LINEAR STABILITY OF TRIANGULAR EQUILIBRIUM POINTS IN THE PHOTOGRAVITATIONAL RESTRICTED THREE BODY PROBLEM WITH POYNTING -ROBERTSON DRAG WHEN BOTH PRIMARIES ARE OBLATE SPHEROID

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Abstract. We have examined the linear stability of triangular equilibrium points in the photogravitational restricted three body problem with Poynting-Robertson drag when both primaries are considered as radiating as well as oblate spheroid. We have found the position of triangular equilibrium points of our problem. The equations of motion are affected by radiation pressure force, oblateness and P-R drag. With the help of characteristic equation, we have discussed the stability conditions. All classical results involving photogravitational and oblateness in restricted three body problem may be verified from this result.

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1. Introduction

The simplest form of the three-body problem is called the restricted three-body problem, in which a particle of infinitesimal mass moves in the gravitational field of two massive bodies orbiting according to the exact solution of the two-body problem. (The particle with infinitesimal mass, sometimes called a massless particle, does not perturb the motions of the two massive bodies.) There is an enormous literature devoted to this problem, including both analytic and numerical developments. The analytic work was devoted mostly to the circular, planar restricted three-body problem, where all particles are confined to a plane and the two finite

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masses are in circular orbits around their centre of mass. Numerical developments allowed consideration of the more general problem.

In the circular problem, the two finite masses are fixed in a coordinate system rotating at the orbital angular velocity, with the origin (axis of rotation) at the centre of mass of the two bodies. Lagrange showed that in this rotating frame there are five stationary points at which the massless particle would remain fixed if placed there. There are three such points lying on the line connecting the two finite masses: one between the masses and one outside each of the masses. The other two stationary points, called the triangular points, are located equidistant from the two finite masses at a distance equal to the finite mass separation. The two masses and the triangular stationary points are thus located at the vertices of equilateral triangles in the plane of the circular orbit. J.H. Poynting(1903) considered the effect of the absorption and subsequent re-emission of sunlight by small isolated particles in the solar system.The proper relativistic treatment of this problem was formulated by H.P. Robertson(1937) who showed that to first order in $\frac{\vec{V}}{c}$ the radiation pressure force is given by

$$\vec{F} = F_p\left\{\frac{\vec{R}}{R} - \frac{\vec{V}.\vec{R}\vec{R}}{cR^2} - \frac{\vec{V}}{c}\right\} \tag{1}$$

Where $F_p=\frac{3Lm}{16\pi R^2\rho sc}$ denotes the measure of the radiation pressure force, $\vec{R}$ the position vector of P with respect to radiation sours S, $\vec{V}$ the corresponding velocity vector and c the velocity of light. In the expression of F_p, L is luminosity of the radiating body, while m, ρ and s are the mass, density and cross section of the particle respectively.

The first term in equation (1) expresses the radiation pressure. The second term represents the Doppler shift of the incident radiation and the third term is due to the absorption and subsequent re-emission of the incident radiation. These last two terms taken together are the Poynting-Robertson effect. The Poynting-Robertson effect will operate to sweep small particles of the solar system into the Sun at cosmically rapid rate.The motion of an infinitesimal body near one of the Lagrangian points is said to be stable if given a very small displacement and small velocity, the body oscillates for a considerable time around the point. If it departs from the point as the time increases, the motion is said to be unstable. Mathematically, we express that the stability of motion means bounded displacement and velocity with bounded functions of time in the neighborhood of the equilibrium points.

The Poynting-Robertson (P-R) effect is the most important nongravitational and dust particles. The P-R effect may be significant in dealing with other problems in astrophysics.

