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Different control strategies for a newly created supply chain model

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Abstract

Many marketing management models assume that a manufacturer can always supply enough products to meet customer demand. In real situations, however, this is not always true. Demand fulfillment depends on practical factors such as production capacity limits, market uncertainty, and changing supply–demand conditions. Manufacturers cannot simply increase production in direct proportion to demand. Individual customers also cannot directly influence production decisions, but their needs are reflected through retailer orders and overall market feedback.

This paper develops a nonlinear supply chain model incorporating realistic manufacturing capacity limits. Under this framework, production volumes do not expand monotonically with shifting consumer demand, necessitating a departure from traditional linear formulations. We evaluate the system's equilibrium solutions and analyze their stability margins via the Routh–Hurwitz framework. The mathematical analysis indicates that the unchecked system yields unstable trajectories for specific parameter sets. To suppress these variations, we deploy multiple feedback control strategies designed to restore systemic equilibrium. Extensive numerical simulations are presented to verify the

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operational efficacy and robustness of the proposed stabilization schemes within the modified supply chain framework.

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1. Introduction

An industrial supply chain represents an integrated network of business processes and logistics nodes designed for raw material sourcing, processing inventory into finalized merchandise, and distribution to consumer endpoints. Constructing mathematical formulations for these systems serves as a primary tool for contemporary operations planning [1].

Supply chains are inherently complex because they consist of many interconnected components that influence one another. Their behavior is often nonlinear in nature. Because of this complexity, researchers commonly use dynamical system frameworks to study supply chain operations. These frameworks help explain how inventory levels, production rates, and demand fluctuations change over time. As a result, concepts from system dynamics and control theory are widely applied in supply chain analysis, since they make it easier to understand system responses and to design strategies that improve overall performance [2-4].

Nonlinear dynamical systems can show many different types of behavior, including periodic oscillations, bifurcations, and even chaos under certain conditions. Chaotic behavior is especially difficult to study because it is very sensitive to initial conditions and can produce irregular and unpredictable trajectories, even when the underlying rules are deterministic. For this reason, detecting and properly characterizing chaos is not straightforward. Chaos control methods are designed to reduce or regulate such irregular behavior by stabilizing unstable trajectories or steering the system toward more desirable operating states. These control approaches are important for improving reliability and predictability in systems where uncontrolled chaos can otherwise reduce performance.

Several methods have been developed to control chaotic behavior in nonlinear systems. Among the most well-known are the Ott–Grebogi–Yorke (OGY) method [5] and the Pyragas time-delay feedback method [6], both of which are considered foundational approaches in chaos control. More broadly, feedback control remains a central and challenging topic in nonlinear dynamics. It is widely used in engineering, economics, and business systems to regulate system behavior, improve stability, and maintain desired operating conditions [7-10].

Chaos synchronization refers to the phenomenon in which two or more chaotic systems, starting from different initial conditions, adjust their states to eventually display identical or correlated behavior. The pioneering work of Pecora and Carroll [11] introduced this concept and demonstrated its feasibility through numerical and experimental studies. They considered a master–slave configuration and showed that appropriate coupling can force a slave chaotic system to follow a master system. Linear coupling functions were used, and synchronization conditions were established.

Chaos synchronization has found applications in secure communication [12], data encryption [13, 14], and biomedical systems [15, 16]. Sivaganesh, Sweetlin, and Arulgnanam [17] investigated robust synchronization in a unidirectionally coupled, quasiperiodically forced nonlinear electronic circuit with strange non-chaotic attractors. Khan and Poria [18] studied projective synchronization using a bidirectional coupling scheme. Similar synchronization techniques have been applied to well-known chaotic models such as the Lorenz system [19], the Rössler system [20], and the Lorenz–Stenflo system [21].

