

MATHEMATICAL MODELLING OF OBSERVER IN PHYSICAL SYSTEMS

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ABSTRACT. In this paper a mathematical model for a one-dimensional observer by the use of fuzzy sets is studied. A realistic approach to the problem of unity by using of relative metrics on relative manifolds is presented. A driven system of Chua's diode contains an observer as a relative vector field is considered.

Key Words: Chua's diode, Problem of Unity, Relative manifold, Relative vector field, Relative Topology

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1. INTRODUCTION

There exists an ancient unsolved problem in mathematical physics and differential geometry called problem of unity and that is:
is there any unique mathematical model to describe the four essential forces of nature?

An equivalent problem in differential geometry is: is there any method to construct a metric with the signature different with the dimension of a manifold by using of a positive definite metric?

There is a physical fact which can be considered parallel to this problem and that is: in the level of electroweak force [6] we do not need to pay attention to gravity [2] and in the level of gravity force, electroweak is approximately zero. It means for the mathematical modelling of this physical fact the role of observer is very important. Observer in this level is an evaluation function or a fuzzy function [9]. Mathematically a one dimensional observer of the set M is a function $\mu : X \rightarrow [0, 1]$ [3, 7]. The other point is: the mathematical structures of the space depends to observer. In this paper we have an overview on the new results created by the

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observer such as relative topology, relative manifold, relative vector field and relative metric. At the end a new approach to the problem of unity is presented.

2. RELATIVE STRUCTURES

Let μ be an observer of M . Then a μ -topology (or relative topology) [7] is a collection τ_μ of subsets of μ with the following conditions:

- i) $\mu, \chi_\emptyset \in \tau_\mu$, where $\chi_\emptyset(x) = 0$ for all $x \in M$;
- ii) $\lambda_1 \cap \lambda_2 \in \tau_\mu$ whenever $\lambda_1, \lambda_2 \in \tau_\mu$ and $(\bigcap_{i \in I} \lambda_i)(x) = \bigwedge_{i \in I} \lambda_i(x)$ for all $x \in M$;
- iii) $\bigcup_{i \in I} \lambda_i \in \tau_\mu$ whenever $\lambda_i \in \tau_\mu$ and $(\bigcup_{i \in I} \lambda_i)(x) = \bigvee_{i \in I} \lambda_i(x)$ for all $x \in M$.

Elements of τ_μ are called μ -open sets.

Example 2.1 If M is the three dimensional torus $\mathbf{R}^3/\mathbf{Z}^3 = \{[a, b, c] : a, b, c \in (0, 1)\}$, and $\mu([a, b, c]) = a$, then $\tau_\mu = \{\lambda_i : \lambda_i([a, b, c]) = ia, 0 \leq i \leq 1\}$ is a μ -topology.

A relative topology can define different topologies for M . In fact for given $a, b \in \mathbf{R}$ we can associate a topology for M generated by $\{\lambda^{-1}((a, b)), M : \lambda \in \tau_\mu\}$. We denote this topology by $(\tau_\mu)_a^b$.

We assume that C is the set $\{U : U = U_0 \cap \mu^{-1}((a, b)) \text{ for some } a, b \in \mathbf{R} \text{ and } U_0 \in C_0\}$ where C_0 is a subset of $\bigcup_{a, b \in \mathbf{R}} (\tau_\mu)_a^b$. For a fixed natural number n we

assume that D is the set $\{(U, h) : U \in C \text{ and } h \text{ be a one to one mapping on } U \text{ such that } h(U) \text{ is a } C^r \text{ immersed submanifold of } \mathbf{R}^{2n+1}\}$. D is called a C^r μ -structure [5] if:

- i) $\mu(\bigcup_{U \in C} U) = \mu(M)$;
- ii) If $(U, h) \in D$, and $\mu(U) = \mu(V)$, then there exist a one to one and onto mapping $\psi_0 : U \rightarrow V$ and a C^r diffeomorphism $\psi : h(U) \rightarrow f(V)$ such that $f \circ \psi_0 = \psi \circ h$ and the restriction of ψ_0 on $U \cap V$ is the identity mapping.

In this case (M, D) is called a C^r μ -manifold.

If (M, D_1) and (M, D_2) are two C^r μ -manifold and γ -manifold respectively, then we say that $D_1 \leq D_2$ if there is a one to one mapping ϕ from $\bigcup_{U \in C_1} U$ to $\bigcup_{V \in C_2} V$

such that, $(U, h) \in D_1$ implies that $(\phi(U), h \circ \phi^{-1}) \in D_2$ and $\mu(U) = \gamma(\phi(U))$. Two C^r μ -structure D_1 and γ -structure D_2 on M are called equivalent if $D_1 \leq D_2$ and $D_2 \leq D_1$.

Theorem 2.1 [5] If M is a C^r μ -manifold, then there exists a C^r μ -structure A such that if D is another C^r μ -structure with $A \leq D$, then D is equivalent to A .

With assumptions and notations of Theorem 2.1 (M, A, μ) is called a C^r relative manifold if $\mu(M - \bigcup_{(U, h) \in A} U) = \{0\}$.

The C^r μ -structure A is called a C^r μ -atlas.

For a given $p \in M$ an equivalence relation $\sim$ on $A^p = \{(U, h) : (U, h) \in A \text{ \& } p \in U\}$ defined by: $(U, h) \sim (V, f)$ if $f(U \cap V)$ is a submanifold of $f(V)$ of codimension 0, and $h(U \cap V)$ is a submanifold of $h(U)$ of codimension 0. We denote the quotient space of A^p to this equivalence relation by B_0^p . By the use of a choice function we can chose an element of each class and we denote the set of these elements by B^p .