The effect of radiation pressure and P-R drag in the restricted three body problem has been studied by Colombo *et al.*(1966),Chernikov Yu.A.(1970). Schuerman(1980) discussed the position as well as the stability of the Lagrangian equilibrium points when radiation pressure, P-R drag force are included. Murray C.D.(1994) systematically discussed the dynamical effect of general drag in the planar circular restricted three body problem. Liou J.C. *et al.*(1995) examined the effect of radiation pressure, P-R drag and solar wind drag in the restricted three body problem.The classical three body problem has five Lagrangian points. Their location and stability properties are well known. The three collinear points L_1, L_2, L_3 lies on the line joining the two primaries are linearly unstable. The two Lagrangian points L_4, L_5 lie in the orbit of the test particle which are symmetrical about the line joining the primaries, are linearly stable with the condition that the mass parameter μ below Routh's critical value,$\mu_c = 0.0385$. Hence, we are interested to discuss the linear stability of triangular equilibrium points in generalised photogravitational restricted three body problem with Poynting-Robertson drag. After Newton solved the problem of the orbit of single planet around the Sun, the natural next challenge was to find the solution of two planets orbiting the Sun(i.e. three body problem). Many of the best minds in Mathematics and Physics worked on this problem in the last century. Three-Body problem is a continuous source of study, since the discovery of its non-integrability due to Poincare(1892). Regular and chaotic motion have been widely investigated with any kind of tools, from analytical result to numerical explorations. The restricted three-body problem is a modified three-body problem when one of the three masses is set to zero(infinitesimal mass).

We have examined the linear stability of triangular equilibrium points in the photogravitational restricted three body problem with Poynting-Robertson drag when both primaries are considered as radiating as well as oblate spheroids. We have found the position of triangular equilibrium points of our problem. The equations of motion are affected by radiation pressure force, oblateness and P-R drag.

2. Location of Triangular Equilibrium Points

We suppose m_1, m_2, the masses of both primaries respectively. Let these primaries revolve around their centre of mass in circular orbit. Using dimensionless variables and synodic coordinate system (x, y) as in Szebehely(1967). Let $\mu = \frac{m_2}{m_1+m_2}$ then $1 - \mu = \frac{m_1}{m_1+m_2}$ with $m_1 > m_2$ where μ is mass parameter. Then coordinates of m_1 and m_2 are $(-\mu, 0)$ and $(1 - \mu, 0)$ respectively.

To derive the equations of motion, we used relations(1) and used the same techniques as Chernikov(1970) and Schuerman(1980). In dimensionless coordinate system, the dimensionless velocity of light is given by $c = c_d$, which depends on the physical masses of the primaries and the distance between them. Equations of motion are,

$$\ddot{x} - 2n\dot{y} = \Omega_x, \tag{2}$$

$$\ddot{y} + 2n\dot{x} = \Omega_y, \tag{3}$$

where

$$\Omega_x = n^2 x - \frac{(1-\mu)q_1(x+\mu)}{r_1^3}\left(1+\frac{3A_1}{2r_1^2}\right) - \frac{(\mu)q_2(x+\mu-1)}{r_2^3}\left(1+\frac{3A_2}{2r_2^2}\right)$$
$$-\frac{W_1}{r_1^2}\left[\frac{(x+\mu)}{r_1^2}[(x+\mu)\dot{x}+y\dot{y}]+\dot{x}-ny\right]$$
$$-\frac{W_2}{r_2^2}\left[\frac{(x+\mu-1)}{r_2^2}[(x+\mu-1)\dot{x}+y\dot{y}]+\dot{x}-ny\right]$$
$$\Omega_y = n^2 y - \frac{(1-\mu)q_1 y}{r_1^3}\left(1+\frac{3A_1}{2r_1^2}\right) - \frac{(\mu)q_2 y}{r_2^3}\left(1+\frac{3A_2}{2r_2^2}\right)$$
$$-\frac{W_1}{r_1^2}\left[\frac{y}{r_1^2}[(x+\mu)\dot{x}+y\dot{y}]+\dot{y}+n(x+\mu)\right]$$
$$-\frac{W_2}{r_2^2}\left[\frac{y}{r_2^2}[(x+\mu-1)\dot{x}+y\dot{y}]+n(x+\mu-1)\right]$$

where

$$W_1 = \frac{(1-\mu)(1-q_1))}{c_d}, W_2 = \frac{(1-\mu)(1-q_2)}{c_d},$$
$$r_1^2 = (x+\mu)^2 + y^2 \ , r_2^2 = (x+1-\mu)^2 + y^2, \quad n^2 = 1 + \frac{3}{2}(A_1+A_2),$$

