ATTRACTORS FOR RELATIVE SEMI-DYNAMICAL SYSTEMS

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Abstract. In this paper a model for the concept of bornology from the point of view of an observer is presented. By the use of relative bornological spaces, the notion of attractor for relative semi-dynamical systems is considered.

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1. Basic Notions

Relative semi-dynamical systems as a mean for considering dynamics from the point of view of an observer considered in [1].
In this paper we would like to consider attractors for relative semi-dynamical systems. For this goal we need some basic notions. We assume that X is a non-empty set and an observer is a fuzzy set $\mu : X \to [0, 1]$. A topology from the point of view μ, is a collection τ_μ of subsets of μ such that [2]:
i) $\mu, \chi_\emptyset \in \tau_\mu$, where χ is the characteristic function;
ii) $\lambda_1 \cap \lambda_2 \in \tau_\mu$, where $\lambda_1, \lambda_2 \in \tau_\mu$;
iii) $\bigcup_{i \in \Gamma} \lambda_i \in \tau_\mu$ when $\{\lambda_i \ : \ i \in \Gamma\} \subseteq \tau_\mu$.
A relative semi-dynamical system [1] is a (μ, μ)- fuzzy continuous map $f : X \to X$, i.e. $f^{-1}(\eta) \cap \mu \in \tau_\mu$ for all $\eta \in \tau_\mu$, where $f^{-1}(\eta) : X \to [0, 1]$ defined by $f^{-1}(\eta)(x) = \eta(f(x))$.
Now let us to introduce a relative description of the notion of bounded sets.
Definition 1.1 A relative-bornology on X is a set $\beta = \{\lambda_\gamma \in [0, 1]^X \ : \ \gamma \in \Gamma \ \& \ \lambda_\gamma \subseteq \mu\}$ such that:

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i) $\bigcup_{\gamma\in\Gamma} \lambda_\gamma = \mu$;
ii) β is closed under finite union;
iii) If $\eta \in \beta$, and $\gamma \subseteq \eta$ then $\gamma \in \beta$.
An element of β is called a relative, bounded set.
Example 1.1 Let $X = [-1, 1]$, and $\mu(x) = \sqrt{1-x^2}$ moreover let for $y \in [-1, 1]$ $\lambda_y : [-1, 1] \to [0, 1]$ defined by $\lambda_y(x) = \begin{cases} \mu(x) & \text{if } x = y \\ 0 & \text{if } x \neq y \end{cases}$. Then $\beta = \{\gamma \subseteq \lambda_y : y \in [-1, 1]\}$ is a relative bornology.
A relative bornology β is compatible with the relative-topology τ_μ if for all $\eta \in \beta$, there exists $\gamma \in \tau_\mu \cap \beta$ such that $\eta \subseteq \gamma$. In this case β is called a relative topological bornology and elements of $\beta \cap \tau_\mu$ are called relative bounded open sets and (X, τ_μ, β) is called a relative bornological space.
In the rest of this paper we assume that all relative bornologies are topological bornologies.
Theorem 1.1 Let (X, τ_μ, β) be a relative bornological space and let $f : X \to X$ be a one to one relative semi-dynamical system. Then $(X, \tau_\mu, f^{-1}(\beta))$ is a relative bornological space, where $f^{-1}(\beta) = \{f^{-1}(\gamma) : \gamma \in \beta\}$.
Proof. Since $\mu = \bigcup_{\eta\in beta} \eta$ then $\mu = f^{-1}(\mu) = \bigcup_{\eta\in\beta} f^{-1}(\eta)\cdot$

If $\{f^{-1}(\beta_i) : i = 1, \cdots, n\} \subseteq f^{-1}(\beta)$ then $\bigcup_{i=1}^{n} f^{-1}(\beta_i) = f^{-1}(\bigcup_{i=1}^{n} \beta_i) \in f^{-1}(\beta)\cdot$.
If $f^{-1}(\eta) \in f^{-1}(\beta)$ and $\gamma \subseteq f^{-1}(\eta)$ then $\gamma o f^{-1} \subseteq \eta$. So $\gamma o f^{-1} \in \beta$. Hence $\gamma = f^{-1}(\gamma o f^{-1}) \in f^{-1}(\beta)$.
Now if $\eta \in f^{-1}(\beta)$, then $\eta o f^{-1} \in \beta$. Thus there exists $\gamma \in \eta \cap \tau_\mu$, such that $\eta o f^{-1} \subseteq \gamma$. Hence $\eta \subseteq f^{-1}(\gamma)$ and $f^{-1}(\gamma) \in \tau_\mu \cap f^{-1}(\beta) \cdot \square$