Supply chains play a vital role in modern economic activity because they organize how products, services, and information move from their source to final users. In manufacturing, effective supply chain management has a direct impact on productivity and cost efficiency through decisions related to procurement, inventory, and distribution. Retail businesses rely on supply chains to maintain product availability and to respond quickly to changing customer demand. In healthcare, dependable supply chain operation is even more crucial, since timely access to medicines and essential equipment can be critical. Service sectors such as banking depend less on physical goods, but they still rely heavily on well-structured process and information flows. Overall, these examples indicate that supply chain design is not just an operational support activity, but a key contributor to performance and service quality.

A well-known phenomenon in supply chain operations is the bullwhip effect, where small changes in customer demand lead to increasingly large fluctuations in upstream orders. This effect does not come from a single reason. Instead, it arises from several nonlinear factors, such as information delays, forecast updates, and locally optimized decision policies. When demand information is delayed or partially distorted, firms often compensate by placing larger safety orders. This response unintentionally increases variability across the supply network. Because supply chains are governed by feedback relationships, these over-corrections can reinforce each other and generate oscillatory behavior. For this reason, recent research focuses on more transparent and responsive modeling frameworks, especially those that reduce delays and improve the quality of shared information.

From a dynamical systems viewpoint, supply chains naturally exhibit nonlinear behavior. As a result, they may show complex time patterns such as inventory oscillations, demand amplification, and repeated stock imbalances [22-24]. For some parameter values, the interaction among feedback loops, delays, and adjustment rules can even push the system into chaotic regimes, where long-term prediction becomes difficult despite deterministic laws. Control-based approaches offer a structured way to handle this problem by stabilizing unstable system paths and reducing unwanted fluctuations. Suitable control strategies can improve not only stability but also practical performance measures such as lead time, cost, and resource utilization. Therefore, nonlinear dynamical analysis of supply chain models has become an increasingly active and important research area.

The rest of this work is structured as follows. Section 2 outlines the operational significance and core motivation behind this study. Section 3 lays out the dynamic differential formulations tracking the supply chain network. Section 4 reviews the algebraic mechanics of the Routh–Hurwitz stability array. Section 5 elaborates on the design of the active chaos suppression controllers. Finally, Section 6 offers a review of the principal results along with future research pathways.

2. Significance of the research

In the application of chaos theory in supply chain management, controlling chaotic behaviour is very important because the supply chain system consists of many connected components with nonlinear relationships. Chaos theory shows that a small change in initial conditions can produce very different outcomes over time. In large supply chain networks, this sensitivity becomes more visible, and therefore the system behaviour may become irregular if proper control is not applied. In such a situation, the operational results may become unpredictable.

Chaotic behavior in a supply chain increases uncertainty and risk, and it makes fluctuations and disruptions harder to manage. Because of nonlinear feedback and time delays, even small changes can grow over time and strongly affect inventory levels, ordering decisions, and production planning. Chaos control methods are used to reduce this kind of instability by identifying the main causes of irregular behavior and applying suitable control actions. This leads to a more stable and robust supply chain system. If chaos is not controlled, it can also make it difficult to properly optimize important performance measures such as cost, response time, and service reliability.

By using chaos-based control strategies, it is possible to understand the internal dynamic behaviour of the supply chain more clearly and improve system performance in a structured way. Chaos theory also indicates that flexibility and adaptability are necessary properties for such complex systems. With effective control mechanisms, the supply chain can respond faster to demand changes and external disturbances. Therefore, chaos control in nonlinear supply chain models is necessary not only for stability and risk reduction, but also for better performance and decision support. Overall, it supports improved efficiency and resilience of the supply chain.

