A PLANE SYMMETRIC RADIATING MODEL IN BRANS-DICKE THEORY

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Abstract. Field equations in the scalar-tensor theory of gravitation proposed by Brans and Dicke [Phys.Rev.124, 925, 1961] are obtained with the aid of a non-static plane symmetric metric in the presence of a perfect fluid matter distribution. The field equations being highly non-linear, a cosmological model describing disordered radiation is presented. Some physical and kinematical properties of the model are also studied.

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1. Introduction

In the last few decades, there has been considerable interest in alternative theories of gravitation. The most important among them are scalar-tensor theories proposed by Brans and Dicke (1961) and general scalar - tensor theories. The latest inflationary models (Mathiazhagan and Johri, 1984), extended inflation (La and Steinhardt, 1989 and Steinharadt and Accetta, 1990), hyper extended inflation and extended chaotic inflation (Linde, 1990) are based on Brans-Dicke theory and general scalar-tensor theories. In Brans-Dicke theory a scalar f-field is introduced in addition to preudo-Riemannian metric of the space-time V4 occurring in Einstein general relativity; supposedly, this long range field is generated by the whole of matter in the universe according to Mach's principle (Dicke, 1964). Brans-Dicke field equations are

$$G_{ij} = -8\pi\phi^{-1}T_{ij} - \omega(\phi^{-1})(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k})\phi^{-1}(\phi_{i;j} - g_{ij}\phi^{;k}_{;k}) \tag{1.1}$$

and

$$\phi^{;k}_{;\,k} = \frac{8\pi T}{(3+2\omega)} \tag{1.2}$$

where $G_{ij} = R_{ij} - \frac{1}{2}g_{ij}R$ is the Einstein tensor, ω is a dimensionless coupling constant T_{ij} is the stress energy tensor for matter distribution and comma and semicolon denote partial and covariant differentiation respectively. Equations of motion

$$T^{ij}_{;j} = 0 \tag{1.3}$$

is a consequences of the field equations (1) and (2).

Brans-Dicke cosmology has been extensively studied by several authors. The work of Singh and Rai (1983) gives a detailed discussion of Brans-Dicke cosmological models. It is well known that the universe is spherically symmetric and the matter distribution in it is on the whole isotropic and homogeneous. But, during the early stage of evolution, it is unlikely that it could have had such a smoothed out picture. Hence we consider plane symmetry in Brans-Dicke theory which provides an opportunity for the study of in homogeneity and other cosmological implications of the theory. Reddy (1977), Banerjee and Duttachoudhury (1980), Reddy and Rao (1988) Mohanty and Sahoo (2002) and Mishra (2003) are some of the authors who have investigated plane symmetric models in Brans-Dicke and other alternative theories of gravitation. However, non-static plane symmetric cosmological models involving a perfect fluid have not been studied in Brans-Dicke theory of gravitation. In this paper, we present a non-static plane symmetric cosmological model with disordered radiation in Brans-Dicke theory. To get a deterministic solution of the highly non-linear field equations we have chosen a particular form of the Brans-Dicke scalar field ϕ . We have also discussed the physical and Kinematical properties of the model.

2. Metric and Field Equations

We consider the space-time whose geometry is described by the line element

$$ds^2 = e^{2h}(dt^2 - dr^2 - r^2 d\theta^2 - S^2 dz^2) \tag{2.1}$$

wherer, θ, z are usual cylindrical polar coordinates and S and h are functions of time t only. It is well known that this metric is Riemannian (non-flat) and plane symmetric. The non-static plane symmetry assumed, obviously, implies that the Brans-Dicke scalar field ϕ, also, is a function of t only. Perfect fluid with disordered radiation can be regarded as matter having the energy momentum tensor

$$T^i_j = \rho u^i u_j - \delta^i_j p \tag{2.2}$$

characterized by the equation of state

$$\rho = 3p \tag{2.3}$$

In case find

$$T^i_j = 0 (i \neq j)$$

We use commoving coordinates, so that

$$u^4 u_4 = 0 \tag{2.4}$$

Equation (5) gives

$$T^1_1 = T^2_2 = T^3_3 = -p, \; ; T^4_4 = \rho \tag{2.5}$$

Using the metric (4), the Brans-Dicke field equations (1)-(3) in the presence of perfect fluid with disordered radiation ($\rho = 3p$) can be written as,

$$e^{-2h}\left[\frac{S_{44}}{S} + 2h_{44} + \frac{2S_4h_4}{S} + \frac{\omega}{2}(\frac{\phi_4}{\phi})^4 - \frac{h_4\phi_4}{\phi}\right] = 8\pi\phi^{-1}p \tag{2.6}$$

