

COSMOLOGICAL MODEL OF THE UNIVERSE WITH VISCOUS FLUID IN KALUZA-KLEIN THEORY OF GRAVITATION

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ABSTRACT. In this paper we have investigated Kaluza-Klein type 5-dimensional model of universe with a flat geometry filled with viscous fluid in the presence of variable cosmological term Λ . A non-singular solution leading to a variable deceleration parameter is obtained which reduces to the solution of Murphy (1973) in the absence of Λ and also reduces to the solutions of Vishwakarma (2002) in the absence of viscosity.

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1. INTRODUCTION

Over the last two decades, increasing attention has been paid to the theories that unify the fundamental interactions in more than three special dimensions. The story of this kind of theories started in the 20's, when Kaluza (1921) and Klein (1926) augmented the dimensionality of space to describe both gravity and electromagnetism as manifestation of geometry, using the degrees of freedom available from 5-dimension metric tensor (Lovey (1987)). The original idea of Kaluza

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and Klein turned out to be incomplete for several reasons, but it still pervades in one way or another of many unifying schemes currently thought to be viable. A great number of exact cosmological solutions of Einstein field equations with different equation of state and different symmetries, including or not a cosmological constant, has been found with 5D (Demarat and Hanquin, (1985), Davidson and Vozmediano (1985a), (1985b)) and also with arbitrary number of dimension (Lorenz-Petzold (1984), (1985), (1986a), (1986b)). Sahdev (1984), Emelyanov et al. (1986) and Chatterjee and Bhui (1993), have studied physics of the universe in higher-dimensional space-time.

The status of the cosmological constant Λ term has long been discussed (Souriau (1977), Felten (1986), Charlton (1987), Sandage (1988)), whereas it is clearly established in General Relativity (GR) as universal constant (Souriau (1964)). Therefore, similar to Newton constant of Gravitation G , its value has to be estimated from observations. However, such an estimate does not agree by hundred orders of magnitude with its expected value as obtained from quantum field theories (Carroll (2001), Straumann (2002), Padmanabhan (2003)) by assuming that vacuum energy density of quantum fluctuations is the origin of this constant. The aim of the present analysis is to analyse on geometrical grounds this problem, called cosmological constant problems (CCP).

A decaying Λ model on the other hand does not allow an solution of the field equations in the absence of matter and hence is consistent with Mach's idea (Mach, (1983)) that the presence of matter is essential for a meaningful space time geometry. Empty space time cannot be obtained as a solution of Einstein's field equations with a dynamical Λ (Vishwakarma, (2002)). Since the presence of a positive cosmological Λ term introduces repulsive force, it might be a possible source for the observed acceleration of the present Universe. Although with the present quality of supernovae Ia data, the allowed parameter space is wide enough to accommodate decelerating models as well (Vishwakarma (2003)).

The majority of the studies in cosmology involve a perfect fluid. However, observed physical phenomena such as the large entropy per baryon and the remarkable degree of isotropy of the cosmic microwave background radiation suggests analysis of dissipative effects in cosmology. Furthermore, there are several processes which are expected to give rise to viscous effects. These are the decoupling of neutrinos

during the radiation era and the decoupling of radiation and matter during recombination era. Bulk viscosity is associated with the GUT phase transition and string creation. The model studied by Murphy (1973) possessed an interesting feature in that the big bang type of singularity of infinite space time curvature does not occur to be a finite past. However, the relationship assumed by Murphy (1973) between the viscosity coefficient and the matter density is not acceptable at large density. The effect of bulk viscosity on the cosmological evolution has been investigated by a number of authors in the framework of general theory of relativity (Maartens (1995), Zimdahl (1996), Murphy (1973), Santos (1985), Weinberg (1971), Waga (1986), Padmanabhan (1987), Johri (1988), Mohanty (1990), Beesham (1994), Pradhan (1997), Kalyani (1997), Reddy (2000), Chakrabarty (2001)).

The present work is the generalization of the work obtained earlier by Dutta Choudhury and Sil (2006). We have obtained the non-singular model of the universe which reduces to the solutions obtained by Murphy (1973) in the absence of cosmological term Λ and also reduces to those obtained by Vishwakarma (2002) in the absence of Viscosity. We have also obtained the dust and radiation model for $\gamma = 1$ and $\gamma = \frac{3}{4}$.

