

EXPERIMENTAL EVIDENCE FOR THE NUCLEAR WAVE FUNCTION OF ^{15}N DERIVED WITHOUT ANY APPEAL TO MECHANICS

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ABSTRACT. The nuclear wave function of ^{15}N is derived from an irreducible representation of the Lorentz group without any attempt to describe the dynamics of individual nucleons although the space in which they move may be determined by geometry. This wave function in the Figure shows a giant resonance that may be quenched by a very high frequency $\pi/4$ out of phase that will cause the nucleus to freeze. Evidence is provided in the stratosphere where cirrus ice clouds are observed in an environment full of N and cosmic rays but with no aerosols present. Freezing of supercooled water droplets has also been observed in the laboratory.

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1. INTRODUCTION

In this contribution the nuclear wave function of ^{15}N is derived without any appeal whatsoever to mechanics. No attempt is made to understand the nature of nucleons. Only their properties, spin, isospin and parity are considered together with the nuclear shape. However nothing can be said about the position and momentum of individual nucleons in the quadrupole or octupole. This is a collective phenomenon and interactions take place by entanglement [7]. Quarks appear naturally as a consequence of the nuclear geometry and are considered to provide the nuclear binding force via the gluon condensate, but this aspect is dealt with elsewhere as it does not affect the conclusions of this note (cf.[8]).

A hundred years ago Hudson [11] described how the equation of the Kummer surface, that yields a quadrupole, may be found by insisting on Lorentz invariance alone. Or, in other words, by finding irreducible representations (IRs) of the center D of the quaternion or Dirac ring. This is equation (1) that is the core of this paper. Actually the idea of associating a particle with such representations goes back to Dirac [9], Eddington [10], Biedenharn ([2], Ch.4) and Wigner [13]. In fact

Kahan [12] states in his preface that 'Particles may be said to be representations of the Lorentz group, just as the electron may be said to be the equation of Dirac'. An IR of the center D lies in a Kaehler manifold or string space if the atomic number A is odd because of equation (19). Furthermore this representation has a Hodge decomposition arising from the tensor product of (1) with itself that will be discussed in the next section. The decomposition for ^{15}N is equation (14).

In the last section the wave function is calculated by exponentiation after a fibration that factors all those states that have the same quantum numbers of spin, charge and parity thus introducing a gauge. This is equation (23) that is plotted in the accompanying Figure. There is a giant resonance that may be quenched by a frequency $\pi/4$ as shown in the lower half. Such a frequency could be provided by cosmic rays in the stratosphere where there is plenty of ^{15}N and evidence of freezing shown by cirrus ice clouds whose origin is not properly understood [3]. More direct experimental evidence appears in Ref. [1] where there are photographs of cosmic rays passing through a cloud chamber filled with super-cooled water droplets that leave a trail of ice.

2. THE TENSOR PRODUCT

The irreducible representation, or one-form

$$(1) \quad \frac{1}{4}\Psi = (iE_4\psi_1 + E_{23}\psi_2 + E_{14}\psi_3 + E_{05}\psi_4)e$$

of the center D of the Dirac ring was found long ago in [4] and is actually a minimum left ideal or density matrix because Ψ is idempotent (cf.[2], section 7.7) e being a primitive idempotent. Here Eddington's E-numbers are related to the Dirac matrices by

$$(2) \quad \gamma_\nu = iE_{0\nu}, E_{\mu\nu} = E_{\rho\mu}E_{\rho\nu} = -E_{\nu\mu}; E_{\mu\nu}^2 = -1; \mu < \nu = 1, \dots, 5$$

and the commuting operators E_{23} , E_{14} and E_{05} are respectively rotations in 3-space, 4-space and isospace that correspond to the spin σ , parity π and charge T_3 carried by a single nucleon. The parameters ψ_2, ψ_3, ψ_4 are half-angles of rotation and E_4 is the unit 4×4 matrix. To see how E_{14} is related to parity we notice that a rotation through π about t will send x to $-x$ without inverting the time but instead changing to a left-handed coordinate system. Then a many nucleon system is found in the enveloping algebra $A(\gamma)$ of the Dirac ring by constructing tensor products of (1) with itself. The basis elements are the $4^A \times 4^A$ matrices

$$(3) \quad E_{\mu\nu}^l = E_4 \otimes \dots \otimes E_4 \otimes E_{\mu\nu} \otimes E_4 \otimes \dots \otimes E_4$$

with $E_{\mu\nu}$ in the l -th position and the $E_{\mu\nu}^l, E_{\mu\nu}^{l+1}$ commute. The IRs, or minimal left ideals of $A(\gamma)$ are [4]

