

HIGHER DIMENSIONAL COSMOLOGICAL MODEL IN BRANS-DICKE THEORY OF GRAVITATION-I

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ABSTRACT. A spatially homogenous and anisotropic Bianchi type-I cosmological model is examined with five-dimensions in Brans-Dicke (1961) scalar-tensor theory of gravitation. Some properties of the model are also studied.

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Keywords: Scalar-tensor theory, cosmic strings, Bianchi type model

1. INTRODUCTION

The study of higher dimensional physics has a great importance today because of several prominent results obtained in the development of the recent study in superstring theory. The study of string theory is a promising theory unifying all the elementary interactions including gravity. The latest studies of super string and super gravity theories and Weinberg (1986) studied the unification of the fundamental forces with gravity which reveals that the space-time should be different from four.

Brans -Dicke (1961) theory of gravitation is the simplest example of scalar - tensor theories in which a scalar field ϕ has been introduced in addition to the familiar general relativistic metric tensor. In this theory the scalar field has the dimension of inverse of the gravitational constant and its role is confined to its effects on gravitational equations.

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Brans- Dicke (1961) field equations for combined scalar and tensor fields are

$$(1) \quad G_{ij} = -8\pi\phi^{-1}T_{ij} - \omega\phi^{-2} \left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k} \right) - \phi^{-1}(\phi_{i,j} - g_{ij}\phi_{;k}^{,k})$$

and

$$(2) \quad \phi_{;k}^{,k} = 8\pi\phi^{-1}(3 + 2\omega)^{-1}T$$

where $G_{ij} = R_{ij} - \frac{1}{2}g_{ij}R$ is the Einstein tensor, T_{ij} is the stress energy tensor of the matter, ω is the dimensionless coupling constant and comma and semicolon denotes partial and covariant differentiation respectively.

The equations of motion

$$(3) \quad T_{;j}^{ij} = 0$$

are consequences of the field equations (1) and (2).

Several aspects of Brans-Dicke cosmology have been extensively investigated by many authors. The work of Singh and Rai (1983) gives a detailed discussion of Brans -Dicke cosmological models with perfect fluid as source. In particular, spatially homogenous Bianchi type models in the alternative theories of gravitation in the presence of perfect fluid with or without radiation are quite important to discuss the early stages of evolution of the universe. Recently, Rahaman et al. (2005) studied some cosmological models in Lyra (1951) geometry using a special law of variation of Hubble's parameter that yields constant deceleration parameter models of the universe while Reddy and Venkateswaralu(1989),Reddy(2003a,b), Reddy et al. (2006 a,b) discussed Bianchi type cosmological models with cosmic strings and negative constant deceleration parameter in scalar -tensor theories formulated by Saez and Ballester (1985) and Brans and Dicke (1961).

In this paper, we have considered a spatially homogeneous and anisotropic Bianchi type - I cosmological model with five dimensions in Brans - Dicke scalar - tensor theory of gravitation in presence of cosmic string source. This study is important because of the fact that the resulting cosmological model is considered to be analogous with the inflationary model obtained in Brans-Dicke cosmology.

2. METRIC AND FIELD EQUATIONS

The five-dimensional Bianchi type-I line element can be written as

$$(4) \quad ds^2 = dt^2 - A^2dx^2 - B^2dy^2 - C^2dz^2 - D^2dw^2$$

with the metric functions A, B, C, D being functions of time t only. Here the extra coordinate is taken to be space-like.

The energy momentum tensor for cosmic strings is given by

$$(5) \quad T_{ij} = \rho u_i u_j - \lambda x_i x_j,$$

where ρ is the rest energy density of cloud of strings with particles attached to them , λ is the tension density of strings , u_i is cloud four velocity and x_i is the direction of anisotropy.