Since our approach is a relative approach, we only consider the subset $\mu^{-1}((0, 1])$ of M . So for simplicity we assume that M is equal to $\mu^{-1}((0, 1])$. In other words $M = \bigcup_{(U,h) \in A} U$.

A mapping f from M to $\bigcup_{p \in M} \left(\prod_{(U,h) \in B^p} \mathbf{R} \right)$ is called a C^r mapping if $f(p) \in \prod_{(U,h) \in B^p} \mathbf{R}$

and $\pi_h \circ f \circ h^{-1}$ is a C^r function, where $(U, h) \in B^p$ and π_h is the projection mapping on the (U, h) component.

We denote the set of these mappings by $G^r(M)$. Straightforward calculation shows that $G^r(M)$ with the multiplication and addition of functions in components is a vector space on $\mathbf{R}$.

A mapping X from $G^{r+1}(M)$ to $G^r(M)$ is called a relative C^r vector field with observer μ [4] if:

- i) $X(fg) = X(f)g + fX(g)$ for $f, g \in G^{r+1}(M)$;
- ii) $X(af + g) = aX(f) + X(g)$, where $a \in \mathbf{R}$ and $f, g \in G^{r+1}(M)$.

Theorem 2.2 [4] Let $F^{r+1}(M)$ be the set of C^{r+1} functions on M , and let X be a C^r vector field on M . Moreover let $X_p : F^{r+1}(M) \rightarrow \prod_{(U,h) \in B^p} \mathbf{R}$ be defined by

$$X_p(f) = X \left(\prod_{(U,h) \in B^p} f(p) \right), \text{ where } p \in M. \text{ Then } X_p \in T_p^\mu(M).$$

Theorem 2.3 [4] X is a C^r relative vector field on U_p if and only if there exist C^r vector fields X^U defined on U , such that $X = \prod_{(U,h) \in B^p} X^U$.

The previous theorem implies that if X is a C^r relative vector field on M , then the restriction of X on U_p is equal to $\prod_{(U,h) \in B^p} \left(\sum_i x_U^i \frac{\partial}{\partial h_U^i} \right)$, where x_U^i are C^r coordinate functions defined on U .

3. AN EXAMPLE OF RELATIVE VECTOR FIELDS

In this section we construct a relative vector field by using of Chua's diode [1, 8]. The Chua's diode is the system:

$$\dot{X} = AX + \Psi(X)$$

where

$$X = \begin{pmatrix} v_1 \\ v_2 \\ i_3 \end{pmatrix}, \quad A = \begin{pmatrix} -\frac{G}{C_1} & \frac{G}{C_1} & 0 \\ \frac{G}{C_2} & -\frac{G}{C_2} & \frac{1}{C_1} \\ 0 & -\frac{1}{L} & -\frac{R_0}{L} \end{pmatrix}, \quad \Psi(X) = \begin{pmatrix} -\frac{1}{C_1} f(v_1) \\ 0 \\ 0 \end{pmatrix}$$

and $f(\cdot)$ is the piecewise-linear characteristics. In this system C_1 and C_2 are two linear capacitors, R and R_0 are two linear resistors, and L is a linear inductor. We consider the driven system

$$\dot{Y} = AY + \Lambda \Psi(X)$$

where

$$\Lambda = \mu(x)A + \mu(g(x))\Lambda_1, \quad \Lambda_1 = \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix}$$

μ is an observer of $\mathbf{R}^3$, and $g : \mathbf{R}^3 \rightarrow \mathbf{R}^3$ is the mapping $g(X) = \Lambda X$. Since the set $H = \{X \in \mathbf{R}^3 : \Lambda X = X\}$ is a meager set in $\mathbf{R}^3$, then there exists a nonempty subset E of $\mathbf{R}^3$ such that $\mathbf{R}^3 - H = E \cup g(E)$ and $E \cap g(E) = \emptyset$. Now let $M = E \cup g(E)$ and the observer of M be the restriction of μ to M . Then the driven system is a relative vector field up to any μ -structure of M .

4. A NEW APPROACH TO THE PROBLEM OF UNITY

In this section we assume that, B^p is a countable set. In this case a function g^p from $T_p^\mu(M) \times T_p^\mu(M)$ to $\mathbf{R} \cup \{-\infty, \infty\}$ is called a μ -metric if there exist metrics g_U^p on $T_p(U)$ such that $g^p = \sum_{(U,h) \in B^p} \mu(U)g_U^p$.

Relative manifolds can be a suitable model for approaching to the problem of unity. In fact let T be an n -dimensional manifold with the positive definite metric $diag(1, 1, 1, \dots, 1)$. Let $M = \prod_{i \in N} T$, where N is the set of natural numbers, and

$\mu(x) = \frac{1}{i}$ if $x \in T_i$. Then M is a μ -manifold where its differentiable structure is borrowed from differentiable structure of T , by using of intersection of characteristic functions of its open sets with μ . Moreover let $g^0 = \sum_{i \in N} \frac{1}{i} g_i^0$ where g_i^0 is the metric

$diag(-1.0003 + \frac{i}{100}, i, \dots, i)$ on T_i .

Then when $i \leq 100$ we have Lorentz metric on $T = T_i = \mu^{-1}(\frac{1}{i} - \epsilon, \frac{1}{i} + \epsilon)$ ($\epsilon < \min\{(\frac{1}{i} - \frac{1}{i+1}), (\frac{1}{i-1} - \frac{1}{i})\}$) and when $i > 100$ we have positive definite metric on T_i . This means that for $i \leq 100$ we only pay attention to electroweak without any attention to gravity and for $i > 100$ we only consider gravity. So different observers can present different perspectives of the space.

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