$$(4) \quad \Psi^{(A)} = \sum C_{[\lambda]} P_{[\lambda]}$$

where $[\lambda] = [\lambda_1 \lambda_2 \lambda_3 \lambda_4]$ labels each row of $\Psi^{(A)}$ and $\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4$ is a partition of the atomic number A where we have adopted the canonical labelling $N = \lambda_1 + \lambda_2, Z = \lambda_3 + \lambda_4$ for the number of neutrons and protons respectively and

$$(5) \quad s = \frac{1}{2}(A - 2(\lambda_2 + \lambda_3)), p = \frac{1}{2}(2(\lambda_2 + \lambda_4) - A)$$

for spins and parities that will be justified later. $P_{[\lambda]}$ is a projection operator of the same form as (1) and the physics is provided by the coefficient

$$(6) \quad C_{[\lambda]} = i^{\lambda_1} \sum (E_{23}^1 \dots E_{23}^{\lambda_2} E_{14}^{\lambda_2+1} \dots E_{14}^{\lambda_2+\lambda_3} E_{05}^{\lambda_2+\lambda_3+1} \dots E_{05}^{A-\lambda_1})$$

which by virtue of the isomorphism between Clifford multiplication and the exterior product is a three-form in the centralizer D.

To find the Hodge decomposition of (6) we write it

$$(7) \quad C_{[\Lambda]} = i^{\Lambda_1} \sigma_0^{\Lambda_2} \pi_0^{\Lambda_3} T_0^{\Lambda_4} - \sum_{\lambda} i^{\lambda_1} \sigma_0^{\lambda_2} \pi_0^{\lambda_3} T_0^{\lambda_4}$$

where $[\Lambda] = [\Lambda_1 \Lambda_2 \Lambda_3 \Lambda_4]$ is the ground state and σ_0, π_0, T_0 are the de Broglie operators

$$(8) \quad \sigma_0 = (E_{23}^1 + \dots + E_{23}^A) = 2is, \pi_0 = (E_{14}^1 + \dots + E_{14}^A) \\ = 2ip, T_0 = (E_{05}^1 + \dots + E_{05}^A) = i(Z - N) = 2T_3$$

that underlie (4). The summation contains all of those terms arising from repeated indices, eg. $E_{23}^k E_{23}^k, E_{23}^k E_{14}^k, E_{23}^k E_{05}^k$ and $E_{14}^k E_{05}^k$ that yield a single term by (2) and (5). An elementary example is

$$(9) \quad \sigma_0 T_0 = P(E_{23}^i E_{05}^j) + i\{\pi_0\}$$

where P signifies summation over the $A!/(A-n)!$ permutations in the bracket. Then

$$(10) \quad C_{[A-2]101] = i^{(A-2)} P(E_{23}^i E_{05}^j) = i^{(A-2)} (\sigma_0 T_0 - i\pi_0)$$

and if $A = 3, Z = 1, T_0 = i(A - Z) = -i$ then

$$(11) \quad C_{[1101]} = (\sigma_0 + \pi_0)$$

which characterizes the ground state of H-3. The gs. of He-3 is readily obtained by interchanging $\sigma_0 \Leftrightarrow \pi_0$ in (10) to get

$$(12) \quad C_{[(A-2)011]} = i^{(A-2)} (\sigma_0 T_0 - i\sigma_0)$$

so that when $A = 3, Z = 2, T_0 = i$ we find the mirror nucleus

$$(13) \quad C_{[1101]} = (\sigma_0 - \pi_0)$$

which is manifestly CP-invariant because $T_0 \rightarrow T_0$ is accompanied by $\pi_0 \rightarrow -\pi_0$. This process may be continued by adding one nucleon at a time, i.e. by multiplying by σ_0, π_0, T_0 until $\Lambda_2 + \Lambda_3 + \Lambda_4 = A - \Lambda_1$. However the ground state $[\Lambda]$ is unique in that $\Lambda_1 = \Lambda_4 \geq \Lambda_2, \Lambda_3$ otherwise there is no Hodge decomposition. Specifically this

means that all other states with $Z = \lambda_3 + \lambda_4$, $N = \lambda_1 + \lambda_2$ are Zeeman sub-levels of the gs and not so-called excited states arising from the decay products of other nuclei.