We have

$$(6) \quad u^i u_i = -x^i x_i = 1 \text{ and } u^i x_i = 0$$

We consider $\rho = \rho_p + \lambda$, where ρ_p is the rest energy density of particles and x^i to be along x-axis so that

$$(7) \quad x^i = (A^{-1}, 0, 0, 0, 0) \text{ and } u^i u_i = (0, 0, 0, 0, 1).$$

Now, the Brans -Dicke field equations (1) and (2) for the metric (4) with the help of equations (5) - (7) can be written as

$$(8)$$

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{D_{44}}{D} + \frac{B_4 C_4}{BC} + \frac{B_4 D_4}{BD} + \frac{C_4 D_4}{CD} = 8\pi\phi^{-1}\lambda - \frac{\omega}{2} \left(\frac{\phi_4}{\phi} \right)^2 - \frac{\phi_4}{\phi} \left(\frac{B_4}{B} + \frac{C_4}{C} + \frac{D_4}{D} \right) - \frac{\phi_{44}}{\phi}$$

$$(9)$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{D_{44}}{D} + \frac{A_4 C_4}{AC} + \frac{A_4 D_4}{AD} + \frac{C_4 D_4}{CD} = -\frac{\omega}{2} \left(\frac{\phi_4}{\phi} \right)^2 - \frac{\phi_4}{\phi} \left(\frac{A_4}{A} + \frac{C_4}{C} + \frac{D_4}{D} \right) - \frac{\phi_{44}}{\phi}$$

$$(10)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{D_{44}}{D} + \frac{A_4 B_4}{AB} + \frac{A_4 D_4}{AD} + \frac{B_4 D_4}{BD} = -\frac{\omega}{2} \left(\frac{\phi_4}{\phi} \right)^2 - \frac{\phi_4}{\phi} \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{D_4}{D} \right) - \frac{\phi_{44}}{\phi}$$

$$(11)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{A_4 B_4}{AB} + \frac{A_4 C_4}{AC} + \frac{B_4 C_4}{BC} = -\frac{\omega}{2} \left(\frac{\phi_4}{\phi} \right)^2 - \frac{\phi_4}{\phi} \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} \right) - \frac{\phi_{44}}{\phi}$$

$$(12)$$

$$\frac{A_4 B_4}{AB} + \frac{A_4 D_4}{AD} + \frac{B_4 D_4}{BD} + \frac{B_4 C_4}{BC} + \frac{A_4 C_4}{AC} + \frac{C_4 D_4}{CD} = 8\pi\phi^{-1}\rho + \frac{\omega}{2} \left(\frac{\phi_4}{\phi} \right)^2 - \frac{\phi_4}{\phi} \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} + \frac{D_4}{D} \right)$$

$$(13) \quad \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} + \frac{D_4}{D} \right) \phi_4 + \phi_{44} = \frac{8\pi\phi^{-1}}{3+2\omega} (\rho + \lambda)$$

$$(14) \quad \rho_4 + \rho \left(\frac{B_4}{B} + \frac{C_4}{C} + \frac{D_4}{D} \right) + (\rho - \lambda) \left(\frac{A_4}{A} \right) = 0$$

where the suffix 4 following an unknown function denotes ordinary differentiation with respect to time t .

We solve the above set of field equations (8)-(14) with the transformation (Reddy and Venkateswaralu, 1989)

$$(15) \quad A = e^\alpha, B = e^\beta, C = e^\gamma, D = e^\delta, dt = ABCD dT$$

The above set of equations reduces to

$$(16)$$

$$\beta'' + \gamma'' + \delta'' - \beta'\alpha' - \beta'\gamma' - \beta'\delta' - \gamma'\alpha' - \gamma'\delta' - \delta'\alpha' + \frac{\omega}{2} \left(\frac{\phi'}{\phi} \right)^2 + \frac{\phi''}{\phi} - \frac{\phi'\alpha'}{\phi} = 8\pi\phi^{-1}\lambda \exp [2\alpha + 2\beta + 2\gamma + 2\delta]$$

$$(17) \quad \alpha'' + \gamma'' + \delta'' - \alpha'\beta' - \alpha'\gamma' - \alpha'\delta' - \gamma'\beta' - \gamma'\delta' - \delta'\beta' + \frac{\omega}{2} \left(\frac{\phi'}{\phi} \right)^2 + \frac{\phi''}{\phi} - \frac{\phi'\beta'}{\phi} = 0$$