$A_1 = \frac{r_e^2 - r_p^2}{5r^2}, A_2 = \frac{r_e'^2 - r_p'^2}{5r^2}$ be the oblateness coefficient, r_e, r_p, andr'_e, r'_p be the equatorial and polar radii of m_1, m_2 respectively r be the distance between primaries, $q = \left(1 - \frac{F_p}{F_g}\right)$ be the mass reduction factor expressed in terms of the particle's radius a, density ρ and radiation pressure efficiency factor χ (in the C.G.S.system) i.e., $q = 1 - \frac{5.6\times 10^{-5}\chi}{a\rho}$. Assumption $q = constant$ is equivalent to neglecting fluctuation in the beam of solar radiation and the effect of solar radiation, the effect of the planet's shadow, obviously $q \le 1$. Triangular equilibrium points are given by $\Omega_x = 0 = \Omega_y, y \ne 0$, then we have

$$\begin{aligned} x \;=\; & x_0\Bigg[1 - \frac{\delta_1^2\left(A_1 + A_2 - \frac{A_1}{\delta_1^2}\right)}{2x_0\left(1+\frac{5A_1}{2\delta_1^2}\right)} + \frac{\delta_2^2\left\{A_1 + A_2 - \frac{A_2}{\delta_2^2}\right\}}{2x_0\left(1+\frac{5A_2}{2\delta_2^2}\right)} \\ & -\frac{nW_1}{3x_0y_0}\left\{\frac{\delta_1^2+\delta_2^2-1}{2(1-\mu)\left(1+\frac{5A_1}{2\delta_1^2}\right)} + \frac{\delta_2^2}{\mu\left(1+\frac{5A_2}{2\delta_2^2}\right)}\right\} \\ & -\frac{nW_2}{3x_0y_0}\left\{\frac{\delta_1^2+\delta_2^2-1}{2\mu\left(1+\frac{5A_2}{2\delta_2^2}\right)} + \frac{\delta_1^2}{(1-\mu)\left(1+\frac{5A_1}{2\delta_1^2}\right)}\right\}\Bigg] \end{aligned} \quad (4)$$

$$
\begin{aligned}
y \;=\; & y_0\Bigg[1+\frac{\delta_1^2\left(A_1+A_2-\frac{A_1}{\delta_1^2}\right)(\delta_1^2-\delta_2^2-1)}{2y_0^2\left(1+\frac{5A_1}{2\delta_1^2}\right)}-\frac{\delta_2^2\left(A_1+A_2-\frac{A_2}{\delta_2^2}\right)(\delta_1^2-\delta_2^2+1)}{2y_0^2\left(1+\frac{5A_2}{2\delta_2^2}\right)}\\
& +\frac{nW_1}{3y_0^3}\left\{\frac{(\delta_1^2+\delta_2^2-1)(\delta_1^2-\delta_2^2-1)}{2(1-\mu)\left(1+\frac{5A_1}{2\delta_1^2}\right)}+\frac{\delta_2^2(\delta_1^2-\delta_2^2+1)}{\mu\left(1+\frac{5A_2}{2\delta_2^2}\right)}\right\}\\
& +\frac{nW_2}{3y_0^3}\left\{\frac{\delta_1^2(\delta_1^2-\delta_2^2-1)}{(1-\mu)\left(1+\frac{5A_1}{2\delta_1^2}\right)}+\frac{(\delta_1^2+\delta_2^2-1)(\delta_1^2-\delta_2^2+1)}{2(\mu)\left(1+\frac{5A_2}{2\delta_2^2}\right)}\right\}\Bigg]^{1/2} \qquad (5)
\end{aligned}
$$

where (x_0, y_0) are coordinates of L_4 and L_5 in the Photogravitational restricted three body problem. $x_0 = \frac{\delta_1^2-\delta_2^2+1}{2} - \mu, y_0 = \pm\left\{\delta_1^2 - \left(\frac{\delta_1^2-\delta_2^2+1}{2}\right)^2\right\}$ and $\delta_1 = q_1^{1/3}, \delta_2 = q_2^{1/3}$ Eq.(4) and (5) are valid for $W_1 \ll 1, W_2 \ll 1,\ A_1 \ll 1, A_2 \ll 1$.