2. Attractor

Let (X, τ_μ, β) be a relative bornological space and let (λ_n) be a sequence of subsets of μ. The relative upper bound $ls\lambda_n$ is a fuzzy set which has the following property.
For every $\eta \in \beta \cap \tau_\mu$ with $\eta \cap ls\lambda_n \neq$
$chi\emptyset$ there is $n_o \in N$ such that $\eta \cap \lambda_n \neq \chi_\emptyset$ for infinitely $n > no$.
The relative lower bound $li\lambda_n$ is a fuzzy set such that for all $\eta \in \beta \cap \tau_\mu$ with $\eta \cap li\lambda_n \neq \chi_\emptyset$, there exists $no \in N$ such that $\eta \cap \lambda_n \neq \chi_\emptyset$ for all $n > no$.
If $li\lambda_n = ls\lambda_n$, then this common fuzzy set is called the relative topological limit of λ_n, and denoted by $lt\lambda_n$.
Theorem 2.1 Let (X, τ_μ, β) be a relative bornological space and let the relative upper bound and the relative lower bound of the sequence λ_n exists. Moreover let f be a one to one relative semi-dynamical system on X. Then in $(X, \tau_\mu, f^{-1}(\beta))$ we have $lsf^{-1}(\lambda_n) = f^{-1}(ls\lambda_n)$ and $lif^{-1}(\lambda_n) = f^{-1}(li\lambda_n)$.
Proof. Let $f^{-1}(\eta) \in f^{-1}(\beta) \cap \tau_{mu}$ be given such that $f^{-1}(\eta) \cap f^{-1}(ls\lambda_n) \neq \chi_\emptyset$. So there exists $n_o \in N$ such that $\eta \cap \lambda_n \neq \chi_\emptyset$ for infinitely $n > n_o$. Hence $f^{-1}(\eta) \cap f^{-1}(ls\lambda_n) = f^{-1}(\eta \cap ls\lambda_n) \neq f^{-1}(\chi_\emptyset) = \chi_\emptyset$, and for infinitely $n > n_o$ we

have $f^{-1}(\eta) \cap f^{-1}(\lambda_n) = f^{-1}(\eta \cap \lambda_n) \neq \chi_\emptyset$.
This means that $lsf^{-1}(\lambda_n) = f^{-1}(ls\lambda_n)$.
The similar calculation shows that $lif^{-1}(\lambda_n) = f^{-1}(li\lambda_n) \cdot \square$

Definition 2.1 A fuzzy subset ρ of μ is called an attractor for the relative semi-dynamical system $f : (X, \tau_\mu, \beta) \to (X, \tau_\mu, \beta)$ if for all $\chi_\emptyset \neq \eta \in \beta$, $ltf^n(\eta) = \rho$.

Example 2.1 Let $X = R,\ \mu(x) = e^{-x^2}$ and $\tau_\mu = \{\lambda\ :\ \lambda \subseteq \mu\}$. Moreover let $\beta = \{\lambda \subseteq \mu :$ there is $n \in N$ such that $\lambda(x) = 0$ if $x \notin [-n, n]\}$, and $f : R \to R$ defined by $f(x) = \frac{1}{2}(x-1)$. Then $\rho : R \to [0, 1]$ defined by $\rho(x) = \begin{cases} \frac{1}{e} & \text{if } x = 1 \\ 0 & \text{otherwise} \end{cases}$ is an attractor for f.

3. Persistence of Attractors Under Relative Conjugate Relations

Let $X, \tau_\mu, \beta)$ be a relative bornological space and let $f : X \to X$ and $g : X \to X$ be two relative semi-dynamical systems. Then f and g are called μ-conjugate [1] if there exists a μ-homeomorphism $h = X \to X$ such that $hof = goh$.

Theorem 3.1 let f and g be two μ-conjugate relative semi-dynamical systems on a relative bornological space (X, τ_μ, β). Moreover let ρ be an attractor for f. Then $h(\rho) = \rho oh^{-1}$ is an attractor for g in the relative bornological space $(X, \tau_\mu, h(\beta))$.

Proof. Let $h(\eta) \in h(\beta) \cap \tau_\mu$ be given such that $h(\beta) \cap h(\rho) \neq \chi_\emptyset$. Then $\eta \cap \rho \neq \chi_\emptyset$. So for all $\gamma \in \beta$ there exists $n_0 \in N$ such that $\beta \cap f^n(\gamma) \neq \chi_\emptyset$ for infinitely (all) $n > n_0$. Thus $\eta \cap h^{-1}og^n oh(\gamma) \neq \chi_\emptyset$ for infinitely (all) $n > n_o$. Hence $h(\eta) \cap g^n(h(\gamma)) \neq \chi_\emptyset$ for infinitely (all) $n > n_o$. Thus $h(\rho)$ is an attractor for g in the relative bornological space $(X, \tau_\mu, h(\beta)) \cdot \square$

4. Conclusion

The notion of repellor can define similarly by the use of relative bornological spaces. In fact a fuzzy subset $\delta \subseteq \mu$ is called a repellor for a relative semi-dynamical system $f : (X, \tau_\mu, \beta) \to (X, \tau_\mu, \beta)$ if for all $\chi_\emptyset \neq \eta \in \beta, ltf^{-n}(\eta) = \delta\cdot$. The consideration of repellor for relative semi-dynamical systems will be a new research topic.

We bring this paper to end by posing the following question.

Question. Is a fuzzy set an attractor for a relative semi-dynamical system?

References

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