$$(18) \quad \alpha'' + \beta'' + \delta'' - \alpha' \beta' - \alpha' \gamma' - \alpha' \delta' - \beta' \gamma' - \beta' \delta' - \delta' \gamma' + \frac{\omega}{2} \left(\frac{\phi'}{\phi} \right)^2 + \frac{\phi''}{\phi} - \frac{\phi' \gamma'}{\phi} = 0$$

$$(19) \quad \alpha'' + \beta'' + \gamma'' - \alpha' \beta' - \alpha' \delta' - \beta' \gamma' - \beta' \delta' - \gamma' \alpha' - \gamma' \delta' + \frac{\omega}{2} \left(\frac{\phi'}{\phi} \right)^2 + \frac{\phi''}{\phi} - \frac{\phi' \delta'}{\phi} = 0$$

$$(20)$$

$$\beta' \alpha' + \beta' \gamma' + \beta' \delta' + \gamma' \alpha' + \gamma' \delta' + \delta' \alpha' - \frac{\omega}{2} \left(\frac{\phi'}{\phi} \right)^2 + \frac{\phi'}{\phi} (\alpha' + \beta' + \gamma' + \delta') = 8\pi\phi^{-1}\rho \exp[2\alpha + 2\beta + 2\gamma + 2\delta]$$

$$(21) \quad \phi'' = \frac{8\pi\phi^{-1}}{3 + 2\omega} (\lambda + \rho) \exp[2\alpha + 2\beta + 2\gamma + 2\delta]$$

$$(22) \quad \rho' + \rho(\beta' + \gamma' + \delta') + (\rho - \lambda) \alpha' = 0$$

where a superscript prime (') indicates differentiation with respect to T .

3. STRING COSMOLOGICAL MODEL

To obtain a determinate solution of the set of highly non-linear field equations(16)-(22) , we assume that

$$(23) \quad \rho + \lambda = 0$$

i.e. the sum of rest energy density and tension density for a cloud of string vanishes. Using equation (23), the set of equations (16)-(22) reduces to

$$\beta'' + \gamma'' + \delta'' + \frac{\phi'}{\phi} [\beta' + \gamma' + \delta'] = 0.$$

$$\gamma'' - \beta'' - \frac{\phi' \beta'}{\phi} + \frac{\phi' \gamma'}{\phi} = 0.$$

$$\delta'' - \gamma'' - \frac{\phi' \gamma'}{\phi} + \frac{\phi' \delta'}{\phi} = 0.$$

$$\alpha'' + \frac{\phi' \alpha'}{\phi} = 8\pi\phi^{-1}\rho \exp[2\alpha + 2\beta + 2\gamma + 2\delta]$$

$$\phi'' = 0$$

$$8\pi\rho \exp[2\alpha + \beta + \gamma + \delta] = k_5 \text{ where } k_5 = 8\pi k$$

Which further gives an exact solution given by

$$A = e^\alpha = \exp \left[\frac{k_5 (aT + b)^{k_1 + k_2 + k_3 + a/a}}{(k_1 + k_2 + k_3 + a)^2} \right] (aT + b)^{k_4/a}$$

$$B = e^\beta = (aT + b)^{k_3/a}$$

$$C = e^\gamma = (aT + b)^{k_2/a}$$

$$D = e^\delta = (aT + b)^{k_1/a}$$

$$(24) \quad \phi = aT + b$$

$$(25) \quad 8\pi\rho = -8\pi\lambda = k_5 \left[\exp \left\{ \frac{(-2k_5) (aT + b)^{\frac{k_1 + k_2 + k_3 + a}{a}}}{[k_1 + k_2 + k_3 + a]^2} \right\} \right] [aT + b]^{-[\frac{k_1 + k_2 + k_3 + 2k_4}{a}]}$$

where the constants $a \neq 0$, $b \neq 0$ and $k_i, i = 1, 2, 3, 4$ satisfy the relation

$$(k_1 k_2 + k_2 k_3 + k_3 k_4 + k_4 k_1) + a (k_1 + k_2 + k_3 + k_4) - \left(\frac{wa^2}{2} \right) = 0.$$

