

**$Z = \left(\frac{t}{z}\right)$ -TYPE PLANE WAVE SOLUTION OF E-M EQUATIONS
COEXISTING WITH NULL FLUID AND NULL CURRENT IN
SPACE-TIME V_5**

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ABSTRACT. In this paper, we have shown that generalized plane wave metric be the solution of Einstein-Maxwell equations in the presence of null fluid and null currents in higher dimensional space-time V_5 for $Z = \left(\frac{t}{z}\right)$ -type plane wave and also discussed some particular cases.

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1. INTRODUCTION

The plane gravitational waves g_{ij} are mathematically exposed by H. Takeno [8] in general relativity and shown that these waves are transformable into each other by using suitable transformation. T. Singh [7] has investigated that the generalized

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plane wave metric be the solution of Einstein-Maxwell field equation in the presence of null fluid and null currents for $Z = \left(\frac{t}{z}\right)$ -type plane wave. In similar way Chirde et al.[3] shown that the generalized space-time obtained by Metkar et al.[4], be the solution of Einstein-Maxwell field equation coexisting with null fluid and null currents for $Z = \left(\frac{t}{z}\right)$ -type plane wave. Adhav et al.[1] reformulated Takeno's definition of plane wave in higher dimension and investigated space-times. Patil et al.[5] shown that the space-times for $Z = (z - t)$ and $Z = \left(\frac{t}{z}\right)$ -type waves in five dimension are transformable in to each other. Ambatkar et al.[2] further studied five dimensional space-time for $Z = (z - t)$ and $Z = \left(\frac{t}{z}\right)$ -type waves investigated by Adhav et al.[1] and obtained plane wave solution of Einstein-Maxwell equations for both type waves. Patil et al.[6] deduced the generalized form of $Z = \left(\frac{t}{z}\right)$ -type plane wave in higher dimensional space-time by taking $E = E(x, y, u, Z)$ as

$$(1) \quad ds^2 = -Adx^2 - 2Ddx dy - Bdy^2 - 2Gdx du - 2Hdy du - Fdu^2 \\ - Z^2 (C - E) dz^2 - 2ZEdz dt + (C + E) dt^2,$$

where A, B, C, D, F, G and H are the function of $Z = \left(\frac{t}{z}\right)$ and $E = E(x, y, u, Z)$. The Einstein-Maxwell field equations in the presence of null fluid and null currents are given by

$$(2) \quad R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi (E_{ij} + L_{ij}).$$

Here,

$$(3) \quad E_{ij} = \frac{1}{4}g_{ij}F_{kl}F^{kl} - F_{il}F_{jm}g^{lm},$$

be the energy momentum tensor of an electromagnetic field F_{ij} and,

$$(4) \quad L_{ij} = hv_i v_j,$$

is the energy tensor for the null fluid of the energy density $h = h(x, y, u, Z)$ with v^i velocity satisfying

$$(5) \quad g_{ij}v^i v_j = 0,$$

where g_{ij} being the metric tensor. And the Maxwell's equations are

$$(6) \quad F_{ij;k} + F_{jk;i} + F_{ki;j} = 0.$$

The electric five currents j^i are given by,

$$(7) \quad J^i \equiv F^i_{;k} = \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^k} (\sqrt{-g} F^{ik}),$$

satisfying

$$(8) \quad g_{ik} J^i J^k = 0.$$

In this paper, we have shown that the space-time (1) be the solution of the field equation (2) in the presence of null fluid and null currents and also discussed some particular cases.

2. SOLUTION OF FIELD EQUATIONS

From equation(1) we have

$$(9) \quad g_{11} = -A, \quad g_{12} = g_{21} = -D, \quad g_{13} = g_{31} = -G, \quad g_{22} = -B, \quad g_{23} = -H,$$

$$g_{33} = -F, \quad g_{44} = Z^2(C + E), \quad g_{45} = -ZE, \quad g_{55} = C + E.$$

$$(10) \quad g^{11} = \frac{(BF - H^2)}{m}, \quad g^{12} = \frac{-(DF - GH)}{m}, \quad g^{13} = \frac{(DH - BG)}{m},$$

$$g^{21} = \frac{-(DF - GH)}{m}, \quad g^{22} = \frac{(AF - G^2)}{m}, \quad g^{23} = \frac{-(AH - DG)}{m},$$

$$g^{31} = \frac{-(DH - BG)}{m}, \quad g^{32} = \frac{-(AH - DG)}{m}, \quad g^{33} = \frac{(AB - D^2)}{m},$$

$$g^{44} = \frac{-(C + E)}{C^2 Z^2}, \quad g^{45} = \frac{-E}{C^2 Z}, \quad g^{55} = \frac{(C + E)}{C^2}$$

where $m = (-ABF + AH^2 + D^2 - 2DGH + BG^2)$.