3. Model Formulation

The immediate order volume originating from end-consumers is dynamically linked to retailer activity and is shaped by historical demand fulfillment rates, with the manufacturing tier extending long-term logistics coordination to the retail channel via distributors to buffer demand volatility. This interaction is formulated as

$$x_i = \sigma_1 y_{i-1} - \sigma_2 x_{i-1} + \alpha w_{i-1}, \tag{1}$$

where x, y and w represent the customer demand, retailer inventory, and manufacturer production quantity, respectively. The parameters σ_1, σ_2 and α represent the delivery efficiency of retailer, ratio of end customer demand and delivery efficiency from manufacturer. In reality, it is clear that the dependency between the end-customer, retailer, distributor and manufacturer are not linear in nature. In this case retailer needs to accept the effect of end-customer and distributor before making his order and retailer need to make the demand from end-customer. This fact is modelled by equation

$$y_i = \beta x_{i-1} - x_{i-1}z_{i-1} - \gamma y_{i-1}, \tag{2}$$

where the z variable represent the inventory of distributor and the parameters β and γ are the distortion effect from end-customer and demand rate of retailer from distributor. Furthermore, since downstream replenishment requests depend heavily on consumer buying patterns, the distribution tier must evaluate the combined signals from both retailers and consumers to optimize routing choices. This dynamic is captured by

$$z_i = x_{i-1}y_{i-1} - kz_{i-1}, \tag{3}$$

where k is the ratio of demand by retailer and end-customer both. Mainly end-customers demand effect on manufacturer, obviously via retailer and distributor. So rate of change of manufacturer in current time depends on the demand on end-customer. So this situation is modelled by equation

$$w_i = \eta x_{i-1} - \zeta w_{i-1}, \tag{4}$$

where η is the demand rate of end-customer to manufacturer. Effective of the demand of end-customer to manufacturer is less since their demand is covered by retailer which is already dispatched to retailer from manufacturer via distributor. In this case, the parameter η represents the positive demand rate from the end-customer to the manufacturer, while the parameter ζ represents the supply rate from the manufacturer end.

Equation (1) represents the quantity demanded by end-customer, the inventory level of retailer and distributor is represented by equation (2) and (3) respectively and the quantity produced by manufacturer represented by equation (4). Considering very small time interval, the above four discrete time difference equations (1) to (4) represent the following non-linear continuous supply chain model

$$\begin{aligned}
\dot{x} &= \sigma_1 y - \sigma_2 x + \alpha w,\\
\dot{y} &= \beta x - \gamma y - xz,\\
\dot{z} &= xy - kz,\\
\dot{w} &= \eta x - \zeta w.
\end{aligned} \tag{5}$$

In the model (5) we observe that inventory of any objects are dependent on the demand of it. That is inventory increases with the increase of demand from end-customer. However in reality the demand can not increase monotonically for the boundedness of the manufacturer. Because for mass production of the commodities needs huge number of raw materials which costs huge money and also need a large amount of ware house, so it is not possible to produce a huge amount of commodities at a time. That is why manufacturer may not supply any quantity of commodities on demand. So we may need to tiny change of constant delivery efficiency parameter α and the supplier rate from manufacturer end parameter ζ. For boundedness of the parameter α and ζ, we treat α as $\alpha = f(w) = \frac{a}{1+bw}$ and $\zeta = \frac{c}{1+bw}$, where b is the saturation rate of demand with the increase of manufacturer item and a and c are suitable constants. Therefore the model (5) becomes into the following rectified realistic model

$$\begin{aligned} \dot{x} &= \sigma_1 y - \sigma_2 x + \frac{aw}{1+bw}, \\ \dot{y} &= \beta x - \gamma y - xz, \\ \dot{z} &= xy - kz, \\ \dot{w} &= \eta x - \frac{cw}{1+bw}. \end{aligned} \tag{6}$$

Values of the newly created chaotic supply chain system parameters are given below

Table 1

Values of the supply chain system parameters

Parameters	σ_1	σ_2	a	b	k	β	γ	η	c
Values	28	10	1	0.01	2.6	28	2.5	1	1

4. Algorithmic Framework of the Routh-Hurwitz Stability Criterion

To evaluate whether the root spectrum of a characteristic equation lies exclusively within the open left-half complex plane, we implement the classical algebraic protocol established by the Routh–Hurwitz stability criterion. Consider a general characteristic polynomial of degree m expressed as:

$$A_0 s^m + A_1 s^{m-1} + A_2 s^{m-2} + \cdots + A_m = 0 \tag{7}$$

Evaluating system stability involves mapping the polynomial coefficients into a structured matrix layout known as the Routh array. The subsequent array entries are computed recursively from the two immediate preceding rows. Ultimately, asymptotic stability margins are verified by checking the signs of the components along the leading column of the completed array.

