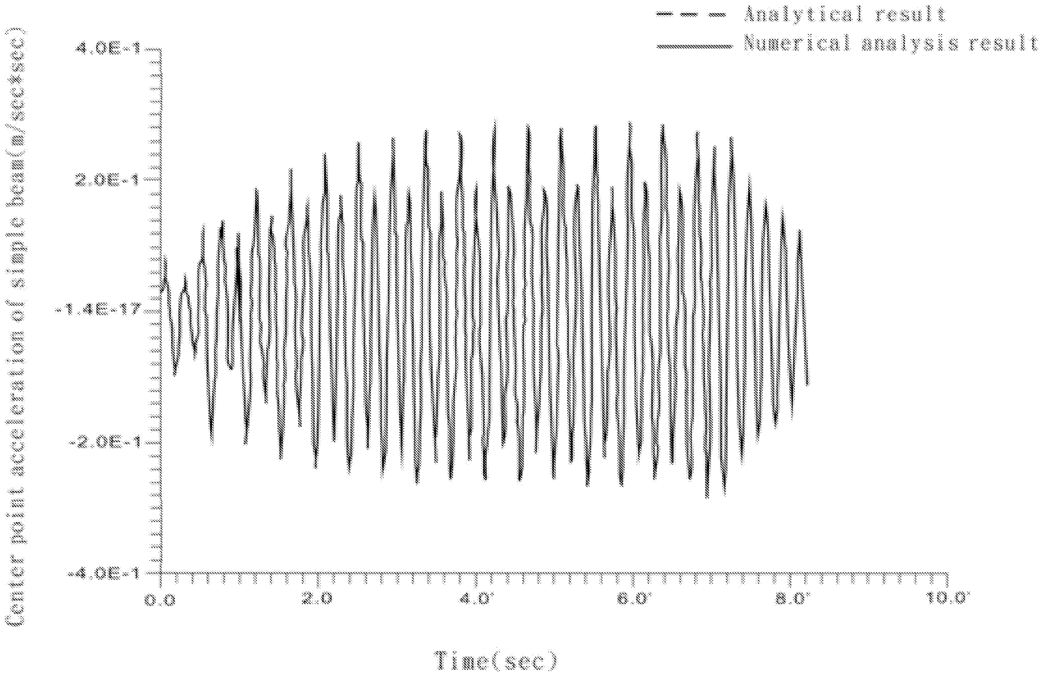


FIGURE 13. The center point acceleration for 30-meter single car passing over a simple-supported beam with 210 km/hr

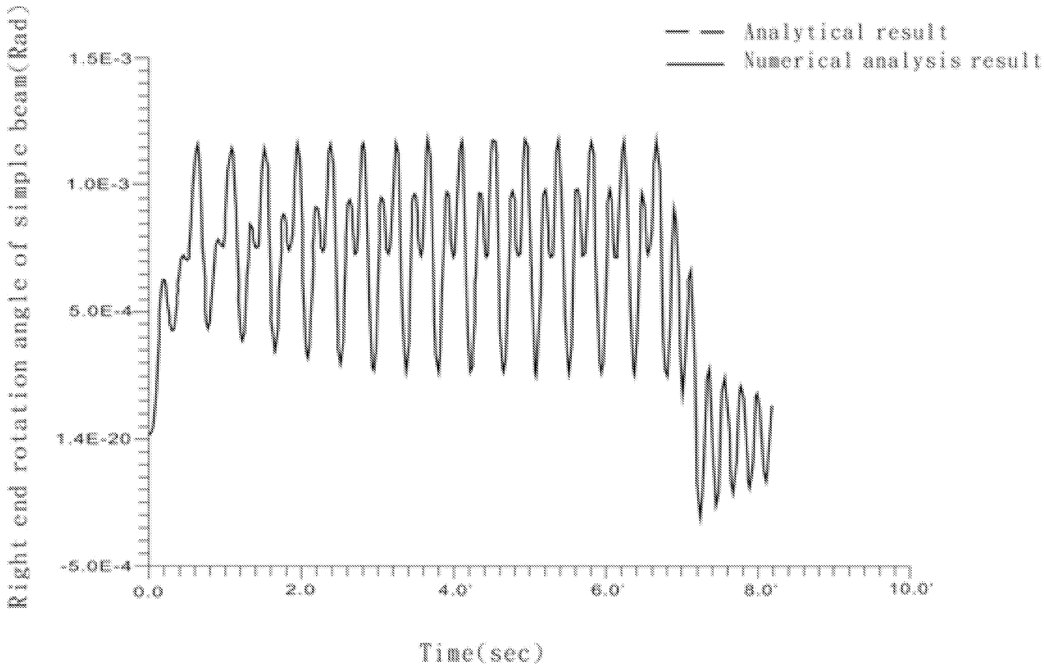


FIGURE 14. The right end rotation angle for 30-meter single car passing over a simple-supported beam with 210 km/hr

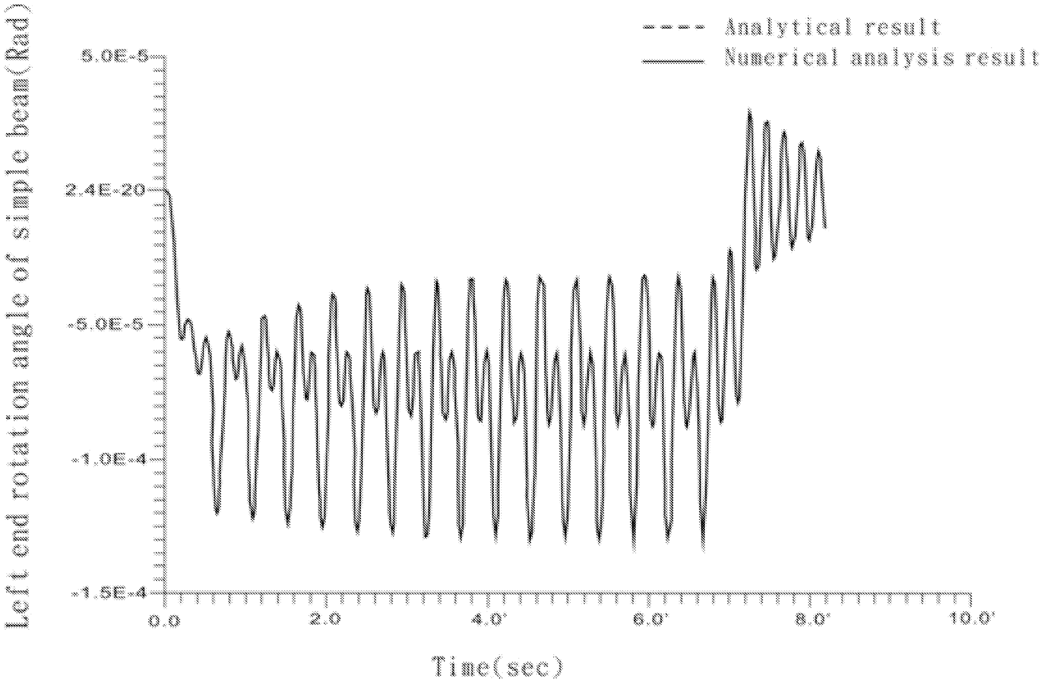


FIGURE 15. The left end rotation angle for 30-meter single car passing over a simple-supported beam with 210 km/hr