2. MODEL AND FIELD EQUATIONS

Let us consider the Kaluza-Klein type metric of the form

$$(1) \quad ds^2 = dt^2 - R^2(t) \left[\frac{dr^2}{(1 - kr^2)} + r^2(d\theta^2 + \sin^2\theta d\phi^2) + (1 - kr^2)d\psi^2 \right],$$

where $R(t)$ is the scale factor and $k = 0, -1$, or $+1$ is the curvature parameter for flat, open and closed universe, respectively. The universe is assumed to be filled with distribution of matter represented by energy-momentum tensor for viscous fluid given by

$$(2) \quad T_{\mu\nu} = (\bar{p} + \rho)u_\mu u_\nu - \bar{p}g_{\mu\nu},$$

The effective pressure is $\bar{p} = p - \xi\nu_{;k}^k$, where p is the fluid pressure, ξ viscosity coefficient and ρ the energy density of the fluid, u_μ is the five-velocity vector such that $u_\mu u^\mu = 1$.

The Einstein field equations with time-dependent cosmological constant is given by

$$(3) \quad R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -T_{\mu\nu} + \Lambda(t)g_{\mu\nu} ,$$

where $R_{\mu\nu}$ is the Ricci tensor and $\Lambda(t)$ being the variable cosmological constants. The divergence of Eq. (2), taking into account the Bianchi identity, gives

$$(4) \quad (T_{\mu\nu} + \Lambda g_{\mu\nu})^{;\nu} = 0.$$

Using comoving coordinates

$$(5) \quad u_\mu = (1, 0, 0, 0, 0),$$

in Eq.(2) and with line element (1), Einstein's field Eq.(3) yields

$$(6) \quad \rho = \frac{6\dot{R}^2}{R^2} - \Lambda(t),$$

$$(7) \quad \bar{p} = -\frac{3\ddot{R}}{R} - \frac{3\dot{R}^2}{R^2} + \Lambda(t) ,$$

since Eq.(6) and (7) involves five unknowns viz. ρ , p , ξ , Λ and R we have assumed three relations among the unknown in order to solve the equations. We assume

$$(8) \quad p = (\gamma - 1) \rho ,$$

$$(9) \quad \xi = \alpha \rho ,$$

$$(10) \quad \Lambda = n H^2 ,$$

where $H \equiv \frac{\dot{R}}{R}$ is the Hubble parameter, α and n are positive.

Using Eq. (8) and (10), Eq. (7) can be written as

$$(11) \quad \frac{3\ddot{R}}{R} + \frac{3\dot{R}^2}{R^2} = -\rho \left[(\gamma - 1) - 3\alpha \frac{\dot{R}}{R} \right] + n \frac{\dot{R}^2}{R^2} .$$

Now using Eq. (10) and (11), Eq. (6) takes the form

$$(12) \quad (6 - n) \frac{\dot{R}^2}{R^2} = \rho + \Lambda .$$

Now for matter energy density ρ to be positive, $n \leq 6$. Eliminating ρ from Eq. (11) and (12) we obtain the following equations

$$(13) \quad R^2 \ddot{R} + aR\dot{R}^2 - b\dot{R}^3 = 0 ,$$

where,

$$(14) \quad a = \frac{\gamma}{3}(6-n) - 1 ,$$

$$(15) \quad b = \alpha(6-n) .$$

In view of $\alpha \geq 0$ and $n \leq 6$, it is apparent that $b \geq 0$ and also $(a+1) \geq 0$. The general solution of Eq. (13) is

$$(16) \quad A \frac{R^{a+1}}{a+1} + bR = a(t-t_0) ,$$

where A and t_0 are arbitrary constants of integration. However, the constant t_0 can be absorbed by suitable scale transformation of the time-coordinate so that the Eq. (16) can be written as

$$(17) \quad A \frac{R^{a+1}}{a+1} + bR = at .$$

The physical and dynamical variables in terms of scale factor R can be written as

