HIGHER DIMENSIONAL STRING COSMOLOGICAL MODEL WITH DYNAMICAL COSMOLOGICAL TERM Λ

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ABSTRACT. We have analysed Kaluza Klein type five dimensional cosmological model by considering three different forms of variable Λ: $\Lambda \sim \left(\frac{\dot{R}}{R}\right)^2$, $\Lambda \sim \left(\frac{\ddot{R}}{R}\right)$ and $\Lambda \sim \rho$. It is found that, the connecting free parameters of the models with cosmic matter and vacuum energy density parameters are equivalent, in the context of higher dimensional space time.

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1. INTRODUCTION

Strings plays a significant role in the study of early universe. It is believed that cosmic strings give rise to density perturbations which leads to the formation of galaxies Zeldovich (1980), these strings posses stress enerand are coupled to the gravitational field. The detailed accounts of various features of cosmic strings are discussed by Vilenkin (1981), Gott(1985), and Garfinkle (1985). The general relativistic formalism of cosmic strings are given by Letelier (1979) and Stachel (1980). The relativistic strings can also be used i) to describe the large scale anisotrpy [Kaiser et. al. (1986), Traschen et. al. (1986), Brandenberger et. al. (1986)] of the universe, ii) to explain the observed two point correlation function for cluster of galaxies Turok (1985) (Abel clusters), and iii)to describe extended objects in general relativity Letelier (1979), Tian (1986), Garfinkle (1985), Garriga et. al. (1987). The

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Kaluza-Klein theories presents an attractive way to unify all guage interaction with gravity Duff et. al. (1986). One of the problem facing these theories if to explain why the extra-dimensional space is do small, since otherwise it would be obdserved. Some aythors explain entproduction Alvarez et. al. (1983), Barr et. al. (1984) and inflation Abbott et. al. (1984), Shafi et. al. (1983) as a result contraction of the extra space.Mann and Vincent (1985) have investigated the cosmological character of 5 dimensional (KK type) solution of Einstein's equation nd shown that the vaccum solution of these equations gives rise to an effective radiation density connected with the existence of extradimension. Now in the present work the five dimensional model, which if just ordinary Kaluza-Klein theory aimed to unify electromagnetism and gravity, the strings lie along the fifth dimension; as no more extra dimensions are included, this should be taken as a toy model. Thus the resulting space is a four dimensional spacetime and an extra compatified dimension. The model of the cloud ordered strings i. e. a ,"cloud" formed by a distribution of geometric strings lying along the same direction, has already been used in cosmology. The fact that all strings lie along fifth dimension might be produced by the same process that brought about the compactification of that dimension.

The cosmological constant problem can be expressed as the discrepancies between negligible values Λ has for the present universe and the values 10^{50} times larger expected by the Glashow Salam Weinberg model or by Grand Unified Theory (GUT) where it should be 10^{107} times larger. The cosmological term Λ is then small at the present epoch simply because the universe is too old. The problem in this approach is to determine the right depending of Λ on R and t . A current of thought holds the view that the cosmological term is not really constant, but its value decreases as the universe expands. Some of the discussions on cosmological problem and on cosmology with time varying cosmological constant, Ratra and Peebles (1988) , Dolgov (1983, 1990, 1997) and Sahni and Staro Binsky (2000) pointedout that in absence of any interaction with a matter or radiation the cosmological constant remains constant. However in presence of interaction with a matter or radiation a solution of Eienstein's equations and the assume equations of covariant conservation of trace energy with a time varying Λ can be found. [Rajeev (1983), Maia et. al. (1994), Moffat (1996), Hoyle et. al. (1997), John et. al (1997)] have derived analytical decay laws in other theories of gravitation from modified version of Eienstein's action. In most such papers, however no exact solution for Λ is obtained; the intend is nearly to demonstrate that the decay of the effective coal term is possible in the principle. A number of authors have constructed a model of more phenomenological character in which specific decay laws are postulated for Λ within general relativity. This law provides reasonable solutions to the cosmological puzzles presently known. One of the motivations for introducing Λ term is to reconcile the age parameter and the density parameter of the universe with recent observational data.

