

NONEXISTENCE OF PLANE SYMMETRIC STRING COSMOLOGICAL MODEL IN BRANS-DICKE THEORY OF GRAVITATION

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ABSTRACT. Field equations in Brans-Dicke scalar-tensor theory of gravitation are obtained with the aid a non-static plane symmetric metric in the presence of massive string source. It is shown that a non-static string model, in this theory, does not exist when the sum of the energy density and the tension density of the cosmic string vanishes.

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1. INTRODUCTION

Brans and Dicke (1961) formulated a scalar-tensor theory of gravitation in which gravity is mediated by a long range scalar field ϕ in addition to the usual metric tensor field g_{ij} present in Einstein's theory. In this theory the scalar field ϕ has the dimension of the universe of the gravitational constant G . The field equations in Brans-Dicke theory are

$$(1) \quad R_{ij} - \frac{1}{2}g_{ij}R = -8\pi\phi^{-1}T_{ij} - \omega(\phi^{-1})(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k})\phi^{-1}(\phi_{;i;j} - g_{ij}\phi^{,k}_{;k})$$

and

$$(2) \quad \phi^{,k}_{;k} = \frac{8\pi T}{(3+2\omega)}$$

where ω is a dimensionless coupling constant T_{ij} is the stress energy tensor for matter distribution and comma and semicolon denote partial and covariant differentiation respectively. Brans-Dicke theory is consistent with local observations in the solar system as long as the coupling parameter ω between the scalar and the

tensor fields is equal to or even greater than 500 (Willis, 1980). In the limit $\omega \rightarrow \infty$ Brans-Dicke theory reduces to Einstein's theory. Also the equation

$$(3) \quad T_{ij}^{ij} = 0$$

is a consequences of the field equations (1) and (2).

Recently, considerable interest has been focused on cosmic string and string cosmological models. Cosmic string are topologically stable objects which might be formed during a phase transitions in the early universe (Kibble, 1976). The gravitational effects of such objects are of particular interest since they are considered as possible 'seeds' for galaxy formation and gravitational lenses (Zeldovich, 1980). The gravitational effects of cosmic strings have been extensively discussed by Vilenkin (1985), Letelier (1983), Gott (1985), Stachel (1980) and Krori et al. (1990). Relativistic string models in the context of Bianchi space-time have been obtained by Krori et al. (1990), Banerjee et al. (1990) while Tikekar et al. (1994) have presented a class of cylindrically symmetric model in string cosmology. Bhattacharjee and Baruah (2001) studied the problem of cosmic string taking Bianchi type cosmologies with a self interacting scalar field. Barros et al. (1997), Gundlach (1990), Barros and Romero (1995) and Sen et al. (1997) are some of the authors who have investigated several aspects of cosmic string Brans-Dicke theory of gravitation. Very recently, Reddy (2003a, b) has obtained Bianchi type-I string cosmological models in Brans-Dicke and Saez-Ballester (1985) Scalar-tensor theories of gravitation when the sum of the energy density and the tension density of the cosmic string source vanishes.

The present investigation is concerned with non-static plane symmetric string cosmological model in Brans-Dicke theory of gravitation. Such investigation in Brans-Dicke theory in the presence of cosmic string source has not been undertaken so far.

2. METRIC AND FIELD EQUATIONS

We consider the space-time whose geometry is described by the line element

$$(4) \quad ds^2 = e^{2h}(dt^2 - dr^2 - r^2 d\theta^2 - S^2 dz^2)$$

where r, θ, z are usual cylindrical polar coordinates and S and h are functions of time t only. It is well known that this metric is Riemannian (non-flat) and plane symmetric. The non-static plane symmetry assumed, obviously, implies that the scalar field, also, is a function of t only. The energy momentum tensor for cosmic string source is

$$(5) \quad T_{ij} = \rho u_i u_j - \lambda X_i X_j$$

where ρ is the rest energy density of cloud of strings with particles attached to them, λ the tension density of the strings, u^i the cloud four-velocity and X^i is the direction of anisotropy. We have

$$u^i u_i = -X^i X_i$$

and

$$(6) \quad u^i X_i = 0$$

We consider

$$\rho = \rho_p + \lambda$$

where ρ_p is the particle density in the cloud. The energy conditions imply $\rho > 0$, leaving the sign of the string tension density λ unrestricted. We take X^i to be along z-axis, so that

$$(7) \quad X^i = (0, 0, e^{-h} S^{-1}, 0)$$

and

$$u^i u_i = (0, 0, 0, 1)$$

Bran-Dicke field equations (1), (2) and (3) for the metric (4) with the help of equations (5), (6) and (7) can be written as

$$(8) \quad \frac{S_{44}}{S} + 2h_{44} + \frac{2S_4 h_4}{S} + \frac{\omega}{2} \left(\frac{\phi_4}{\phi} \right)^4 - \frac{h_4 \phi_4}{\phi} = 0$$