Using (1), the non-vanishing components of Ricci tensor are as follows,

$$(11) \quad \frac{R_{44}}{Z^2} = -\frac{R_{45}}{Z} = R_{55} = \frac{W}{z^2}$$

$$(12) \quad R_{14} = -ZR_{15} = \frac{E_x}{Cz}, \quad R_{24} = -ZR_{25} = \frac{E_y}{Cz}, \quad R_{34} = -ZR_{35} = \frac{E_u}{Cz}$$

$$(13) \quad R = 0,$$

where

$$W = -\frac{1}{m} \left[(BF - H^2) \dot{p} + (AF - G^2) \dot{Q} + (AB - D^2) \dot{R} \right]$$

$$+ \frac{1}{m} \left[2(DF - GH) \dot{S} + 2(AH - DG) \dot{T} - (DH - BG) \dot{U} \right],$$

$$\dot{P} = \frac{1}{2} \left[\bar{A} - z^2 E_{xx} \right] + \frac{1}{4m} \left[(BF - H^2) \bar{A}^2 + (AF - G^2) \bar{D}^2 + (AB - D^2) \bar{G}^2 \right]$$

$$\begin{aligned}
& -\frac{1}{4m} [2(DF - GH) \bar{A}\bar{D} + 2(AH - DG) \bar{D}\bar{G} - (DH - BG) \bar{A}\bar{G}] - \frac{\bar{A}}{2C} \left(\bar{C} - \frac{E}{Z} \right), \\
\dot{Q} &= \frac{1}{2} \left[\bar{\bar{A}} - z^2 E_{xx} \right] + \frac{1}{4m} [(BF - H^2) \bar{A}^2 + (AF - G^2) \bar{D}^2 + (AB - D^2) \bar{G}^2] \\
& -\frac{1}{4m} [2(DF - GH) \bar{A}\bar{D} + 2(AH - DG) \bar{D}\bar{G} - (DH - BG) \bar{A}\bar{G}] - \frac{\bar{A}}{2C} \left(\bar{C} - \frac{E}{Z} \right), \\
\dot{R} &= \frac{1}{2} \left[\bar{\bar{A}} - z^2 E_{xx} \right] + \frac{1}{4m} [(BF - H^2) \bar{A}^2 + (AF - G^2) \bar{D}^2 + (AB - D^2) \bar{G}^2] \\
& -\frac{1}{4m} [2(DF - GH) \bar{A}\bar{D} + 2(AH - DG) \bar{D}\bar{G} - (DH - BG) \bar{A}\bar{G}] - \frac{\bar{A}}{2C} \left(\bar{C} - \frac{E}{Z} \right), \\
\dot{S} &= \frac{1}{2} \left[\bar{\bar{A}} - z^2 E_{xx} \right] + \frac{1}{4m} [(BF - H^2) \bar{A}^2 + (AF - G^2) \bar{D}^2 + (AB - D^2) \bar{G}^2] \\
& -\frac{1}{4m} [2(DF - GH) \bar{A}\bar{D} + 2(AH - DG) \bar{D}\bar{G} - (DH - BG) \bar{A}\bar{G}] - \frac{\bar{A}}{2C} \left(\bar{C} - \frac{E}{Z} \right), \\
\dot{T} &= \frac{1}{2} \left[\bar{\bar{A}} - z^2 E_{xx} \right] + \frac{1}{4m} [(BF - H^2) \bar{A}^2 + (AF - G^2) \bar{D}^2 + (AB - D^2) \bar{G}^2] \\
& -\frac{1}{4m} [2(DF - GH) \bar{A}\bar{D} + 2(AH - DG) \bar{D}\bar{G} - (DH - BG) \bar{A}\bar{G}] - \frac{\bar{A}}{2C} \left(\bar{C} - \frac{E}{Z} \right), \\
\dot{U} &= \frac{1}{2} \left[\bar{\bar{A}} - z^2 E_{xx} \right] + \frac{1}{4m} [(BF - H^2) \bar{A}^2 + (AF - G^2) \bar{D}^2 + (AB - D^2) \bar{G}^2] \\
& -\frac{1}{4m} [2(DF - GH) \bar{A}\bar{D} + 2(AH - DG) \bar{D}\bar{G} - (DH - BG) \bar{A}\bar{G}] - \frac{\bar{A}}{2C} \left(\bar{C} - \frac{E}{Z} \right).
\end{aligned}$$

where $E_x = \frac{\partial E}{\partial x}$ and $E_y = \frac{\partial E}{\partial y}$ and the bar (-) over the letter denotes the ordinary derivative w.r.t. Z etc.

