STRING COSMOLOGICAL MODEL WITH BULK VISCOSITY IN HIGHER DIMENSIONAL SPACE TIME

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ABSTRACT. We have investigated the bulk viscous fluid string dust cosmological model in the higher dimensional space-time. To obtain a determinate solution, it is assumed that the coefficient of bulk viscosity is a power function of the scalar of expansion $\tau = A\theta^m(t)$ and the scalar of expansion is proportional to shear scalar, which leads to a relation between metric potentials $A = KR^n$ where A and R are function of time. It is also shown that bulk viscosity plays an important role on the evolution of the universe. The physical and geometrical aspects of the model are also discussed.

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1. INTRODUCTION

The general relativistic treatment of strings was pioneered by Letelier(1979, 1983) and Stachel (1980). The main reason to study the clouds of strings model are as a test of consistency for some particular field theories based on string models and other models that use, string as a basic element. We must have a reasonable behaviour of the gravitational field produced by the strings. We know that the universe can be represented by a collection of extended objects (galaxies). So a string dust cosmology gives us a model, to investigate this fact. Also the existence of the strings in the early universe can be used to introduce density fluctuation that might shed some light on the problem of galaxy formation.

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On the other hand cosmological model of fluid with viscosity plays a significant role in the study of evolutions of the universe. It is well known that an early stage of universe when neutrino decoupling occurs, the matter behaves like a viscous fluid. The coefficient of viscosity is known to decrease as the universe expands. Bulk viscosity associated with the Grand Unified Theory (GUT) phase transition can lead to an inflationary universe scenario [Langacker(1981), Waga et. al.(1986), Pacher(1987)] and string creation. It is known that the introduction of bulk viscosity Weinberg (1972) can avoid the big bang singularity [Murphy (1973), Belinskii(1975), Golda et. al.(1983)]. Bulk viscosity can provide phenomenological description of quantum particle creation in a bulk viscous model Barrow (1988). Further interesting models that arises when bulk viscosity is introduced are eternally oscillating models, in which entropy increases with each cycle Heller (1983), and closed models, which do not always collapse to big crunch Barrow (1988). Viscous string dust cosmological model have been discussed by Raj Bali and Upadhayay (2003) , Bali and Dave(2002), Xing Xang Wang (2004). Bianchi Type I & III string cosmological model with and without source free magnetic field have discussed respectively by Banerjee et al. (1990) and Tikekar and Patel (1992). Many authors have studied string cosmological model with bulk viscosity Murphy (1973), Raj Bali and Upadhayay (2003), Xing Xang Wang (2004), Nightangle (1973), Baneerjee et. al. (1985), Santos et. al (1985), Caderni and Fabri (1977, 1979), Harko (1994), Mak et. al. (1997).

Recently Yilmaz (2006) has studied quark matter coupled to the string cloud and domain walls in the five dimensional Kaluaza-Klein space time. An inhomogeneous cosmological model with massive string as a source term is developed by Chaterjee (1993) in a Kaluza-Klein type space time where the usual space exhibits spherical symmetry and the extra dimension is taken in the form of a circle.Some authors have studied Kaluaza-Klein models with different matters (Fukui(1990), Overduin(1997), Chi(1990)). So it will be interesting to study string dust cosmological model with bulk viscosity in the five dimensional Kaluaza-Klein space time.

In this paper we have investigated the string dust cosmological model associated with/without bulk viscosity in higher dimensional space time. To get a determine solution, we have used a condition $\sigma \propto \theta$ where σ is shear and θ is scalar expansion which leads to $A = KR^n$ where n is a constant. The physical and geometrical aspects of the model are also discussed.