Vishwakarma(2001) has studied the magnitude-redshift relation for the type Ia supernovae data and the angular size-redshift relation for the updated compact radio sources data Gurvits (1999) by considering four variable Λ-models:$\Lambda \sim R^{-2}$, $\Lambda \sim H^{-2}$, $\Lambda \sim \rho$ and $\Lambda \sim t^{-2}$. Recently Ray and Mukhopadhyay (2004) have solved Einstein's equations for specific dynamical models of the cosmological term Λ in the form: $\Lambda \sim \left(\frac{\dot{R}}{R}\right)^2$, $\Lambda \sim \left(\frac{\ddot{R}}{R}\right)$ and $\Lambda \sim \rho$ and shown that the models are equivalent in the framework of flat RW space time. In this context, the aim of the present work is based on recent available observational information. In this paper the implication of cosmological model with cosmological term of three different forms: $\Lambda = \alpha \left(\frac{\dot{R}}{R}\right)^2$, $\Lambda = \beta \left(\frac{\ddot{R}}{R}\right)$ and $\Lambda = \gamma\rho$, where α, β and γ are free parameters, are analyzed within the framework of higher dimensional space time. The paper is organized as follows. The Einstein's field equations and their general solutions for different Λ-dependent models are presented in Sections 2 and 3 respectively. The equivalence of different forms of dynamical cosmological term Λ are discussed in Section 4.

2. Field equations and solutions

We consider Kaluza-Klein type the line element

$$ds^2 = -dt^2 + R^2(t)\left(dx^2 + dy^2 + dz^2\right) + A^2(t)\,d\psi^2 \tag{1}$$

where R and A are the functions of t alone.
The energy momentum tensor for bulk viscous string dust is taken as

$$T_i^j = \rho\nu_i\nu^j - \lambda x_i x^j \tag{2}$$

with

$$\nu^i\nu_i = -x^i x_i = -1 \tag{3}$$

and

$$\nu^i x_i = 0 \tag{4}$$

where ρ is rest energy density of the cloud of strings with particles attached to them, $\rho = \rho_p + \lambda$ with ρ_p being the rest energy density of particles, λ the tension density of the cloud of strings. The vector ν^i describes the cloud four velocity and x^i represents a direction of anisotropy, i. e. direction of strings.
The Einstein's field equations

$$G_i^j = -T_i^j + \Lambda(t) \tag{5}$$

where G_i^j is the Einstein's tensor and we choose the unit such as $c = 1$, $8\pi G = 1$ and $\Lambda(t)$ is the variable cosmological term. For the metric (1), equation (5) leads to

$$\frac{3\dot{R}^2}{R^2} + \frac{3\dot{R}\dot{A}}{RA} = \rho + \Lambda(t) \tag{6}$$

$$\frac{3\ddot{R}}{R} + \frac{3\dot{R}^2}{R^2} = \lambda + \Lambda(t) \tag{7}$$

$$\frac{2\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{2\dot{R}\dot{A}}{RA} + \frac{\ddot{A}}{A} = \Lambda(t) \tag{8}$$

where the dot denotes derivative with respect to time t.
Similarly, the expansion factor θ amd the shear scalar σ are given by

$$\theta = 3\frac{\dot{R}}{R} + \frac{\dot{A}}{A} \tag{9}$$

$$\sigma^2 = \frac{3}{8}\left(\frac{\dot{R}}{R} - \frac{\dot{A}}{A}\right)^2 \tag{10}$$

Equations (6) to (8) are three equations of five unknowns $R, A, \rho, \lambda, \Lambda$. To get a determinate solution, we assume a supplementary condition

$$A = KR^n \tag{11}$$

where K, n are real constants.
Then the equations (6)-(8) together with equation (11) leads to

$$3(1+n)\frac{\dot{R}^2}{R^2} = \rho(t) + \Lambda(t) \tag{12}$$

$$3\frac{\ddot{R}}{R} + 3\frac{\dot{R}^2}{R^2} = \lambda(t) + \Lambda(t) \tag{13}$$

$$(n+2)\frac{\ddot{R}}{R} + (n^2+n+1)\frac{\dot{R}^2}{R^2} = \Lambda(t) \tag{14}$$

The above equations are to be solved for $\rho(t), \lambda(t), R(t)$ and $\Lambda(t)$. The solution of the system (12)-(14) can be obtained for different cases and they are solved only when we impose the condition on one of the variables which can be seen in the following cases. In each case the solution is completely determined subject to the conditions specified at $t = t_0$ as $R(t_0) = R_0$ and $\dot{R}(t_0) = \dot{R}_0$. Now we make the dependent variables as dimensionless as follows $x = \dot{R}_0(t - t_0)/R_0$, $\bar{R}(x) = R(t)/R_0$, $\bar{\rho}(x) = \left(\frac{R_0}{\dot{R}_0}\right)^2 \rho(t)$, $\bar{\lambda}(x) = \left(\frac{R_0}{\dot{R}_0}\right)^2 \lambda(t)$ and $\bar{\Lambda}(x) = \left(\frac{R_0}{\dot{R}_0}\right)^2 \Lambda(t)$. Thus $\frac{d\bar{R}}{dx} = \bar{R}'(x) = \frac{\dot{R}(t)}{\dot{R}_0}$ and $\frac{d^2\bar{R}}{dx^2} = \bar{R}''(x) = \frac{\ddot{R}(t)R_0}{(\dot{R}_0)^2}$. So that the initial conditions at $t = t_0$ corresponds to at $x = 0$ reduce to $\bar{R}(0) = 1$ and $\left.\frac{d\bar{R}}{dx}\right|_{x=0} = \bar{R}'(0) = 1$. Hence, without loss of generality, by suppressing the overline symbol, the above equations (12) to (14) reduce to