In the case of ^{15}N , $[\Lambda] = [4434]$ and after some work it can be shown that the Hodge decomposition with $T_0 = -i$ is

$$(14) \quad C_{[4434]} = \frac{1}{144}((\sigma_0^7 + \pi_0^7) + 4(\sigma_0\pi_0^6 + \sigma_0^6\pi_0) + 12(\sigma_0^5\pi_0^2 + \sigma_0^2\pi_0^5) \\ + 18(\sigma_0^3\pi_0^4 + \sigma_0^4\pi_0^3) + (281(\sigma_0^5 + \pi_0^5) + 842(\sigma_0\pi_0^4 + \sigma_0^4\pi_0) \\ + 1692(\sigma_0^3\pi_0^2 + \sigma_0^2\pi_0^3)) + (20089(\sigma_0^3 + \pi_0^3) \\ + 403729(\sigma_0\pi_0^2 + \sigma_0^2\pi_0)) + (315009(\sigma_0 + \pi_0))$$

from which eigenvalues may readily obtained for any state $[\lambda_1\lambda_2\lambda_3\lambda_4]$ characterized by $\lambda_3 + \lambda_4 = P = 7$ and $\lambda_1 + \lambda_2 = N = 8$ with spins and parities given by (5). These eigenvalues are

$$(15) \quad e = \{85750; 12250(2); 4150(2); 3574; 2650(2); \\ 1750(3); 1110(3); 1050(4); 630(2)\}$$

with the degeneracies shown in brackets and must be normalized before exponentiating by (20). We subtract $630 \times (\text{unitmatrix})$ from -e and divide by 1120 to get

$$(16) \quad A = \{76; 83/8(2); 22/7(2); 92/35; 101/56(2); 1(3); 3/7(3); 3/8(3); 0(2)\}$$

where the ground state $[4434]$ eigenvalue is $92/35$.

The matrices σ_0, π_0 in (14) are $4^A \otimes 4^A$ -dimensional but may be expressed in terms of Lie operators of the infinitesimal rotation group $\text{so}(3)$ by (17) after a fibration that introduces the Yang-Mills field discussed in the next section. The same notation will be used because after the matrix (18) has been written in a bi-symmetric form by interchanging rows and columns the eigenvalues are unchanged which establishes the fibration.

3. THE WAVE FUNCTION OF ^{15}N AND CLOUD PHYSICS

To find the wave function we have exponentiate (14) after expressing it in terms of the matrices

$$(17) \quad \sigma_i = E_N \otimes^P \gamma_i +^N \gamma_i \otimes E_P, \pi_i = E_N \otimes^P \gamma_i -^N \gamma_i \otimes E_P, i = 1, 2, 3$$

where $^P\gamma_i, ^N\gamma_i$ are $(P+1), (N+1)$ -dimensional Lie operators of $\text{so}(3)$ and E_P, E_N are $(P+1)-, (N+1)-$ dimensional unit matrices [5]. The operator σ_i may be recognized as the well-known angular momentum matrix for a coupled system of P protons and N neutrons and π_i is a parity operator. In fact σ_i, π_i are the generators of $\text{SO}(4)$ and are a fibration of the $4^A \times 4^A$ matrix representation if E_{23}, E_{14} consisting of all those states which have the same quantum numbers of spin, parity and charge. There is a beautiful de Rham decomposition into isobaric

multiplets each of which is characterized by (17). In this way a Yang-Mills gauge field is introduced with connections on a fiber bundle.

To proceed further we consider rotations E_{23} about the z- axis and write $2\sigma_1 = \sigma_0$, $2\pi_1 = \pi_0$ in (8) then (14) becomes

$$(18) \quad M = \frac{1}{72}((E_N \otimes (35\gamma_P^7 + 2818\gamma_P^5 + 60461\gamma_P^3 + 315009\gamma_P) \\ + \gamma_N^2 \otimes (15\gamma_P^5 + 1110\gamma_P^3 + 19895\gamma_P) \\ + \gamma_N \otimes (9\gamma^3 + 571\gamma_P) + 5\gamma_N^6 \otimes \gamma_P)$$

where now $\gamma_P = 2\{^P\gamma_1\}$, $\gamma_N = 2\{^N\gamma_1\}$.

We will need to exponentiate an irreducible subspace m of the matrix M which is a complex projective space P^3 characterized by

$$(19) \quad m = \begin{pmatrix} 0 & X \\ -X^T & 0 \end{pmatrix}$$

where X is a real bisymmetric matrix with rows labelled by $[\lambda]_k = [\lambda_1 \lambda_2 \lambda_3 \lambda_4]_k$, and m is a Kaehler manifold. The eigenvalues λ_k of X are precisely those of (16) which justifies the fibration (17) and canonical labelling (5). Then the exponential formula [6] states that with $X=\mu$