Step 1: Establishing the Base Rows

The introductory two rows are populated directly using alternating coefficients from the characteristic equation, serving as the foundational vectors for higher-order iterations.

Row 1	A_0	A_2	A_4	...
Row 2	A_1	A_3	A_5	...

Step 2: Recursive Evaluation of the Third Row

The elements of the third row are calculated via cross-determinant operations utilizing elements from the first two rows.

Row 1	A_0	A_2	A_4	...
Row 2	A_1	A_3	A_5	...
Row 3	B_1	B_3	B_5	...

The intermediate scalars B_1 and B_3 are defined using the negative determinant ratio form as follows:

$$B_1 = -\frac{1}{A_1}\begin{vmatrix} A_0 & A_2 \\ A_1 & A_3 \end{vmatrix} = \frac{A_1A_2 - A_0A_3}{A_1}$$

$$B_3 = -\frac{1}{A_1}\begin{vmatrix} A_0 & A_4 \\ A_1 & A_5 \end{vmatrix} = \frac{A_1A_4 - A_0A_5}{A_1}$$

Step 3: Constructing the Fourth Row Sequence

Higher-order rows are sequentially evaluated by shifting the computational focus to the immediate preceding row pairs (Rows 2 and 3).

Row 1	A_0	A_2	A_4	...
Row 2	A_1	A_3	A_5	...
Row 3	B_1	B_3	B_5	...
Row 4	C_1	C_3	C_5	...

The corresponding row values C_1 and C_3 are calculated using the localized matrix structures from Rows 2 and 3:

$$C_1 = -\frac{1}{B_1}\begin{vmatrix} A_1 & A_3 \\ B_1 & B_3 \end{vmatrix} = \frac{B_1A_3 - A_1B_3}{B_1}$$

$$C_3 = -\frac{1}{B_1}\begin{vmatrix} A_1 & A_5 \\ B_1 & B_5 \end{vmatrix} = \frac{B_1A_5 - A_1B_5}{B_1}$$

This recursive pipeline is systematically repeated to populate all lower rows up to the terminal index m. According to the Routh–Hurwitz theorem, a necessary and sufficient condition for the system to exhibit asymptotic convergence is that every entry in the first column shares an identical algebraic sign (conventionally strictly positive, assuming a positive initialization parameter $A_0 > 0$). Any sign reversal within this main column indicates that eigenvalues have crossed into the right-half plane, confirming that the corresponding equilibrium state is unstable.

5. Control of chaos in supply chain model

In this section, we derive the equilibrium solution of the model (6). It is obvious that equation (6) has three fixed points $E_0 = (0,0,0,0)$, $E_1 = (a_1, a_2, a_3, a_4)$ and $E_2 = (-a_1, -a_2, a_3, -a_4)$, where

$$a_1 = \sqrt{\frac{k\gamma(\sigma_2 c - a\eta) - kc\sigma_1\beta}{\sigma_2 c - a\eta}},$$

$$a_2 = \frac{1}{c\sigma_1}\sqrt{k\gamma(\sigma_2 c - a\eta)^2 - kc\sigma_1\beta(\sigma_2 c - a\eta)},$$

$$a_3 = \frac{\gamma(\sigma_2 c - a\eta) - c\sigma_1\beta}{c\sigma_1} \text{ and } a_4 = \frac{\eta\sqrt{k\gamma(\sigma_2 c - a\eta) - kc\sigma_1\beta}}{c\sqrt{\sigma_2 c - a\eta} - b\eta\sqrt{k\gamma(\sigma_2 c - a\eta) - kc\sigma_1\beta}}.$$

The Jacobian matrix of the system (6) at (x, y, z, w) is given by $J = \begin{bmatrix} -\sigma_2 & \sigma_1 & 0 & \frac{a}{(1+bw)^2} \\ \beta - z & -\gamma & -x & 0 \\ y & x & -k & 0 \\ \eta & 0 & 0 & -\frac{c}{(1+bw)^2} \end{bmatrix}$. The Jacobian matrix of the system (6) at equilibrium point $E_0 = (0,0,0,0)$ is given by $J(0,0,0,0) = \begin{bmatrix} -\sigma_2 & \sigma_1 & 0 & a \\ \beta & -\gamma & 0 & 0 \\ 0 & 0 & -k & 0 \\ \eta & 0 & 0 & -c \end{bmatrix}$.