$$3(1+n)\frac{R'(x)^2}{R^2} = \rho(x) + \Lambda(x), \tag{15}$$

$$3 \ + 3\frac{R'(x)^2}{R^2} = \lambda(x) + \Lambda(x), \tag{16}$$

$$(2+n)\frac{R''(x)}{R} + (n^2+n+1)\frac{\dot{R}(x)^2}{R^2} = \Lambda(x), \tag{17}$$

subject to initial conditions $R(0) = 1$, $R'(0) = 1$.

3. Different Forms of Λ

3.1. **Model for Λ=constant.** Solving equation (17) for $R(x)$ subject to the given initial conditions, we get

$$R = \left(c_1 e^{\alpha x} + c_2 e^{-\alpha x}\right)^{\frac{n+2}{n^2+2n+3}}, \quad \Lambda > 0$$

$$= \left(c_1 \cos \alpha x + c_2 \sin \alpha x\right)^{\frac{n+2}{n^2+2n+3}}, \quad \Lambda < 0 \tag{18}$$

where $\alpha^2 = \frac{|\Lambda|(n^2+2n+3)}{(n+2)^2}$, $c_1 = \frac{(n^2+3n+5)}{2(n+2)}$ and $c_2 = \frac{-(n^2+n>+1)}{2(n+2)}$.

The energy density ρ and string tension density λ, from equation (15) and (16) are given by

$$\rho = \frac{3(1+n)\Lambda}{n^2+2n+3}\left(\frac{c_1 e^{\alpha x} - c_2 e^{-\alpha x}}{c_1 e^{\alpha x} + c_2 e^{-\alpha x}}\right)^2 - \Lambda, \quad \Lambda > 0$$

$$= \frac{3(1+n)\Lambda}{n^2+2n+3}\left(\frac{c_2 \cos \alpha x - c_1 \sin \alpha x}{c_2 \sin \alpha x + c_1 \cos \alpha x}\right)^2 - \Lambda, \quad \Lambda < 0 \tag{19}$$

$$\lambda = \frac{3(1-n^2)\Lambda}{(n+2)(n^2+2n+3)}\left(\frac{c_1 e^{\alpha x} - c_2 e^{-\alpha x}}{c_1 e^{\alpha x} + c_2 e^{-\alpha x}}\right)^2 - \frac{n-1}{n+2}\Lambda, \quad \Lambda > 0$$

$$= \frac{3(1-n^2)\Lambda}{(n+2)(n^2+2n+3)}\left(\frac{c_2 \cos \alpha x - c_1 \sin \alpha x}{c_2 \sin \alpha x + c_1 \cos \alpha x}\right)^2 - \frac{n-1}{n+2}\Lambda, \quad \Lambda < 0 \tag{20}$$

The expansion θ and the shear scalar σ, from equation (9) and (10)are given by

$$\theta = \frac{(n+3)(n+2)\alpha}{n^2+2n+3}\left(\frac{c_1 e^{\alpha x} - c_2 e^{-\alpha x}}{c_1 e^{\alpha x} + c_2 e^{-\alpha x}}\right), \quad \Lambda > 0$$

$$= \frac{(n+3)(n+2)\alpha}{n^2+2n+3}\left(\frac{c_2 \cos \alpha x - c_1 \sin \alpha x}{c_2 \sin \alpha x + c_1 \cos \alpha x}\right), \quad \Lambda < 0 \tag{21}$$

$$\sigma^2 = \frac{3(1-n)^2(n+2)^2\alpha^2}{(n^2+2n+3)^2}\left(\frac{c_1 e^{\alpha x} - c_2 e^{-\alpha x}}{c_1 e^{\alpha x} + c_2 e^{-\alpha x}}\right)^2, \quad \Lambda > 0$$

$$= \frac{3(1-n)^2(n+2)^2\alpha^2}{(n^2+2n+3)^2}\left(\frac{c_2 \cos \alpha x - c_1 \sin \alpha x}{c_2 \sin \alpha x + c_1 \cos \alpha x}\right)^2, \quad \Lambda < 0 \tag{22}$$

3.2. **Model for $\Lambda \sim \frac{R''}{R}$.** If we consider $\Lambda = \beta\frac{\ddot{R}}{R}$, where β is a constant. Then equation (17) subject to the given initial conditions, leads to

$$R = (X+1)^{\frac{1}{X+1}}\left[c_3 x + c_4\right]^{\frac{1}{X+1}} \tag{23}$$

where $X = \frac{n^2+n+1}{2+n-\beta}$, $c_3 = 1$ and $c_4 = (X+1)^{-1}$.