3. Stability of Triangular Equilibrium Points

It is well known that the restricted three-body problem has triangular equilibrium points. These points are linearly stable for values of the mass parameter, μ, below Routh's critical value, $\mu_c = 0.0385$ as Deprit(1965). We suppose $\alpha = Ae^{\lambda t}$,$\beta = Be^{\lambda t}$ be the small displacements from Lagrangian points(x_0, y_0) and $A, B,\ \lambda$ are parameters, $x = x_0 + \alpha, \quad y = y_0 + \beta$ and therefore the equations of perturbed motion corresponding to the system of Eq.(1),(2) may be written as,

$$\ddot{\alpha} - 2n\dot{\beta} = \Omega_x^0 + \alpha\Omega_{xx}^0 + \beta\Omega_{xy}^0 + \dot{\alpha}\Omega_{x\dot{x}}^0 + \dot{\beta}\Omega_{x\dot{y}}^0 \qquad (6)$$

$$\ddot{\beta} + 2n\dot{\alpha} = \Omega_y^0 + \alpha\Omega_{yx}^0 + \beta\Omega_{yy}^0 + \dot{\alpha}\Omega_{y\dot{x}}^0 + \dot{\beta}\Omega_{y\dot{y}}^0 \qquad (7)$$

Above system of equations can be written as,

$$(\lambda^2 - \lambda\Omega_{x\dot{x}}^0 - \Omega_{xx}^0)A + [-(2n + \Omega_{x\dot{y}}^0)\lambda - \Omega_{xy}^0]B = 0 \qquad (8)$$

$$[(2n - \Omega_{y\dot{x}}^0)\lambda - \Omega_{yx}^0]A + (\lambda^2 - \lambda\Omega_{y\dot{y}}^0 - \Omega_{yy}^0)B = 0 \qquad (9)$$

Now Eq.(8) and (9) has singular solution if,

$$\begin{vmatrix} \lambda^2 - \lambda\Omega_{x\dot{x}}^0 - \Omega_{xx}^0 & -(2n + \Omega_{x\dot{y}}^0)\lambda - \Omega_{xy}^0 \\ (2n - \Omega_{y\dot{x}}^0)\lambda - \Omega_{yx}^0 & \lambda^2 - \lambda\Omega_{y\dot{y}}^0 - \Omega_{yy}^0 \end{vmatrix} = 0$$

$$\Rightarrow \quad \lambda^4 + a\lambda^3 + b\lambda^2 + c\lambda + d = 0 \qquad (10)$$

At the equilibrium points the system of Eq.(1),(2) gives

$$a = 3\left(\frac{W_1}{r_1^2} + \frac{W_2}{r_2^2}\right) \qquad (11)$$

$$
\begin{aligned}
b \quad = \quad & 2n^2 - \frac{(1-\mu)q_1}{r_1^3}\left(1+\frac{11A_1}{2r_1^2}\right) - \frac{\mu q_2}{r_2^3}\left(1+\frac{11A_2}{2r_2^2}\right) \\
& -\frac{W_1}{r_1^4}\Big[3(x+\mu)\dot{x} + 3y\dot{y} - \Big\{(2(x+\mu)^2+y^2)(\ddot{x}+\ddot{y}) \\
& -\left\{(x+\mu)^2 - y^2\right\}\ddot{y}\Big\}\Big] - \frac{W_2}{r_2^4}\Big[3(x+\mu-1)\dot{x} \\
& +3y\dot{y} - \Big\{(2(x+\mu-1)^2+y^2)(\ddot{x}+\ddot{y}) \\
& -\left\{(x+\mu-1)^2 - y^2\right\}\ddot{y}\Big\}\Big] + 2\left(\frac{W_1}{r_1^2}+\frac{W_2}{r_2^2}\right)^2 + \frac{W_1W_2}{r_1^4 r_2^4}y^2
\end{aligned}
\tag{12}
$$