The electromagnetic wave solution of generalized Maxwell's equations (6) yields the following non-vanishing components of F_{ij} ,

$$(14) \quad zF_{14} = -tF_{15} = -\sigma, \quad zF_{24} = -tF_{25} = \rho, \quad zF_{34} = -tF_{35} = \eta,$$

where σ , ρ and η are arbitrary function of x, y, u and Z .

And the corresponding cauchy-Riemann equations are as follows,

$$(15) \quad \frac{\partial \rho}{\partial x} + \frac{\partial \sigma}{\partial y} = 0, \quad \frac{\partial \sigma}{\partial u} + \frac{\partial \eta}{\partial x} = 0, \quad \frac{\partial \eta}{\partial y} - \frac{\partial \rho}{\partial u} = 0.$$

The F_{ij} given by (13) and (14) represent the transverse electromagnetic waves propagating along the positive direction of z -axis and is null,

$$(16) \quad \Rightarrow F_{ij} F^{ij} = 0.$$

From equation (7) the five current vectors J^i has the following components,

$$J^1 = J^2 = J^3 = 0$$

and

$$(17) \quad ZJ^4 = J^5 = \frac{-1}{mCt} \left[2(AH - DG) \left(\frac{\partial \sigma}{\partial x} - \frac{\partial \sigma}{\partial x} \right) + (DH - BG) \left(\frac{\partial \sigma}{\partial x} + \frac{\partial \sigma}{\partial x} \right) \right] \\ + \frac{1}{mCt} \left[(BF - H^2) \frac{\partial \sigma}{\partial x} + (AF - G^2) \frac{\partial \sigma}{\partial x} + (AB - D^2) \frac{\partial \sigma}{\partial x} \right] \\ - \frac{2}{mCt} \left[(DF - GH) \left(\frac{\partial \sigma}{\partial x} + \frac{\partial \sigma}{\partial x} \right) \right].$$

The J^i given by (17) satisfies (7) and hence it is a null current vectors also J^i satisfies,

$$(18) \quad F_{ik}J^k = 0,$$

which shows that the Lorentz force vanishes.

From (4), the non-vanishing components of the electromagnetic tensor E_{ij} are as follows,

$$(19) \quad \frac{E_{44}}{Z^2} = \frac{-E_{45}}{Z} = E_{55} = \frac{-1}{mt^2} [(BF - H^2) \sigma^2 + (AF - G^2) \rho^2 + (AB - D^2) \eta^2] \\ - \frac{1}{mt^2} [-2(DF - GH) \sigma \rho - 2(AH - DG) \rho \eta - (DH - BG) \eta \sigma].$$

Now, as per Takeno's (1961), we take

$$(20) \quad V = -Z + k,$$

where $k = \text{constant}$. Define, $v_i = \frac{\partial V}{\partial x^i}$

$$(21) \quad v_i = \left(0, 0, 0, \frac{t}{z^2}, -\frac{1}{z} \right),$$

So that,

$$(22) \quad v^i = \left(0, 0, 0, -\frac{1}{Ct}, -\frac{1}{Cz} \right),$$

be the velocity of null fluid current. The velocity v^i satisfies (5) hence it is null.

From (4), the non-vanishing components of the energy tensor for the null fluid are as under,

$$(23) \quad \frac{L_{44}}{Z^2} = -\frac{L_{45}}{Z} = L_{55} = \frac{h}{z^2},$$

where h is the non-negative energy density of the null fluid, function of x, y, u, Z . Using (10), (11), (12), (17) and (19), the field equation (2) becomes,

$$(24) \quad \left[\frac{\bar{m}}{2m} - \frac{\bar{m}}{4m^2} - \frac{\bar{m}\bar{C}}{2m} - \frac{\sum}{2m} + \frac{E\bar{m}}{2mCZ} \right] \\ - \frac{z^2}{2m} [(BF - H^2) E_{xx} + (AF - G^2) E_{yy} + (AB - D^2) E_{uu}] \\ + \frac{z^2}{2m} [2(DF - GH) E_{xy} + 2(AH - DG) E_{xu} + 2(DH - BG) E_{yu}] \\ = \frac{8\pi}{mZ^2} [(BF - H^2) \sigma^2 + (AF - G^2) \rho^2 + (AB - D^2) \eta^2] \\ - \frac{8\pi}{mZ^2} [2(DF - GH) \rho\sigma + 2(AH - DG) \rho\eta + 2(DH - BG) \eta\sigma + hmZ^2],$$

$$(25) \quad \text{where, } \sum = 2D(\bar{G}\bar{H} - \bar{D}\bar{F}) + 2G(\bar{D}\bar{H} - \bar{B}\bar{G}) - 2H(\bar{D}\bar{G} - \bar{A}\bar{H}) \\ - A(\bar{B}\bar{F} - \bar{H}^2) + B(\bar{A}\bar{F} - \bar{G}^2) + F(\bar{A}\bar{B} - \bar{D}).$$

3. SOME PERTICULAR CASES

Case I: Field of null fluid.