The energy density ρ and string tension density λ, from equation (15) and (16) are given by

$$\rho = \frac{3+3n+\beta X}{(X+1)^2}\left[\frac{1}{(x+c_4)^2}\right] \tag{24}$$

$$\lambda = \frac{\beta X - 3X + 3}{(X+1)^2}\left[\frac{1}{(x+c_4)^2}\right] \tag{25}$$

The expansion θ and the shear scalar σ, from equation (9) and (10)are given by

$$\theta = \frac{n+3}{X+1}\left[\frac{1}{(x+c_4)}\right] \tag{26}$$

$$\sigma^2 = \frac{3(1-n)^2}{8(X+1)^2}\left[\frac{1}{(x+c_4)^2}\right] \tag{27}$$

The deceleration parameter $q$is given by

$$q = \frac{-\ddot{R}R}{\dot{R}^2} = \frac{n^2+n+1}{\beta-(n+2)}, \beta \neq n+2 \tag{28}$$

$$\Lambda(x) = \frac{-\beta X}{(X+1)^2}\left[\frac{1}{(x+c_4)^2}\right] \tag{29}$$

3.3. **Model for $\Lambda \sim \left(\frac{R'}{R}\right)^2$.** If we use $\Lambda = \alpha\left(\frac{\dot{R}}{R}\right)^2 = \alpha H^2$, where α is a constant and H is hubble parameter $H = \frac{\dot{R}}{R}$. Then equation (17) subject to the given initial conditions, leads to

$$R = (m+1)^{\frac{1}{m+1}}[c_5 x + c_6]^{\frac{1}{m+1}} \tag{30}$$

where $m = \frac{n^2+n+1-\alpha}{n+2}$, $c_5 = 1$ and $c_6 = (m+1)^{-1}$.

The energy density ρ and string tension density λ, from equation (15) and (16) are given by

$$\rho = \frac{3(1+n)\alpha}{(m+1)^2}\left[\frac{1}{(x+c_6)^2}\right] \tag{31}$$

$$\lambda = \frac{3-\alpha-3m}{(m+1)^2}\left[\frac{1}{(x+c_6)^2}\right] \tag{32}$$

The expansion θ and the shear scalar σ, from equation (9) and (10)are given by

$$\theta = \frac{n+3}{m+1}\left[\frac{1}{(x+c_6)}\right] \tag{33}$$

$$\sigma^2 = \frac{3(1-n)^2}{8(m+1)^2}\left[\frac{1}{(x+c_6)^2}\right] \tag{34}$$

The deceleration parameter $q$is given by

$$q = \frac{-\ddot{R}R}{\dot{R}^2} = \frac{n^2+n+1-\alpha}{n+2}, \tag{35}$$

$$\Lambda(x) = \frac{\alpha}{(m+1)^2}\left[\frac{1}{(x+c_6)^2}\right] \tag{36}$$

3.4. **Model for $\Lambda \sim \rho$.** If we set $\Lambda = \gamma\rho$, where γ is a free parameter. Then equation (14) subject to the given initial conditions, leads to

$$R = (Y+1)^{\frac{1}{Y+1}}[c_7 x + c_8]^{\frac{1}{Y+1}} \tag{37}$$

where $Y = \frac{(n^2+n+1)(1+\gamma)-3(1+n)\gamma}{(1+\gamma)(n+2)}$, $c_7 = 1$ and $c_8 = (Y+1)^{-1}$.

The energy density ρ and string tension density λ, from equation (15) and (16) are given by

$$\rho = \frac{3(1+n)}{(1+\gamma)(Y+1)^2}\left[\frac{1}{(x+c_8)^2}\right] \tag{38}$$

$$\lambda = \frac{3\left[1 - Y - \frac{1+n}{1+\gamma}\right]}{(Y+1)^2}\left[\frac{1}{(x+c_8)^2}\right] \tag{39}$$

The expansion θ and the shear scalar σ, from equation (9) and (10)are given by

$$\theta = \frac{n+3}{Y+1}\left[\frac{1}{(x+c_8)}\right] \tag{40}$$

$$\sigma^2 = \frac{3(1-n)^2}{8(Y+1)^2}\left[\frac{1}{(x+c_8)^2}\right] \tag{41}$$