$$
\begin{aligned}
c \quad = \quad & -\frac{3W_1}{r_1^2}\Big[n^2 + \frac{2A_1q_1(1-\mu)(x+\mu)^2y^2}{r_1^9} + \frac{A_1q_1(x+\mu)}{r_1^5} + \frac{\mu q_2 y^2}{r_1^2 r_2^5} \\
& +\frac{\mu 5A_2q_2y^2}{2r_1^2r_2^7} + \frac{A_2q_2\mu}{r_2^5}\Big] - \frac{3W_2}{r_2^2}\Big[n^2 + \frac{2A_2q_2\mu(x+\mu-1)^2y^2}{r_2^9} \\
& +\frac{A_2q_2}{r_2^5}\mu + \frac{q_1(1-\mu)y^2}{r_1^5r_2^2} + \frac{5A_1(1-\mu)q_1y^2}{2r_1^7r_2^2} + \frac{A_1q_2(1-\mu)}{r_1^5}\Big]
\end{aligned}
\tag{13}
$$

$$
\begin{aligned}
d \quad = \quad & \Big\{(n^2+f_1-f_2)(n^2+f_1'-f_2) \\
& +A_1\Big[\frac{3}{2}(n^2+f_1-f_2)\frac{(1-\mu)q_1}{r_1^5}\left(\frac{5y^2}{r_1^2}-1\right) \\
& +\frac{3}{2}(n^2+f_1'-f_2)\frac{(1-\mu)q_1}{r_1^5}\left(\frac{5(x+\mu)^2}{r_1^2}-1\right)\Big] \\
& +A_2\Big[\frac{3}{2}(n^2+f_1-f_2)\frac{\mu q_2}{r_2^5}\left(\frac{y^2}{r_2^2}-1\right) \\
& +\frac{3}{2}(n^2+f_1'-f_2)\frac{\mu q_2}{r_2^5}\left(\frac{(x+\mu-1)^2}{r_2^2}-1\right)\Big]\Big\} \\
& -\Big\{9\left(\frac{q_1(1-\mu)(x+\mu)}{r_1^5} + \frac{q_2\mu(x+\mu-1)}{r_2^5}\right)^2 y^2 \\
& +\frac{225}{4}\left(\frac{q_1(1-\mu)(x+\mu)A_1}{r_1^7} + \frac{q_2\mu(x+\mu-1)A_2}{r_2^7}\right)y^2 \\
& +45\left(\frac{q_1(1-\mu)(x+\mu)}{r_1^5} + \frac{q_2\mu(x+\mu-1)}{r_2^5}\right)y^2 \\
& \left(\frac{q_1(1-\mu)(x+\mu)A_1}{r_1^7} + \frac{q_2\mu(x+\mu-1)A_2}{r_2^7}\right)\Big\}
\end{aligned}
\tag{14}
$$

where

$$f_1 = \frac{3(1-\mu)(x+\mu)^2 q_1}{r_1^5}, \quad f_2 = \frac{(1-\mu)q_1}{r_1^3} + \frac{\mu q_2}{r_2^3}$$

$$f_1' = \frac{3(1-\mu)q_1 y^2}{r_1^5} + \frac{3\mu q_2 y^2}{r_2^5}$$

In classical case the characteristic roots are $\lambda^4 + \lambda^2 + \frac{27}{4}\mu(1-\mu)$. We can write the four roots of the classical characteristic equation as,

$$\lambda_j = \pm zi, \quad \text{where,} \quad z^2 = \frac{1}{2}\Big\{1 \mp \big[1 - 27\mu(1-\mu)\big]^{1/2}\Big\}, \quad j = 1, 2, 3, 4. \tag{15}$$

