

CHAOS SYNCHRONIZATION

ÖMÜR UMUT, ‡SWARUP PORIA, ‡RAJAT PATRA

†DEPARTMENT OF MATHEMATICS, FACULTY OF ARTS & SCIENCES
ABANT IZZET BAYSAL UNIVERSITY BOLU, TURKEY

‡DEPARTMENT OF MATHEMATICS, MIDNAPORE COLLEGE
MIDNAPORE (WEST), WEST BENGAL, INDIA
E-MAIL:UMUT_O@IBU.EDU.TR, SWARUP_P@YAHOO.COM

(Received: 2 June 2006)

ABSTRACT. In this paper, we use nonlinear control method for synchronization of two identical chaotic generalized Lotka-Volterra system. The determination of the controller is based on the Lyapunov stability theory. Numerical simulation results are shown for demonstration.

AMS Classification: 65P20

Keywords: generalized Lotka-Volterra system, nonlinear control.

1. INTRODUCTION

Historically, the analysis of synchronization phenomena in the evolution of dynamical systems has been a subject of active investigation since the earlier days of physics. It was started in the 17th century with the finding of Huggens that two very weakly coupled pendulum clocks become synchronize in phase. Other early found examples of synchronization are the synchronized lightning of fireflies, or the peculiarities of adjacent organ pipes which can almost reduce one another to silence or speak in absolute unison [1]. The existence of chaos in natural and manmade systems is a well establishes fact. Since Pecora and Carroll introduced a method [3] to synchronize two identical chaotic systems with different initial conditions, chaos synchronization become very important topic in the nonlinear science. Over the last decade, chaos synchronization has received increasing attention due to its potential application in many areas such as secure communication, information processing, biological systems, chemical reactions and neural networks. There exist a large variety of approaches for chaos synchronization such as the feedback method [4], backstepping design technique [7], adaptive control approach [8], generalized synchronization [5] and many others. Huang etal. [2] proposed the method

of chaos synchronization via nonlinear control. In this letter, we use nonlinear control method to synchronize two identical generalized Lotka-Volterra systems and determine the controller based on Lyapunov stability theory.

2. SYNCHRONIZATION VIA NONLINEAR CONTROL METHOD

Let us consider the following drive system described by

$$(2.1) \quad \dot{\mathbf{x}} = A\mathbf{x} + Bf(\mathbf{x})$$

where $\mathbf{x} \in R^n$ is the state vector, $A \in R^{n \times n}$, $B \in R^n$ are matrices and vectors of system parameters, and $f : R^n \rightarrow R^n$ a nonlinear function.

Let the controlled response system is given by

$$(2.2) \quad \dot{\mathbf{y}} = A_1\mathbf{y} + B_1g(\mathbf{y}) + U$$

where $\mathbf{y} \in R^n$ denotes the state vector of the response system, $A_1 \in R^{n \times n}$, $B_1 \in R^n$ are matrices and vectors of this controlled response system parameters, $U \in R^n$ is an additive control, and $g : R^n \rightarrow R^n$ is a nonlinear function.

The problem of synchronization is to design a controller U which synchronizes the states of both the drive and response systems.

Let us define $e = y - x$. Now subtracting (1) from (2) we obtain

$$(2.3) \quad \dot{e} = A_1y + B_1g(y) - Ax - Bf(x) + U$$

The drive and response systems are synchronize if $\lim_{t \rightarrow \infty} e \rightarrow 0$. Now our aim is to choose U such that we can find a Lyapunov function i.e. a continuously differentiable real valued function $V(e)$ with the following properties : (i) $V(e) > 0$ for all $e \neq e^*$ and $V(e^*) = 0$ where e^* is a fixed point of the system (3). (ii) $V(e) < 0$ for all $e \neq e^*$. Then e^* is globally asymptotically stable, for all initial conditions, $e(t) \rightarrow e^*$ as $t \rightarrow \infty$ and the states of drive and response system are synchronized asymptotically globally.