The deceleration parameter $q$is given by

$$q = \frac{(n^2+n+1)(1+\gamma)-3(1+n)\gamma}{(1+\gamma)(n+2)} \tag{42}$$

$$\Lambda(x) = \frac{3(1+n)\gamma}{(1+\gamma)(Y+1)^2}\left[\frac{1}{(x+c_8)^2}\right] \tag{43}$$

4. Equivalence of Three Forms of Λ

Now, let us find out the interrelations between α, β and γ and hence the equivalence of the different forms of the dynamical cosmological terms $\Lambda \sim \left(\frac{R''}{R}\right)$, $\Lambda \sim \left(\frac{R'}{R}\right)^2$, and $\Lambda \sim \rho$.

¿From equation (19), after differentiating it and the dividing by R, we get

$$c_3 x + c_4 = \frac{c_3}{(X+1)H}, \tag{44}$$

where H is the Hubble parameter.

Using equation (41) in equation (21) and also from the definition of the cosmic matter-density parameter $\Omega(=\rho/3(1+n)H^2)$, we get

$$\Omega_{m\beta} = 1 + \frac{\beta X}{3(1+n)}, \tag{45}$$

where $\Omega_{m\beta}$ is the cosmic matter energy density parameter for the β-related dynamic Λ-model.

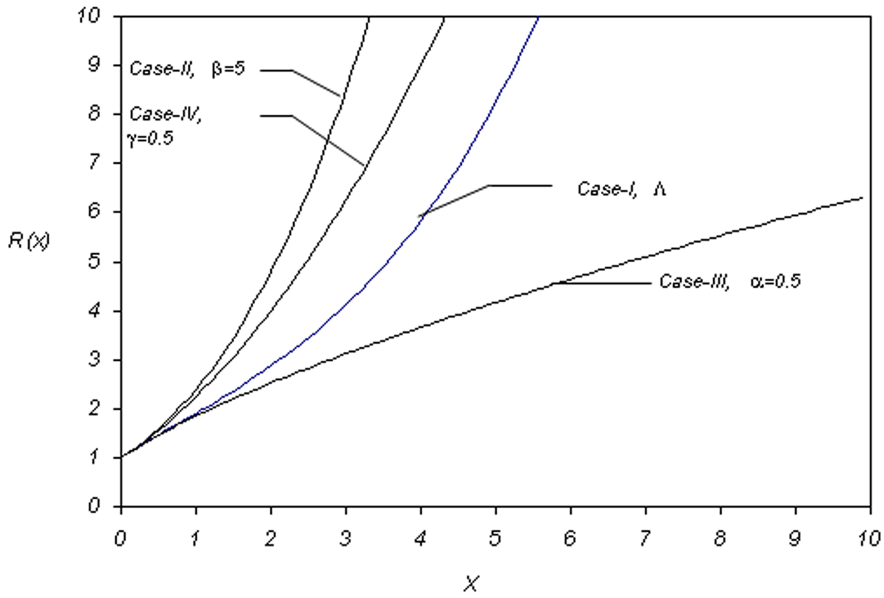

Fig.1: The behaviour of R(X) for n=0.5 in different cases.

Using equation (43) in equation (28) and also from the definition of the cosmic vacuum-energy density parameter $\Omega_\Lambda = \Lambda/3(1+n)H^2$, we have

$$\Omega_{\Lambda\beta} = -\frac{\beta X}{3(1+n)}, \tag{46}$$

where, in the similar fashion, $\Omega_{\Lambda\beta}$ is the cosmic vacuum-energy density parameter for the β-related dynamic Λ-model.

¿From equation (45) and equation (46), we obtain

$$\Omega_{m\beta} + \Omega_{\Lambda\beta} = 1, \tag{47}$$

which is the relation between the cosmic matter- and vacuum-energy density parameters. Similarly from equation (30), equation (31) and equation (36) we obtain

$$\Omega_{m\alpha} = 1 - \frac{\alpha}{3(1+n)}, \tag{48}$$

$$\Omega_{\Lambda\alpha} = \frac{\alpha}{3(1+n)} \tag{49}$$

where $\Omega_{m\alpha}$ and $\Omega_{\Lambda\alpha}$ are respectively the cosmic matter and vacuum energy density parameters for the α-related dynamic Λ-model.