Due to P-R drag and oblateness,we assume the solution of the characteristic Eq.(10) of the form

$$\lambda = \lambda_j(1 + e_1 + e_2 i) = \pm[-e_2 + (1 + e_1)i]z$$

where e_1, e_2 are small real quantities. We have in first order approximation

$$\lambda^2 = [-(1+2e_1) - 2e_2 i]z^2, \quad \lambda^3 = \pm[3e_2 - (1+3e_1)i]z^3, \quad \lambda^4 = [(1+4e_1) + 4e_2 i]z^4$$

Substituting these values in characteristic Eq.(10) neglecting products of e_1 or e_2 with a, b, c, we get

$$e_1 = \frac{-d + 2n^2 z^2 - z^4}{2z^2(2z^2 - 2n^2)} \tag{16}$$

$$e_2 = \frac{\mp cz \pm az^3}{2z^2(2z^2 - 2n^2)} \tag{17}$$

If $e_2 \neq 0$,then the resulting motion of particle displaced from equilibrium points is asymptotically stable only when all the real parts of λ are negative. For stability, we require $Re(\lambda) < 0$. Rate of growth or decay is determined by the largest real part of the roots of the characteristic equation, where $Re(\lambda)$ denotes the real parts of λ and given by

$$Re(\lambda) = \frac{c - az^2}{2(2z^2 - 1)} \tag{18}$$

we have,

$$z^2 = \frac{1}{2}\Big\{1 \mp \big[1 - 27\mu(1-\mu)\big]^{1/2}\Big\} \tag{19}$$

Taking positive sign ,

$$z^2 = 1 - \frac{27}{4}\mu(1-\mu)$$

We consider

$$Re(\lambda) < 0 \quad \text{then} \quad \Big\{c - a\big[1 - \frac{27}{4}\mu(1-\mu)\big]\Big\}\Big\{1 - \frac{27}{2}\mu(1-\mu)\Big\}^{-1} < 0$$

$$\Rightarrow \quad c + \frac{27}{4}\mu(1-\mu)(2c - a) < a \tag{20}$$

Taking negative sign,

$$Re(\lambda) = -\Big[c - a\frac{27}{4}\mu(1-\mu)\Big]\Big[1 - \frac{27}{2}\mu(1-\mu)\Big]^{-1} < 0$$

$$\Rightarrow 0 < c + \frac{27}{4}\mu(1-\mu)(2c-a) \tag{21}$$

From above inequalities we get,

$$0 < c + \frac{27}{4}\mu(1-\mu)(2c-a) < a \tag{22}$$

$$0 < c < a \quad \text{as} \quad \mu \to 0 \qquad \text{Murray C.D.(1994)} \tag{23}$$

Inequality(23) is necessary condition for stability of triangular equilibrium points L_4 and L_5.

4. Conclusion

(i) When $W_1 = W_2 = 0$, then

$$a = 0 \tag{24}$$

$$b = 2n^2 - \frac{(1-\mu)q_1}{r_1^3}\left(1 + \frac{11A_1}{2r_1^2}\right) - \frac{\mu q_2}{r_2^3}\left(1 + \frac{11A_2}{2r_2^2}\right) \tag{25}$$