Now the characteristic equation of $J(0,0,0,0)$ is given by $\lambda^4 + c_1\lambda^3 + c_2\lambda^2 + c_3\lambda + c_4 = 0$, where

$$\begin{aligned} c_1 &= \sigma_2 + \gamma + k + c, \\ c_2 &= \sigma_2(\gamma + k + c) - \beta\sigma_1 + \gamma(k + c) - a\eta + ck, \\ c_3 &= \sigma_2(k\gamma + kc + c\gamma) - \sigma_1\beta(k + c) - a\eta(\gamma + k) + kc\gamma, \\ c_4 &= k(c(\sigma_2\gamma - \sigma_1\beta) - a\gamma\eta). \end{aligned} \tag{8}$$

Using the values of the parameters from the Table 1 and applying the MATLAB software package for R-H criterion we observe $B_1 = -544.39$, hence equilibrium point $(0,0,0,0)$ is in unstable state. The uncontrolled system exhibits irregular and persistent oscillations, indicating the presence of chaotic dynamics, as illustrated in Figure 1.

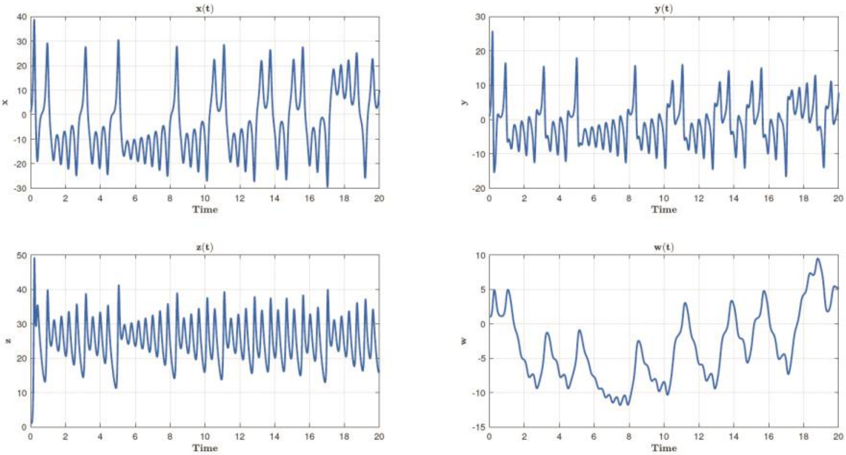

Figure 1
Time evolution of the uncontrolled supply chain model exhibiting chaotic oscillations.

Chaotic systems can often be stabilized by introducing appropriately designed feedback terms. The main objective of a control strategy is to alter the system dynamics so that trajectories converge to a selected equilibrium state. In this work, we consider three different feedback approaches: dislocated feedback control, enhancing feedback control, and conventional linear feedback control. Each method is analyzed theoretically and validated numerically to demonstrate its effectiveness in suppressing chaotic oscillations.