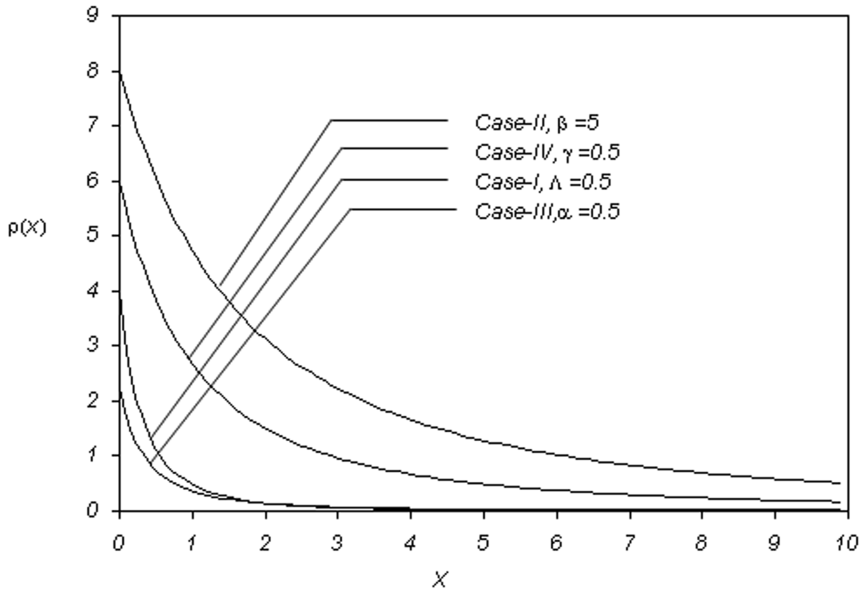

Fig.2: The behavipour of density for n=0.5 in different cases

Adding equation (47) and equation (48), we get

$$\Omega_{m\alpha} + \Omega_{\Lambda\alpha} = 1, \tag{50}$$

which is again the relation between the cosmic matter- and vacuum-energy density parameters for a flat ($k = 0$) universe.

In the same manner from equation (37), equation (38) and equation (43) the cosmic matter and vacuum energy density parameter $\Omega_{m\gamma}$ and $\Omega_{\Lambda\gamma}$ respectively for the model $\Lambda \sim \rho$ also satisfy the relation

$$\Omega_{m\gamma} + \Omega_{\Lambda\gamma} = 1, \tag{51}$$

where

$$\gamma = \frac{\Omega_{\Lambda\gamma}}{\Omega_{m\gamma}}. \tag{52}$$

Thus, from the relations (47), (50) and (51) without any loss of generality we can set

$$\Omega_{m\alpha} = \Omega_{m\beta} = \Omega_{m\gamma} = \Omega_m, \tag{53}$$

$$\Omega_{\Lambda\alpha} = \Omega_{\Lambda\beta} = \Omega_{\Lambda\gamma} = \Omega_\Lambda, \tag{54}$$

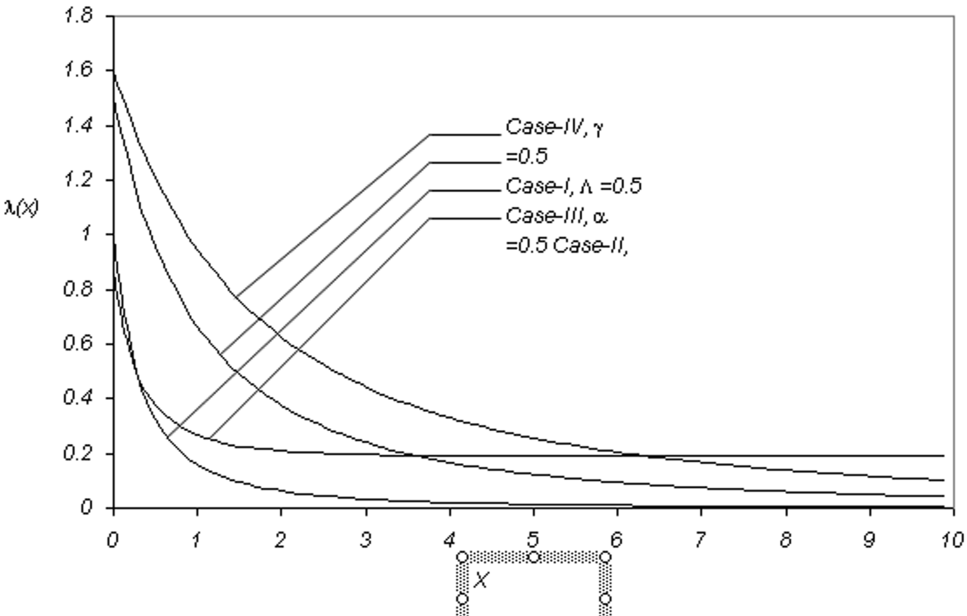

Fig.3: The behaviour of λ(x) for =0.5 in different cases

where Ω_m and Ω_Λ are respectively the cosmic matter and vacuum energy density parameters which in absence of any curvature satisfy the general relation

$$\Omega = \Omega_m + \Omega_\Lambda = 1. \tag{55}$$

Now from equation (49) and equation (55) we obtain

$$\alpha = 3(1+n)\Omega_\Lambda. \tag{56}$$

Also dividing equation (45) by equation (46) and using equation (53) and equation (54) β is obtained as

$$\beta = \frac{n+2}{n^2+n+2} - \left(1 - \frac{1}{n^2+n+2}\right)\frac{3(1+n)\Omega_\Lambda}{\Omega_\Lambda + \Omega_m} \tag{57}$$