$$c = 0$$

$$\begin{aligned} d = & \Bigg\{(n^2 + f_1 - f_2)(n^2 + f_1' - f_2) \\ & + A_1\Bigg[\frac{3}{2}(n^2 + f_1 - f_2)\frac{(1-\mu)q_1}{r_1^5}\left(\frac{5y^2}{r_1^2} - 1\right) \\ & + \frac{3}{2}(n^2 + f_1' - f_2)\frac{(1-\mu)q_1}{r_1^5}\left(\frac{5(x+\mu)^2}{r_1^2} - 1\right)\Bigg] \\ & + A_2\Bigg[\frac{3}{2}(n^2 + f_1 - f_2)\frac{\mu q_2}{r_2^5}\left(\frac{y^2}{r_2^2} - 1\right) \\ & + \frac{3}{2}(n^2 + f_1' - f_2)\frac{\mu q_2}{r_2^5}\left(\frac{(x+\mu-1)^2}{r_2^2} - 1\right)\Bigg]\Bigg\} \\ & - \Bigg\{9\left(\frac{q_1(1-\mu)(x+\mu)}{r_1^5} + \frac{q_2\mu(x+\mu-1)}{r_2^5}\right)^2 y^2 \\ & + \frac{225}{4}\left(\frac{q_1(1-\mu)(x+\mu)A_1}{r_1^7} + \frac{q_2\mu(x+\mu-1)A_2}{r_2^7}\right)y^2 \\ & + 45\left(\frac{q_1(1-\mu)(x+\mu)}{r_1^5} + \frac{q_2\mu(x+\mu-1)}{r_2^5}\right)y^2 \\ & \left(\frac{q_1(1-\mu)(x+\mu)A_1}{r_1^7} + \frac{q_2\mu(x+\mu-1)A_2}{r_2^7}\right)\Bigg\} \end{aligned} \tag{26}$$

(ii) For classical case $A_1 = A_2 = 0$, $q_1 = q_2 = 1$, $W_1 = W_2 = 0$, $n = 1$. Then from inequality (23) we have

$$1 > 27\mu(1-\mu) \quad \Rightarrow \quad \mu < .03852$$

(iii) When $W_1 \neq 0, W_2 \neq 0$, $A_1 = A_2 = 0$, $n = 1$. Then

$$a = 3\left(\frac{W_1}{r_1^2} + \frac{W_2}{r_2^2}\right) \tag{27}$$

$$\begin{aligned} b \;=\; & 2 - \frac{(1-\mu)q_1}{r_1^3} - \frac{\mu q_2}{r_2^3} \\ & - \frac{W_1}{r_1^4}\Big[3(x+\mu)\dot{x} + 3y\dot{y} - \Big\{(2(x+\mu)^2 + y^2)(\ddot{x} + \ddot{y}) \\ & - \left\{(x+\mu)^2 - y^2\right\}\ddot{y}\Big\}\Big] - \frac{W_2}{r_2^4}\Big[3(x+\mu-1)\dot{x} \\ & + 3y\dot{y} - \Big\{(2(x+\mu-1)^2 + y^2)(\ddot{x} + \ddot{y}) \\ & - \left\{(x+\mu-1)^2 - y^2\right\}\ddot{y}\Big\}\Big] + 2\left(\frac{W_1}{r_1^2} + \frac{W_2}{r_2^2}\right)^2 + \frac{W_1 W_2}{r_1^4 r_2^4} y^2 \end{aligned} \tag{28}$$

$$c \;=\; -\frac{3W_1}{r_1^2}\left[1 + \frac{\mu q_2 y^2}{r_1^2 r_2^5}\right] - \frac{3W_2}{r_2^2}\left[1 + \frac{q_1(1-\mu)y^2}{r_1^5 r_2^2}\right] \tag{29}$$

$$\begin{aligned} d \;=\; & \Big\{(1 + f_1 - f_2)(1 + f_1' - f_2)\Big\} \\ & - \left\{9\left(\frac{q_1(1-\mu)(x+\mu)}{r_1^5} + \frac{q_2\mu(x+\mu-1)}{r_2^5}\right)^2 y^2\right\} \\ & - \left\{\frac{225}{4}\left(\frac{q_1(1-\mu)(x+\mu)}{r_1^5} + \frac{q_2\mu(x+\mu-1)}{r_2^5}\right) y^2\right\} \end{aligned} \tag{30}$$

as in Chernikove Yu.A.(1970)

If we consider the P-R drag force then $c < 0$. This dose not satisfy the necessary condition of stability. Hence motion is unstable in the linear sense.

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