$$(9) \quad e^{-2h} \left(2h_{44} + h_4^2 - \frac{h_4 \phi_4}{\phi} - \frac{S_4 \phi_4}{S \phi} + \frac{\omega}{2} \left(\frac{\phi_4}{\phi} \right)^4 \right) = 8\pi \phi^{-1} \lambda$$

$$(10) \quad e^{-2h} \left(\frac{2S_4 h_4}{S} + 3h_4^2 + \frac{h_4 \phi_4}{\phi} - \frac{\omega}{2} \left(\frac{\phi_4}{\phi} \right)^4 - \frac{\phi_{44}}{\phi} \right) = 8\pi \phi^{-1} \rho$$

$$(11) \quad e^{-2h} \left(\phi_{44} + 2h_4 \phi_4 + \frac{S_4 \phi_4}{S} \right) = \frac{8\pi(\rho + \lambda)}{(3 + 2\omega)}$$

$$(12) \quad \rho_4 + \rho(3h_4 + \frac{S_4}{S}) - \lambda(h_4 + \frac{S_4}{S}) = 0$$

where the suffix 4 following an unknown function denotes ordinary differentiation with respect to time t .

3. SOLUTIONS AND THE MODEL

When $\lambda = \rho = 0$ and $\phi = \text{constant}$, it can be easily verified that the field equation (8)-(12) yield the solution

$$(13) \quad S = e^{(-\frac{3h}{2})}, \quad e^{(\frac{h}{2})} = \frac{1}{2}(C_1 A^{-1} t + C_2)$$

where C_1, C_2 and A are non-zero constant of integration. Then the metric (4) with S and h given by (13) describes the non-static empty space-time discussed by Bera (1969) in Einstein's theory.

When $\rho = 0 = \lambda$, the field equations (8) -(12) admit the exact vacuum solution, obtained by Reddy and Rao (1988) in Brans -Dicke theory of gravitation, which is given by

$$(14) \quad S = e^{(\frac{C_2}{C_3})h}, \quad e^{(2 + \frac{C_1 + C_2}{C_3})h} = (2 + \frac{C_1 + C_2}{C_3})(C_3 A^{-1} t + C_4)$$

$$(15) \quad \phi = \phi_0 e^{(\frac{C_1}{C_2})h}$$

where A, ϕ_0 and C_i 's are constants of integration and C_i 's are related by

$$6C_3^2 + 4C_3 C_2 + 6C_3 C_1 - \omega C_1^2 + 2C_2 C_1 = 0$$

and $C_3, A \neq 0$.

The field equations (8) -(12) being highly non-linear, to get a determinate solution in Brans -Dicke theory with string source, we assume (Reddy, 2003a).

$$(16) \quad \rho + \lambda = 0$$

i.e. the sum of rest energy density and tension density for a cloud of string vanishes. In this particular case, the field equations (8) -(12) give us the solution

$$(17) \quad S = e^{(\frac{C_2}{C_3})h}$$

$$e^{(\frac{C_1}{C_2} + \frac{C_2}{C_3} + 2)h} = (\frac{C_1}{C_2} + \frac{C_2}{C_3} + 2)(C_1 A^{-1}t + C_4)$$

$$(18) \quad \phi = \phi_0 e^{(\frac{C_1}{C_2})h}$$

$$(19) \quad 8\pi\rho = -8\pi\lambda = e^{(\frac{C_1}{C_2}-2)h} \left[C_1^2 \left(3 + \frac{C_2}{C_3} - C_1 \right) + \frac{C_1^3}{C_2} \left(\frac{\omega}{2} \frac{C_1}{C_2} - 3 \right) \right]$$

where A, ϕ_0 and C_i 's are constants of integration. Now, using the fact that the solution given by (17)-(19) should necessarily satisfy each of the field equations (8) -(12), we observe $C_1 = 0$ is the only possibility. This immediately implies $\rho = \lambda = 0$ and $\phi = \phi_0$ and hence the solution (16) reduces to empty flat model of Einstein's theory. Thus, the non-static plane symmetric string cosmological mode with the assumption (16) ,does not exist in Brans-Dicke theory. However, Reddy (2003a), with the assumption (16), has obtained Bianchi type-I string cosmological model in Brans-Dicke theory.

4. CONCLUSIONS

Reddy (2003c) has shown that Bianchi string cosmologies do not exist in Rosen's (1973) bimetric theory of gravitation. Reddy (2003a, 2003b) has obtained Bianchi type-I string cosmological models in Brans-Dicke and Saez-Ballester theories when the sum of energy density and the tension density of the string is zero. Here we have shown that with the above condition non-static plane symmetric string cosmological model does not exist in Brans -Dicke theory. It is observed that, in this particular case, we, obtained only the empty flat model of Einstein's theory.

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