Similarly from equation (52), equation (53) and equation (54)

$$\gamma = \frac{\Omega_\Lambda}{\Omega_m}. \tag{58}$$

Hence from equation (55) we get

$$\Omega = (1+\gamma)\Omega_m = 1, \tag{59}$$

which is an other relation of total cosmic energy density in the case of flat universe. It can easily be shown that the particular solution of $\Lambda \sim \left(\frac{\dot{R}}{R}\right)^2$ model for dust

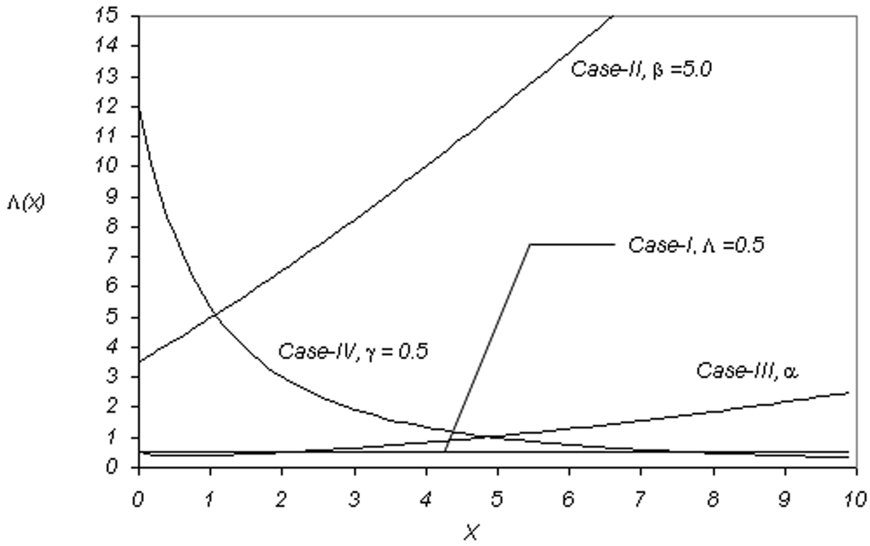

Fig.4;The behaviour of Λ(x) for for n=0.5 in different cases.

and radiation cases become identical with their corresponding counterparts for the other models in term of the dependency of R, ρ and Λ on time t when expressed in terms of Ω_m and Ω_Λ. Therefore, these results imply that in view of Ω_m and Ω_Λ there are no distinctive features so that one can differentiate between the different forms of dynamic cosmological model viz.$\Lambda \sim \left(\frac{\dot{R}}{R}\right)^2$, $\Lambda \sim \left(\frac{\ddot{R}}{R}\right)$ and $\Lambda \sim \rho$. Thus, starting from any one of our Λ-models, since they are equivalent, we can arrive at the other relations.

Now from equation (56), after using equation (46) and equation (54), one can obtain

$$\alpha = \frac{n^2+n+1}{\beta-(n+2)}\beta. \tag{60}$$

Also from equation (56), after using equation (58) and equation (59), we get

$$\alpha = \frac{3(1+n)\gamma}{1+\gamma}. \tag{61}$$

Thus from equation (60) and equation (61), we find that the parameters involve in three dynamical relations are connected by

$$\alpha = \frac{n^2+n+1}{\beta-(n+2)}\beta = \frac{3(1+n)\gamma}{1+\gamma}. \tag{62}$$

It is observed from above equation that the three forms $\Lambda = \alpha\left(\frac{\dot{R}}{R}\right)^2$, $\Lambda = \beta\frac{\ddot{R}}{R}$ and $\Lambda = \gamma\rho$ are equivalent and the three parameters α, β and γ are connected by the relation (62), which indicates that it is possible to find out the identical physical features of others if any one of the phenomenological Λ relation is known and hence these relations are not independent of each other. In cosmology there are many ways to specify the distance between two points, because in the expanding universe, the distance between comoving objects are constantly changing and earth-bound observers look back in time as they look out in distance. In the following sections we discussed some cosmological distances for the case $\Lambda \sim \left(\frac{\dot{R}}{R}\right)^2$.