2. Geometry and the Field Equations

We consider five dimensional line element in the form

$$ds^2 = -dt^2 + R^2\,(t)\left(dx^2 + dy^2 + dz^2\right) + A^2\,(t)\,d\psi^2 \tag{1}$$

where R and A are the functions of t alone.
The energy momentum tensor for bulk viscous string dust is taken as

$$T_i^j = \rho\nu_i\nu^j - \lambda x_i x^j - \tau\theta(g_i^j + \nu_i\nu^j) \tag{2}$$

with

$$\nu^i\nu_i = -x^i x_i = -1 \tag{3}$$

and

$$\nu^i x_i = 0 \tag{4}$$

where ρ is rest energy density of the cloud of strings with particles attached to them, $\rho = \rho_p + \lambda$ with ρ_p being the rest energy density of particles, λ the tension density of the cloud of strings, $\theta = \nu_{;l}^{\ l}$ is the scalar of expansion and τ the coefficient of bulk viscosity. The vector ν^i describes the cloud four velocity and x^i represents a direction of anisotropy, i. e. direction of strings.
The Einstein's field equations

$$G_i^j = -T_i^j \tag{5}$$

where G_i^j is the Einstein's tensor and we choose the unit such as $c = 1$, $8\pi G = 1$. For the metric (1), Eq. (5) leads to

$$\frac{3\dot{R}^2}{R^2} + \frac{3\dot{R}\dot{A}}{RA} = \rho \tag{6}$$

$$\frac{2\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{2\dot{R}\dot{A}}{RA} + \frac{\ddot{A}}{A} = \tau\theta \tag{7}$$

$$\frac{3\ddot{R}}{R} + \frac{3\dot{R}^2}{R^2} = \lambda + \tau\theta \tag{8}$$

where the dot denotes derivative with respect to time t.
Equations (6) to (8) are three equations of five unknowns $R, A, \rho, \lambda, \tau$. To get a determinate solution, we assume a supplementary condition

$$A = KR^n \tag{9}$$

where K, n are real constants.
Then the Eqs. (6)-(8) together with Eq.(9) leads to

$$3(1+n)\frac{\dot{R}^2}{R^2} = \rho(t) \tag{10}$$

$$(n+2)\frac{\ddot{R}}{R} + (n^2+n+1)\frac{\dot{R}^2}{R^2} = \tau(t)(n+3)\frac{\dot{R}}{R} \tag{11}$$

$$3\frac{\ddot{R}}{R} + 3\frac{\dot{R}^2}{R^2} = \lambda(t) + \tau(t)(n+3)\frac{\dot{R}}{R} \tag{12}$$

3. General Solution of the Field Equations

In order to obtain an explicit solution, we assume that the coefficient of the bulk viscosity is a power function of the scalar of expansion

$$\tau = a\,\theta^m(t) \tag{13}$$

where a and m are positive constants.
Using (13) in (11) we get

$$\frac{\ddot{R}}{R}+\left(\frac{n^2+n+1}{n+2}\right)\frac{\dot{R}^2}{R^2}=a\frac{(n+3)^{m+1}}{n+2}\left(\frac{\dot{R}}{R}\right)^{m+1}. \tag{14}$$

To solve (14) we denote $\dot{R}=X$, then $\ddot{R}=X\frac{dX}{dR}$, and equation (14) can be reduced to the first order differential equation

$$\frac{dX}{dR}+\left(\frac{n^2+n+1}{n+2}\right)\frac{X}{R}=a\frac{(n+3)^{m+1}}{n+2}\frac{X^m}{R^m}. \tag{15}$$

Equation (15) is a Bernoulli's equation ($m\neq 1$) and this can readily be integrated to get its solution as

$$X=\frac{dR}{dt}=[MR^{1-m}+CR^{(m-1)\alpha}]^{\frac{1}{1-m}} \tag{16}$$

where C is the constant of integration and

$$\alpha=\frac{n^2+n+1}{n+2}, \tag{17}$$

$$M=a\left[\frac{(n+3)^{m+1}}{(n+2)(\alpha+1)}\right]. \tag{18}$$

with the help of equation (16), the line element (1) reduces to

$$\begin{aligned} ds^2=&-\left[MR^{1-m}+CR^{(m-1)\alpha}\right]^{\frac{2}{m-1}}dR^2+R^2(t)\left(dx^2+dy^2+dz^2\right)\\ &+K^2R^{2n}(t)\,d\psi^2 \end{aligned} \tag{19}$$