5.1 *Dislocated feedback control*

In dislocated feedback control, the demand variable (x) is often multiplied by a coefficient as the feedback gain and is added to the right hand side of second equation where retailer takes demand from end-customer. For studying dislocated feedback control of supply chain model, we consider

$$\begin{aligned} \dot{x} &= \sigma_1 y - \sigma_2 x + \frac{aw}{1+bw}, \\ \dot{y} &= \beta x - \gamma y - xz - px, \\ \dot{z} &= xy - kz, \\ \dot{w} &= \eta x - \zeta w, \end{aligned} \tag{9}$$

where p is the feedback coefficient. The Jacobian matrix of the system (9) at equilibrium point $E_0 = (0,0,0,0)$ is given by $J(0,0,0,0) = \begin{bmatrix} -\sigma_2 & \sigma_1 & 0 & a \\ \beta - p & -\gamma & 0 & 0 \\ 0 & 0 & -k & 0 \\ \eta & 0 & 0 & -c \end{bmatrix}$, then the characteristic equation is $\lambda^4 + g_1\lambda^3 + g_2\lambda^2 + g_3\lambda + g_4 = 0$ where

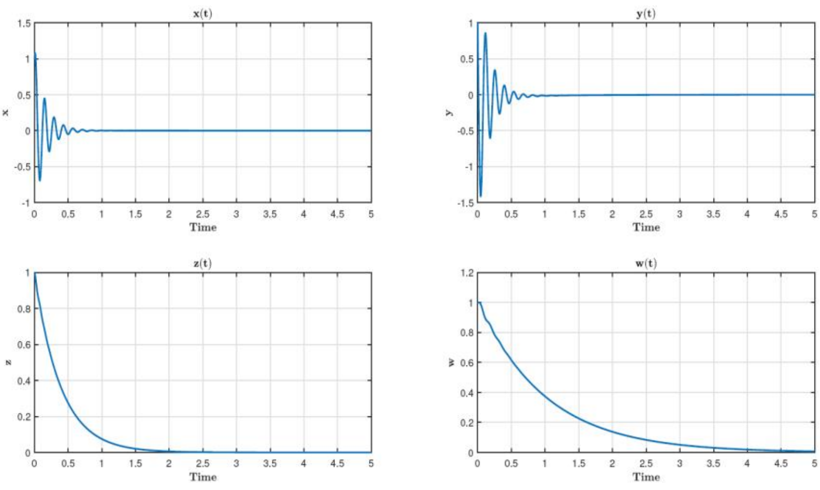

Figure 2
Time evolution of the supply chain model under dislocated feedback control.

$$\begin{aligned} g_1 &= \sigma_2 + \gamma + k + c, \\ g_2 &= \sigma_2(\gamma + k + c) - \sigma_1(\beta - p) + k(\gamma + c) - a\eta + \gamma c, \\ g_3 &= \sigma_2(k\gamma + kc + c\gamma) - \sigma_1\beta(k + c) - a\eta(\gamma + k) \\ &\quad + kc\gamma + \sigma_1 p(k + c), \\ g_4 &= k(c(\sigma_2\gamma - \sigma_1\beta) - a\gamma\eta) + pk\sigma_1 c. \end{aligned} \tag{10}$$

Obviously, when $p > 27.5$, then from R-H criterion, we observe that $A_0 = 1, A_1 = 16.1, B_1 = 53.44, C_1 = 60.34$ and $D_1 = 22.10$ for choice of feedback coefficient $p = 27.5$. Hence the system (9) will gradually converge to the unstable equilibrium point $E_0(0,0,0,0)$. Therefore the unstable fixed point $E_0(0,0,0,0)$ is stabilized by the dislocated feedback control method here. Figure 2 shows that the dislocated feedback controller successfully suppresses chaotic oscillations and stabilizes the system at the origin.

5.2 *Enhancing feedback control*

Sometimes difficult to control chaotic model by using only one feedback variable and in that case feedback gain is always a large quantity. So we consider here multiple variables multiplied by a proper coefficient as the feedback gain. This type of method is called enhancing feedback control. For our supply chain model the control method is considered as follows

$$\begin{aligned} \dot{x} &= \sigma_1 y - \sigma_2 x + \frac{aw}{1+bw} - p_1 x, \\ \dot{y} &= \beta x - \gamma y - xz - p_1 y, \\ \dot{z} &= xy - kz, \\ \dot{w} &= \eta x - \frac{cw}{1+bw}, \end{aligned} \tag{11}$$