$$(18) \quad \rho = (6-n)a^2[AR^{a+1} + bR]^{-2} ,$$

$$(19) \quad p = (\gamma-1)\rho, \quad \xi = \alpha\rho ,$$

$$(20) \quad \Lambda = na^2[AR^{a+1} + bR]^{-2} .$$

Here, we shall restrict A as $A > 0$ because as $A < 0$, $A = -|A|$ and ρ will become infinity at $R = (\frac{b}{A})^{\frac{1}{a}}$ which is unreal. Study of asymptotic behaviour of Eq.(17) reveals that as $t \rightarrow \infty$, $R \rightarrow \infty$. Thus for $A \geq 0$, the model has no singularity at a finite time in the past or future. But this non-singular behaviour is only possible in presence of viscosity i.e. $\alpha \neq 0$. In absence of viscosity $\alpha = 0$, and hence the model becomes singular and goes to that discussed by Vishwakarma (2002) such that

$$(21) \quad R \sim t^{\frac{1}{(1+a)}}, \rho \sim t^{-2}, \Lambda \sim t^{-2} ,$$

which can be verified easily from Eq. (17)-(20). Also when $n = 0$, $\Lambda = 0$ and the model goes over to that of Murphy (1973). The Hubble parameter $H \equiv \frac{\dot{R}}{R}$ and the deceleration parameter $q \equiv -\frac{\ddot{R}}{RH^2}$ in our model have been found as

$$(22) \quad H = a[AR^{a+1} + bR]^{-1} ,$$

and

$$(23) \quad q = \frac{AaR^a}{(b + R^a A)} .$$

Now $q = a$ if $b = 0$ i.e. in absence of viscosity ξ , else $q >= < 0$ if $a >= < 0$.

2.1. Case I. $A > 0, a > 0$

Now from Eq. (22), it follows that $q >= < 0$ according as $R >= < 0$. The constant parameter $n = \frac{\Lambda}{H^2}$ has an upper bound as $n < (6 - \frac{3}{\gamma})$.

2.1.1. Special case: $\gamma = \frac{4}{3}$

For the above value of γ i.e. in radiation model, the value of ' a ' is found as $a = \frac{1}{9}(15 - 4n)$ for $n \leq \frac{15}{4}$. The values of all physical parameters can be found from Eq. (18), (19), (22) and (23) for above values of a .

2.2. Case II. $A > 0, a < 0$

The deceleration parameter in this case derived from Eq.(23) is given by

$$(24) \quad q = - \left[\frac{A|a|R^{-|a|}}{b + R^{-|a|}A} \right] ,$$

where $a = -|a|$. In this case the expansion is accelerated throughout the evolution. However the acceleration converges to $q = -|a|$ at the later stage. The parameter n is bound within limits $6 - \frac{3}{\gamma} < n \leq 6$.

2.2.1. Special Case: $\gamma = 1$ (Dust case i.e. $p = 0$)

For this value of γ the value of physical parameters can be found from Eq. (18),(19),(22) and (23) by putting $a = \frac{(3-n)}{3}$ when $3 < n \leq 6$.

2.3. Case III. $A = 0$

The scale factor R can now be written as

$$(25) \quad R = \left(\frac{a}{b}\right) t.$$

The physical parameters are found as follows.

$$(26) \quad H = \frac{1}{t} = \sqrt{\frac{\Lambda}{n}},$$

$$(27) \quad \Lambda = \frac{n}{t^2},$$

$$(28) \quad \rho = \frac{(6-n)}{t^2},$$

$$(29) \quad q = 0.$$

3. Conclusion

We have obtained non-singular model of the Universe by assuming $\Lambda \propto H^2$ in the context of Kaluza-Klein theory of gravitation. We have found out the solution of the field equations by assuming Eq. (9) and (10). Our solutions reduces to the solution obtained earlier by Murphy (1973) for the case $\Lambda = 0$ and also reduces to that of Vishwakarma (2002) in the no-viscosity limit. Moreover, the deceleration parameter is function of the scale factor and changes its sign with evolution. Our solution is generalization of the solution obtained by Murphy (1973) for $\Lambda = 0$ and also that of Vishwakarma (2002) in absence of viscosity in 4-dimension space time. We have also obtained the dust and radiation model for $\gamma = 1$ and $\gamma = \frac{4}{3}$. The introduction of the viscosity not only frees the model from singularity but also gives deceleration parameter which is function of scale factor. For the case $A = 0$ the scale factor is increasing function of time and density and Λ tends to zero as $t \rightarrow \infty$. Similarly in this case we get the age of the universe i.e. $t \propto \frac{1}{H}$ and the deceleration parameter $q = 0$.

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