Under suitable transformation of coordinates, Eq. (19) reduces to

$$\begin{aligned} ds^2=&-\left[MT^{1-m}+CT^{(m-1)\alpha}\right]^{\frac{2}{m-1}}dT^2+T^2(t)\left(dx^2+dy^2+dz^2\right)\\ &+K^2T^{2n}(t)\,d\psi^2 \end{aligned} \tag{20}$$

For the model of equation (20), the ex-pressions for energy density ρ, the cloud string tension density λ, the particle density ρ_p, the scalar of expansion θ, the shear scalar σ and the spatial volume V are respectively given by

$$\rho=3(1+n)\left[M+CT^{(m-1)(\alpha+1)}\right]^{\frac{2}{1-m}} \tag{21}$$

$$\begin{aligned} \lambda=&M(\alpha+1)(1-n)\left[M+CT^{(m-1)(\alpha+1)}\right]^{\frac{1+m}{1-m}}\\ &+3(1-\alpha)\left[M+CT^{(m-1)(\alpha+1)}\right]^{\frac{2}{1-m}} \end{aligned} \tag{22}$$

$$\rho_p=3(\alpha+n)\left[M+CT^{(m-1)(\alpha+1)}\right]^{\frac{2}{1-m}}$$

$$-M(\alpha+1)(1-n)\left[M+CT^{(m-1)(\alpha+1)}\right]^{\frac{1+m}{1-m}} \tag{23}$$

$$\theta=(n+3)\left[M+CT^{(m-1)(\alpha+1)}\right]^{\frac{1}{1-m}} \tag{24}$$

$$\sigma^2=\frac{3}{8}(1-n)^2\left[M+CT^{(m-1)(\alpha+1)}\right]^{\frac{2}{1-m}} \tag{25}$$

$$V=KT^{n+3} \tag{26}$$

It is worth pointing out that the equations (21)-(26) are formally identical to the solutions obtained by Wang(2005)for four dimensional space time in general theory of relativity. The solutions differs only in the values of constants α and M. From equation (21) we observe that reality condition $\rho \geq 0$, can be fulfilled provided $n \geq -1$ and $C \geq 0$, $M \geq 0$ because $\alpha \geq 0$. In case of $m < 1$ from equation (24) scalar expansion θ is infinitely large at $T = 0$ and θ tends to finite value as $T \to \infty$, hence the model represents the shearing non-rotating expanding universe with a *big bang* start. From Equation (21) the energy density $\rho \to \infty$ as $T \to 0$ and $\rho \to 0$ as $T \to \infty$. However, in the case of $m > 1$, expansion factor θ is finite as $T \to 0$ and vanishes as $T \to \infty$. Similarly energy density ρ tends to finite value when $T \to 0$ and ρ vanishes when $T \to \infty$. Therefore the model describes a shearing non rotating expanding universe with a *finite start*. We can see from the above discussion that bulk viscosity plays a significant role in the evolution of the universe. Further more since $\frac{\sigma}{\theta}$ tends to a finite limit as $T \to \infty$, the model does not approach isotropy for large value of T. The shear σ is zero at $n = 1$, hence $n = 1$ is the isotropy condition.

Now let us discuss the bulk viscous string cosmological model for $m = 1$, because the above solutions are not valid for $m = 1$.