5. Discussion

In this paper we have analyzed string dust cosmological models with particle attached to them by considering three different forms of variable Λ in context of KK type higher dimensional space time. The equivalence of those forms is shown in terms of solutions thus obtained.We have also established the relation between α, β and γ, the three parameters of the three forms of Λ which ultimately yields $\Omega_m + \Omega_\Lambda = 1$, the relation between cosmic matter and the vacuum energy density parameters for flat universe. In the perticular cases the behaviour of $R(x)$, $A(x)$, $\lambda(x)$ and $\Lambda(x)$ versus x are expressed in graphical form in fig. 1-4. It can be seen that, in all cases, the nature of $R(x)$ is an increasing function with x where as the nature of $\rho(x)$ and $\lambda(x)$ are decreasing functions of x. But the nature of $\Lambda(x)$ is not same in all the cases, i.e., $\Lambda(x)$ is constant in the first case as per the assumption, it is an increasing function in case -II and case-III but it is a decreasing function in case - IV.

It has already been shown that the three phenomenological Λ- models presented here are equivalent, then causal connection of the universe as indicated by the above model equally implies that the other models of the universe are also causally connected in the framework of higher dimensional space-time in the presence of string.

References

[1] R. B. Abbott, S. M. Barr, and S. D. Ellis, *Phys. Rev.* **D 30**, 720 (1984).
[2] E. Alvarez and M. B. Gavela, *Phys. Rev. Lett.* **51**, 931 (1983).
[3] S. M. Barr and L. S. Brown, *Phys. Rev.* **D 29**, 2779 (1984).
[4] R. Brandenberger and N. Turok, *ibdi* **33**, 2182 (1986).
[5] A. D. Dolgov, in *The Very Early Universe* , eds. Gibbons, G. W., Hawking, S. W., and Siklos, S. T. C. (Cambridge Univerity Press, Cambridge), (1983).
[6] A. D. Dolgov, M. V. Sazhni and Ya. B. Zeldovich *Basics of Modern Cosmology*, Editons Frontiers. (1990).
[7] A. D. Dolgov, *Phys. Rev.* **D 55**, 5881 (1997).
[8] M. J. Duff, B. E. W. Nilsson, and C. N. Pope, *Phys. Rep.* **130**, 1 (1986).
[9] D. Garfinkle, *Phys. Rev.* **D 32**, 1323 (1985). bibitem Garriga J. Garriga and E. Verdaguer, *ibid* **36**, 2250 (1987).
[10] J. R. Gott, *Astrophys. J.* **288**, 422 (1985).
[11] L. I. Gurvits, K. I. Kellermann, and S. Frey, *Astron. Astrophys.* **342**, 378 (1999).
[12] F. Hoyle, G. Burbidge and J. V. Narlikar, *Mon. Not. R. Astron. Soc.* **286**, 173 (1997).

[13] M. V. John and K. B. Joseph, *Class. Quantum Grav.* **14**, 1115 (1997).
[14] N. Kaiser and A. Stebbins, *Nature (London)* **310**, 391 (1984).
[15] P. S. Letelier, *Phys. Rev.* **D 20**, 1294 (1979).
[16] M. D. Maia and G. S. Silva, *Phys. Rev.* **D 50**, 7233 (1994).
[17] R. B. Mann and D. E. Vincent, *Phys. Letts.* **107A no. 2**, 75 (1985).
[18] J. W. Moffat, *Los Alamos report* **astro-ph/9608202**, (1996).
[19] S. G. Rajeev, *Phys. Letts.* **125B**, 144 (1983).
[20] B. Ratra and P. J. E. Peebles, *phys. Rev.* **D 37**, 3406 (1988).
[21] S. Ray, and U. Mukhopadhyay, **astro-ph/0407295** , (2004).
[22] Q. Shafi and C. Wetterich,, *Phys. Lett.* **129B**, 387 (1983).
[23] V. Sahni and A. Starobinsky, *Int. J. Mod. Phys.* **D 9**, 373 gr-qc/9904398 (2000).
[24] J. Stachel, *Phys. Rev.* **D 21**, 217 (1980).
[25] Q. Tian, *ibid* **33**, 3549 (1986).
[26] J. Trashen, N. Turok, and R. Brandenberger, *Phys. Rev.* **D 34**, 919 (1986).
[27] N. Turok , *Phys. Lett.* **55**, 1801 (1985).
[28] A. Vilinkin, *Phys. Rev.* **D 24**, 2982 (1981).
[29] R. G. Vishwakarma, *Class. Quant. Grav.* **18**, 1159 (2001).
[30] Ya. B. Zeldovich, *Mon. Not. R. Astron. Soc.* **192**,663 (1980).