3. SYNCHRONIZATION OF TWO IDENTICAL LOTKA-VOLTERRA SYSTEMS

In 1988, Samardzija et al. [6] proposed a two-predator one prey generalization of the Lotka-Volterra equations, :

$$(3.1) \quad \begin{aligned} \frac{dx}{dt} &= x - xy + Cx^2 - Azx^2 \\ \frac{dy}{dt} &= -y + xy \\ \frac{dz}{dt} &= -Bz + Azx^2 \end{aligned}$$

where A, B and C are three positive parameters.

The system (4) is taken as drive system and the controlled response system is given by

$$(3.2) \quad \begin{aligned} \frac{dx_1}{dt} &= x_1 - x_1 y_1 + C x_1^2 - A z_1 x_1^2 + u_1 \\ \frac{dy_1}{dt} &= -y_1 + x_1 y_1 + u_2 \\ \frac{dz_1}{dt} &= -B z_1 + A z_1 x_1^2 + u_3 \end{aligned}$$

where $U = [u_1, u_2, u_3]^T$ is the controller to be designed.

We define $e_1 = x_1 - x$, $e_2 = y_1 - y$, $e_3 = z_1 - z$

Now from (4) and (5) we obtain

$$(3.3) \quad \begin{aligned} \frac{de_1}{dt} &= e_1 - x_1 y_1 + xy + C x_1^2 - C x^2 - A z_1 x_1^2 + A z x^2 + u_1 \\ \frac{de_2}{dt} &= e_2 + x_1 y_1 - xy + u_2 \\ \frac{de_3}{dt} &= -B e_3 + A z_1 x_1^2 - A z x^2 + u_3 \end{aligned}$$

According to Lyapunov stability theory, when controller satisfies the assumption with $V(e) = \frac{1}{2} e^T e$, a positive definite function and the first derivative of $V(e)$, $\dot{V}(e) < 0$, the two identical Lotka Volterra systems from different initial conditions will synchronize. We consider a Lyapunov function for system (6) as

$$V(e) = \frac{1}{2} e^T e,$$

then,

$$\begin{aligned} \dot{V}(e) &= e_1^2 - e_1 x_1 y_1 + e_1 xy + C e_1 x_1^2 - C e_1 x^2 - A e_1 z_1 x_1^2 + \\ &\quad A e_1 z x^2 + e_1 u_1 - e_2^2 + \\ &\quad e_2 x_1 y_1 - e_2 xy + e_2 u_2 - B e_3^2 + A e_3 z_1 x_1^2 - A e_3 z x^2 + e_3 u_3. \end{aligned}$$

If we choose the control variables such that

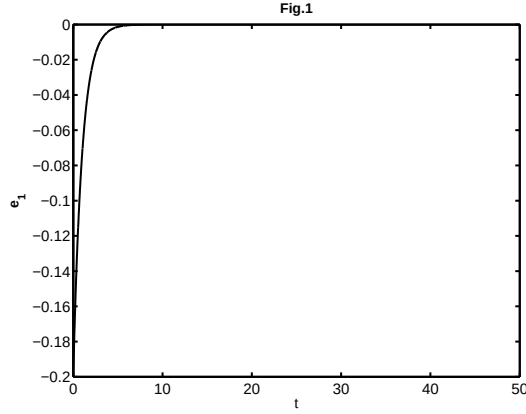
$$(3.4) \quad \begin{aligned} u_1 &= -2e_1 + x_1 y_1 - xy - C x_1^2 + C x^2 + A z_1 x_1^2 - A z x^2 \\ u_2 &= xy - x_1 y_1 \\ u_3 &= A z x^2 - A z_1 x_1^2. \end{aligned}$$