Now Eq. (13) ($m = 1$) i. e.$\tau = a\theta$, together with Eq. (11) can be expressed as

$$\frac{\ddot{R}}{R}+\left[\frac{n^2+n+1}{n+2}\right]\frac{\dot{R}^2}{R^2}=a\frac{(n+3)^2}{(n+2)}\frac{\dot{R}^2}{R^2} \tag{27}$$

The solutions of above equation gives

$$R(t)=(c_1t+c_2)^{\frac{1}{c_0}} \tag{28}$$

where

$$c_0=\frac{\left(n^2+2n+3\right)-a(n+3)^2}{(n+2)}$$

Now from Equation (9) using Equation (28)

$$A(t)=(c_1t+c_2)^{\frac{n}{c_0}} \tag{29}$$

and the other parameters can be obtained as

$$\rho(t)=\frac{3(1+n){c_1}^2}{c_0^2(c_1t+c_2)^2} \tag{30}$$

$$\lambda(t)=\frac{\left[3(2-c_0)-(n+3)^2\right]{c_1}^2}{c_0^2(c_1t+c_2)^2} \tag{31}$$

$$\rho_p = {c_1}^2 \frac{\left[3(1+n) - 3(2-c_0) + (n+3)^2\right]}{c_0^2(c_1 t + c_2)^2} \tag{32}$$

$$\theta = \frac{(n+3)c_1}{c_0(c_1 t + c_2)} \tag{33}$$

$$\sigma^2 = \frac{3}{8}\frac{(1-n)^2 {c_1}^2}{c_0^2(c_1 t + c_2)^2} \tag{34}$$

Now in absence of bulk viscosity i.e. $a = 0$ and $m = 0$ solving Equation (14) for $R(t)$ we get,

$$R = (c_3 t + c_4)^{\frac{1}{b}} \tag{35}$$

where

$$b = \left[\frac{(n^2 + 2n + 3)}{n+2}\right].$$

The density ρ and string tension λ can be expressed as

$$\rho(t) = \frac{3(1+n){c_3}^2}{b^2(c_3 t + c_4)^2} \tag{36}$$

$$\lambda(t) = \frac{3(2-b){c_3}^2}{b^2(c_3 t + c_4)^2} \tag{37}$$

$$\rho_p = 3{c_3}^2 \frac{(1+n) - (2-b)}{b^2(c_3 t + c_4)^2} \tag{38}$$

$$\theta = \frac{(n+3)c_3}{b(c_3 t + c_4)} \tag{39}$$

$$\sigma^2 = \frac{3}{8}\frac{(1-n)^2 {c_3}^2}{b^2(c_3 t + c_4)^2} \tag{40}$$

4. Discussion

We have investigated the Bianchi type I string dust cosmological model with viscous fluid. The expansion of the universe can then seen as a consequence of matter sources. Towards the initial cosmological singularity the universe however looks very different from today. It was essentially the dimensions of the extra space. We have analyzed here the geometric structure of the dust string filled with viscous fluid, in the presence of the extra dimension. The special cases studied that the inclusion of cloud string in extra dimension can be used as a mechanism to make them disappear at later epoch of the universe. In fact, there presence in the extra dimension space is compatible with the dynamical compactification of such a space and the three dimensional expansion of visible universe. We observe from equations (30) and (36) that the energy density $\rho \geq 0$ can be fulfilled from $(n+1) \geq 0$. Therefore for realistic model, we consider $n \geq -1$. The isotropic behaviour can be seen only when $n = 1$ as the shear vanishes for $n = 1$. It is also observed that the string tension density is vanishing at $n = 1$. For the case $m = 1$, one can observed that ρ, λ and ρ_p are proportional to each other i.e.,

$$\rho = K_1\lambda = K_2\rho_p$$

where

$$K_1 = \left[\frac{3(n+1)}{3(2-c_0)-(n+3)^2}\right],$$

$$K_2 = \left[\frac{3(n+1)}{3(n+1)-3(2-c_0)+(n+3)^2}\right]$$

Also the ratio $\bar{\sigma}/\bar{\theta}$ tends to finite limit as $t \to \infty$. Therefore the model is highly anisotropic for large t.

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