Hence five dimensional Bianchi type - I string cosmological model corresponding to the above solution can be written as

$$ds^2 = \exp \left[\frac{2k_5 (aT + b)^{\frac{k_1 + k_2 + k_3 + a}{a}}}{(k_1 + k_2 + k_3 + a)^2} \right] (aT + b)^{\frac{2k_4}{a}} \left[(aT + b)^{2(\frac{k_1 + k_2 + k_3}{a})} dT^2 - dx^2 \right]$$

$$(26) \quad - (aT + b)^{\frac{2k_3}{a}} dy^2 - (aT + b)^{\frac{2k_2}{a}} dz^2 - (aT + b)^{\frac{2k_1}{a}} dw^2$$

where the scalar field in the model is given by equation (24) and the rest energy density, the tension density of the cosmic string is given by equation (25).

4. SOME PHYSICAL PROPERTIES

The model (26) represents an exact five-dimensional Bianchi type - I string cosmological model in the frame work of Brans - Dicke scalar- tensor theory of gravitation when the sum of the rest energy density ρ and the tension density λ of the cosmic strings vanishes . The model has no initial singularities.

The physical quantities that are important in cosmology are proper volume V etc. and have the following expressions for the above model :

$$V = \sqrt{-g} = - (ABCD) = -exp \left[\frac{k_5 (aT + b) \frac{[k_1 + k_2 + k_3 + a]}{a}}{(k_1 + k_2 + k_3 + a)^2} \right] (aT + b)^{\frac{k_1 + k_2 + k_3 + k_4}{a}}$$

which confirms the anisotropic expansion of the above universe with time for $k_5 \geq 0$. In order to obtain more clear and simple picture about the behaviour of the physical and kinematical variables with their simple explicit expressions , we have chosen the constants as $k_1 = k_2 = k_3 = k_4 = a = b = 1, a \neq 0, b \neq 0$. Therefore , the parameters have the following expressions:

$$V = -exp \left[\frac{(T + 1)^4}{16} \right] (T + 1)^4$$

$$\theta = \frac{1}{3} \quad u_{;i}^i = \frac{1}{3} exp \left[\frac{-(T + 1)^4}{16} \right] \left[\frac{1}{4(T + 1)} + \frac{4}{(T + 1)^5} \right]$$

$$8\pi\rho = -8\pi\lambda = exp \left[\frac{-2(T + 1)^4}{16} \right] (T + 1)^{-5}$$

$$8\pi\rho_p = 2exp \left[\frac{-2(T + 1)^4}{16} \right] (T + 1)^{-5} .$$

The scalar field ϕ will be

$$\phi = T + 1.$$

The energy condition indicates that $\rho \geq 0$ and $\rho_p \geq 0$. The rest energy density ρ , the string tension density λ and particle density ρ_p tend to zero as time T increases indefinitely. The spatial volume tends to infinity whereas the expansion scalar θ tend to zero as $T \rightarrow \infty$.

The model (26) can be considered as an inflationary model of string cosmology in Brans-Dicke theory of gravitation. In equation (25) , for $k_5 = 0$, we get $\rho = \lambda = 0$. and the model (26) degenerates into the singularity free Bianchi type-I vacuum model in Brans-Dicke theory.

5. CONCLUSION

Spatially homogenous and anisotropic Bianchi models play a vital role in the understanding of the early stages of evolution of the universe. In this context, the physical systems such as matter distributions with cosmic strings form an important part of investigations. In this paper, we have considered the space-time geometry corresponding to Bianchi type - I model in Brans - Dicke (1961) theory and have obtained a cosmological model in the presence of cosmic string source with five dimensions. A string cosmological model in Brans-Dicke theory of gravitation has been investigated by Reddy (2003) whose work has been extended and studied in five dimensions. An attempt has been made to retain Reddy's (2003) forms of the various quantities. We have noted that all the results of Reddy (2003) can be obtained from our results by assigning appropriate values to the functions concerned.

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