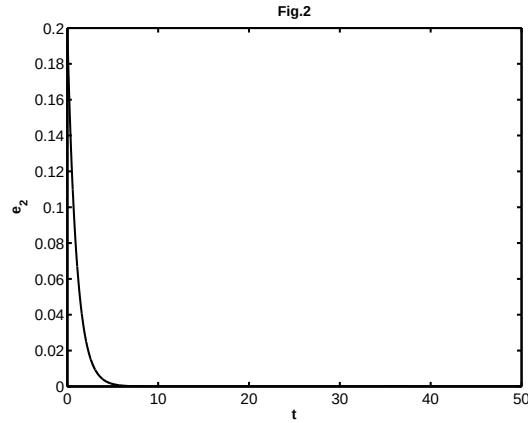
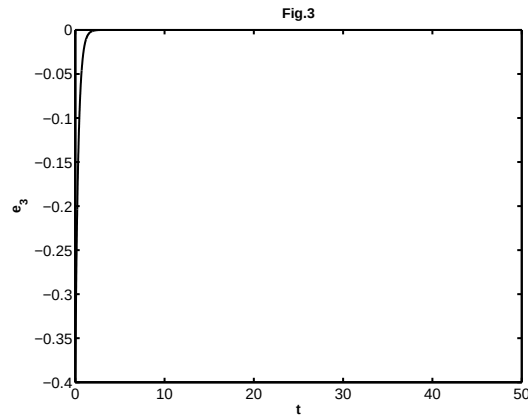
Therefore with these choice of control variables $V(e)$ is positive definite and $\dot{V}(e)$ is negative definite in the neighborhood of the zero solution for the system (6). Therefore the states x_1, y_1 and z_1 of the response system (5) and the states x, y and z of drive system (4) are globally asymptotically synchronized.

4. RESULTS AND DISCUSSIONS

We present here three simulation results. The parameters of generalized Lotka-Volterra system are chosen as follows : $A = 2.98518$, $B = 3.0$ $C = 2.0$ The generalized Lotka-Volterra system behave chaotically under these values of A, B and C . The fourth order Runge- Kutta method with step size .005 is used for solving the coupled driving and driven dynamical system. The initial conditions for the driving system and driven system are given respectively by, $(x(0), y(0), z(0)) = (.1, -0.1, 0.2)$ and $(x_1(0), \bar{y}_1(0), \bar{z}_1(0)) = (-0.01, 0.1, 0.2)$.

Fig.1, Fig.2 and Fig.3 illustrate the time evolution of $e_1 = x_1 - x$, $e_2 = y_1 - y$ and $e_3 = z_1 - z$ respectively. We observe that the synchronization error converges to zero and two chaotic generalized Lotka-Volterra systems are synchronized. In this paper we take two identical chaotic systems for synchronization but this method is equally useful for control and synchronization of two different chaotic systems. The controller is determined using Lyapunov stability theory. Since the Lyapunov exponents are not required for the calculation , this method is very useful for hyperchaotic systems.

FIGURE 1. Time evolution of e_1

FIGURE 2. Time evolution of e_2 FIGURE 3. Time evolution of e_3

REFERENCES

- [1] Boccaletti, S. et al. The synchronization of Chaotic system, *Physics Reports* **366**(2002), pp.1-101.
- [2] Huang, L., Feng, R. and Wang, M. Synchronization of chaotic systems via nonlinear control, *Physics Letters A* **320**(2004)(11),pp.271.
- [3] Pecora LM, Carroll TL. Synchronization in chaotic system, *Phys Rev Lett* **64**(1990),pp.821-824.
- [4] Rulkov, N.F. and Tsimring, L.S. Tracking unstable orbits in chaos using dissipative feedback control, *Physical Review E* **450**(1994),pp.1-.
- [5] Rulkov, N.F., Suschik, M.M. and Tsimring, L.S. Generalized synchronization of chaos in directionally coupled chaotic systems, *Phys Rev E* **51**(1995),pp.980-994.
- [6] Samardzija, N. and Larry D. G. Explosive route to chaos through a fractal torus in a generalized Lotka-Volterra model, *Bulletin of Mathematical Biology* **50**(1988)(5),pp.465-491.

- [7] Wang, C. and Ge, S.S. Adaptive synchronization of uncertain chaotic systems via backstepping design *Chaos, Solitons & Fractals* **12**(2001),pp.1119.
- [8] Yassen, M.T. Adaptive control and synchronization of a modified Chua's circuit system, *Appl. Math. Comput.* **135**(2001),pp.113.