Assuming that the electromagnetic field is not present *i.e.* $\sigma = \rho = \eta = 0$, the field equation (24) reduces to,

$$(26) \quad 8\pi h = \left[\frac{\bar{m}}{2m} - \frac{\bar{m}}{4m^2} - \frac{\bar{m}\bar{C}}{2m} - \frac{\sum}{2m} + \frac{E\bar{m}}{2mCZ} \right] \\ - \frac{z^2}{2m} [(BF - H^2) E_{xx} + (AF - G^2) E_{yy} + (AB - D^2) E_{uu}] \\ + \frac{z^2}{2m} [2(DF - GH) E_{xy} + 2(AH - DG) E_{xu} + 2(DH - BG) E_{yu}].$$

Thus the solution of the field equation (26) is the plane wave g_{ij} given by space-time (1). And the null fluid velocity given by equation (20), (21) and the energy density $h = h(x, y, u, Z)$, where $Z = \left(\frac{t}{z}\right)$.

Case II: Field of null electric current.

In this case, we take $h = 0$ *i.e.* null fluid is not present. Therefore the field equation (23) reduces to,

$$(27) \quad \left[\frac{\bar{m}}{2m} - \frac{\bar{m}}{4m^2} - \frac{\bar{m}\bar{C}}{2m} - \frac{\sum}{2m} + \frac{E\bar{m}}{2mCZ} \right] \\ - \frac{z^2}{2m} [(BF - H^2) E_{xx} + (AF - G^2) E_{yy} + (AB - D^2) E_{uu}]$$

$$\begin{aligned}
 & + \frac{z^2}{2m} [2(DF - GH) E_{xy} + 2(AH - DG) E_{xu} + 2(DH - BG) E_{yu}] \\
 & = \frac{8\pi}{mZ^2} [(BF - H^2) \sigma^2 + (AF - G^2) \rho^2 + (AB - D^2) \eta^2] \\
 & - \frac{8\pi}{mZ^2} [2(DF - GH) \rho\sigma + 2(AH - DG) \rho\eta - 2(DH - BG) \eta\sigma + hmZ^2].
 \end{aligned}$$

Hence in this case the solution of the field equation (27) consist of the metric tensor g_{ij} given by (1), F_{ij} given by (14), (15) and four current vector J^i given by (17).

Case III: Takeno's plane wave solution.

Using equation (11) and E is function of Z only, then the equation (3) implies,

$$(28) \quad E_x = E_y = E_u = 0, \quad E_{xx} = E_{xy} = E_{yy} = E_{uu} = E_{xu} = E_{yu} = 0.$$

Using (28) the field equation (24) becomes,

$$\begin{aligned}
 (29) \quad & \left[\frac{\bar{m}}{2m} - \frac{\bar{m}}{4m^2} - \frac{\bar{m}\bar{C}}{2m} - \frac{\sum}{2m} + \frac{E\bar{m}}{2mCZ} \right] \\
 & = \frac{8\pi}{mZ^2} [(BF - H^2) \sigma^2 + (AF - G^2) \rho^2 + (AB - D^2) \eta^2] \\
 & - \frac{8\pi}{mZ^2} [2(DF - GH) \rho\sigma + 2(AH - DG) \rho\eta - 2(DH - BG) \eta\sigma + hmZ^2].
 \end{aligned}$$

Here A, B, C, D, E, F, G and H are function of $Z = (\frac{t}{z})$ only. Our result reduces to, the solution of Einstein-Maxwell's equations obtained by Ambatkar et al.[1]. In this case the solutions exist only when σ, ρ, η and h are the function of Z alone, which reduces identically to zero. Further by choosing $u = 0$, then (1) reduces to, Takeno's [3] form of plane gravitational wave of the type $Z = (\frac{t}{z})$ i.e. g_{ij} 's are functions of Z only and the solutions exist only when σ, ρ, η and h are the functions of Z alone which reduces J^i identically to zero.

4. CONCLUSION

The g_{ij} 's given by (1) are satisfies Einstein-Maxwell field equations (23) in the presence of null field and null currents in five dimensional space-time for $Z = (\frac{t}{z})$ -type plane wave and we discussed as above some particular cases gives the field equation (26), (27) and (29).

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