$$e^{-2h}\left(2h_{44} + h_4^2 - \frac{h_4\phi_4}{\phi} - \frac{S_4\phi_4}{S\phi} + \frac{\omega}{2}(\frac{\phi_4}{\phi})^4\right) = 88\pi\phi^{-1}p \tag{2.7}$$

$$e^{-2h}\left(\frac{2S_4h_4}{S} + 3h_4^2 + \frac{h_4\phi_4}{\phi} - \frac{\omega}{2}(\frac{\phi_4}{\phi})^4 - \frac{\phi_{44}}{\phi}\right) = 8\pi\phi^{-1}(3p) \tag{2.8}$$

$$\phi_{44} + 2h_4\phi_4 + \frac{S_4\phi_4}{S} = 0 \tag{2.9}$$

$$p_4 + \frac{4}{3}\,p\,(3h_4 + \frac{S_4}{S}) = 0 \tag{2.10}$$

where the suffix 4 following an unknown function denotes ordinary differentiation with respect to time t.

3. Solutions and the Model

It may be mentioned here that the field equation (9) - (13), when $\rho = p = o$ and $\phi =$ constant yield the non-static empty space - time in Einstein's theory discussed by Bera (1969). Also, when $\rho = p = o$, the above field equations give the Brans - Dicke non-static plane symmetric vaccum solution obtained by Reddy and Rao (1988). Here we wish to find a solution of the field equations (9) - (13) for the case of disordered radiation in Brans-Dicke theory. It can be seen that the field equations (9) - (13) reduce to the following set of equations.

$$e^{2h}S_4\phi_4 = c_1, \quad e^{2h}S_4\phi = c_2, \quad e^{4h}S^{\frac{4}{3}}p = c_3 \tag{3.1}$$

which will in turn will give us

$$\phi = S^{(\frac{C_1}{C_2})} \tag{3.2}$$

where C_i's are constants of integration. The field equations (14) and (15) being highly non-linear, to get a determinate solution we assume

$$\phi = (at + b), \quad a \neq 0 \tag{3.3}$$

Where a and b are constants. With this assumption the field equations admit a closed form exact solution given by

$$exp^{2h} = \frac{C_1}{a}(at + b)^{-3}, \quad S = (at + b)^3, \quad \rho = 3p = \frac{9a}{C_1}(at + b)^2 \tag{3.4}$$

where the constants of integration are related as

$$(15 + 2\omega) + 96\lambda = 0, \frac{c_2}{c_1} = 3, \quad c_1 \neq 0 \tag{3.5}$$

This solution can be transformed through a proper choice of coordinates and the metric (4) can be written (redefining the constants suitably) in the form.

$$ds^2 = T^{-3}(dT^2 - dr^2 - r^2d\theta^2 - T^3dz^2) \tag{3.6}$$

This represents the non-static plane symmetric radiating cosmological model in Brans - Dicke theory of gravitation. Some

4. physical properties of the Model

The distribution in the model (19) is given by

$$\rho = 3p = 9T^2 \tag{4.1}$$

and the scalar field in the model is

$$\phi = T \tag{4.2}$$

The flow vector u^i of the distribution is given by

$$u^1 = u^2 = u^3 = u_1 = u_2 = u_3 = 0.$$

$$u^4 = T^{-3/2}, \quad u_4 = T^{-3/2} \tag{4.3}$$

The physical and Kinematical parameters for the model (19) are ; Spatial volume

$$V = (-g)^{1/2} = r(T)^{-9/2} \tag{4.4}$$

Expansion scalar

$$\theta = u^i_{;\,i}\frac{1}{3} = -\frac{3}{2}T^{1/2} \tag{4.5}$$

Shear scalar

$$\sigma^2 = \sigma_{ij}\sigma^{ij} = \frac{3}{8}\ T \tag{4.6}$$

The deceleration parameter

$$q = 3\theta^{-2}[\theta_{ij}u^i + \frac{1}{3}\theta^2] = -\frac{2}{3} \tag{4.7}$$

The energy conditions for the model (19), i.e. $\rho > 0, \ \ p > o$ are satisfied and the scalar field, energy density and the pressure in the model have no initial singularities while the model (19) has initial singularities. From equations (23) and (24) it is interesting to note that the universe (19) contracts with time and hence there is a big crunch in this universe. The negative value of the deceleration parameter q shows that the model inflates. However the model does not admit rotation and acceleration.

5. Conclusions

Here we have presented an exact plane symmetric non-static solution in Brans-Dicke theory of gravitation in the presence of perfect fluid distribution with disordered radiation. Our solution, in a way, is a generalization of the solutions obtained by Bera (1969) in Einstein's theory and Reddy and Rao (1988) in the case of Brans - Dicke vacuum. We hope that our model throws some light for a better understanding of the Brans-Dicke cosmology.

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