$$(20) \quad e^{\mu t} = \mu \sum_{k=0,1,\dots}^n \frac{F_k(\mu) \cos \gamma_k t}{i\gamma_k F(i\gamma_k)} + i \sum_{k=1,2,\dots}^n \frac{F_k(\mu) \sin \gamma_k t}{F_k(i\gamma_k)}$$

where the coupling constants are

$$(21) \quad F(\mu) = \mu(\mu^2 + 1)(\mu^2 + \gamma_2^2) \dots (\mu^2 + \gamma_n^2) = 0, F_0(\mu) \\ = F(\mu)/\{\mu\}, F_k(\mu) = F(\mu)/(\mu^2 + \gamma_k^2), F_j(\mu)F_k(\mu) = 0$$

and

$$(22) \quad K_k(\mu) = \frac{i\gamma(K_k)(F_k)(\mu)}{(\mu)F_k(i\gamma_k)}$$

is idempotent. Equation (22) follows by differentiating (20) at t=0 so that $(\sum)K_k(\mu)$ is a partition of unity. In practice we evaluate (20) at a specific element $[\mu^l]_{jk}$ of μ^l . In this way $exp(\mu t)$ yields rows and columns of wave functions and we select the central element $\mu_{11,11}$ of the bisymmetric matrix as characterizing the gs.

Using (16) we find that the wave function of $l^5 N$ is

$$(23) \quad X1 = X/\sin(9t/16) = \frac{1}{4096}(360 \sin(3t/7) \\ + 1400 \sin(t) + 81 \sin(92t/35) \\ + 400 \sin(22t/7) + 1225 \sin(76t))$$

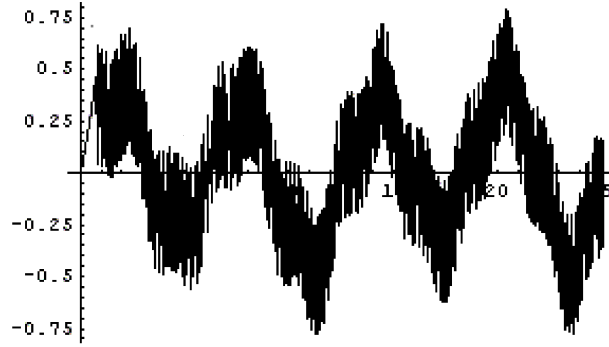


FIGURE 1

Here the oscillations corresponding to the eigenvalues $3/8$; $101/56$; $83/8$ all have zero coefficients and degeneracies have been ignored, but (23) can be checked by (22) when $\mu = \mu_{11,11}$. The factor $\sin(9t/16)$ comes from the translation $630/1120$ of the eigenvalues (15) and introduces a twist but does not affect the conclusions of this paper. Division by 1120 is simply a frequency change.

It is apparent that there is a giant resonance $\sin(76t)$ that accounts for 96.85 % of X1 as shown on the top half of the enclosed Figure. In the lower portion this resonance has been quenched by adding another oscillation $\pi/4$ out of phase. Since 76 is approximately 29 times greater than the gs frequency of $92/35$ this could be in the cosmic ray range

Physically the giant resonance is caused by an extra neutron looking for a home among the decay products $d + 2(^6Li)$ of ^{14}N which calculations show has no giant resonance. It thus vibrates back and forth at cosmic ray frequency between the two 3He nuclei found in the decay $^6Li \rightarrow ^3He + ^3H$. the nucleus ^{16}O decays to ^{15}N which is stable, after a half-life of 122.2 seconds and ^{13}N is also unstable while ^{16}O has rotational symmetry.

Thus ^{15}N is the only candidate for the creation of cirrus clouds by freezing in the lower stratosphere of super-cooled water droplets by cosmic rays that otherwise cannot be accounted for because there are no aerosols at that height [3].. Also there are photographs in Ref.[1] of ice formation by cosmic rays in a cloud chamber filled with super-cooled water droplets. It is even possible that ^{15}N is one of the key building blocks of life on Earth!

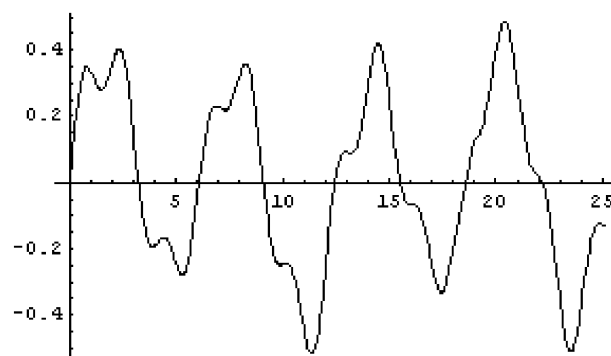


FIGURE 2

4. ACKNOWLEDGEMENT

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