where p_1 is the feedback coefficient. The Jacobian matrix of the system (11) at equilibrium point $E_0 = (0, 0, 0, 0)$ is given by $J(0,0,0,0) = \begin{bmatrix} -\sigma_2 - p_1 & \sigma_1 & 0 & a \\ \beta & -\gamma - p_1 & 0 & 0 \\ 0 & 0 & -k & 0 \\ \eta & 0 & 0 & -c \end{bmatrix}$, then the characteristic equation is $\lambda^4 + h_1\lambda^3 + h_2\lambda^2 + h_3\lambda + h_4 = 0$ where

$$\begin{aligned} h_1 &= \sigma_2 + \gamma + k + c + 2p_1, \\ h_2 &= \sigma_2(\gamma + k + c + p_1) - \sigma_1\beta + k(\gamma + c + 2p_1) \\ &\quad + \gamma c - a\eta + p_1(\gamma + 2c + p_1), \\ h_3 &= (c + k)\Delta + ck(\sigma_2 + \gamma + 2p_1) - a\eta(\gamma + k + p_1), \\ h_4 &= ck\Delta - a\eta k(\gamma + p_1), \end{aligned} \tag{12}$$

where $\Delta = (\sigma_2 + p_1)(\gamma + p_1) - \sigma_1\beta$. Obviously, when $p_1 > 22.45$, then from R-H criterion, we observe that $A_0 = 1, A_1 = 62.10, B_1 = 264.37, C_1 = 311.46$ and $D_1 = 83.20$ for the choice of $p_1 = 23$. Hence the system (11) will gradually converge to the unstable equilibrium point $E_0(0,0,0,0)$. Therefore the unstable fixed point $E_0(0,0,0,0)$ is stabilized by the enhancing feedback control method. As shown in Figure 3, the enhancing feedback controller also stabilizes the system effectively.

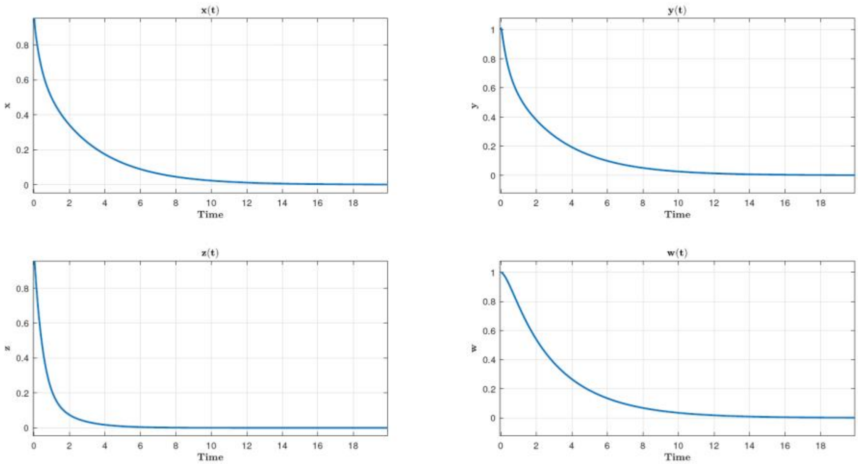

Figure 3
Time evolution of the supply chain model under enhancing feedback control.

5.3 *Linear feedback control*

We introduce the conventional linear feedback control to drive the chaotic trajectory $(x(t), y(t), z(t), w(t))$ to a desired unstable equilibrium point (a_1, a_2, a_3, a_4). Assume that the controlled version of system (6) is

$$\begin{aligned} \dot{x} &= \sigma_1 y - \sigma_2 x + \frac{aw}{1+bw} - u_1 \\ \dot{y} &= \beta x - \gamma y - xz - u_2 \\ \dot{z} &= xy - kz - u_3 \\ \dot{w} &= \eta x - \frac{cw}{1+bw} - u_4. \end{aligned} \tag{13}$$

where u_1, u_2, u_3 and u_4 are the external control inputs. Consider here $u_1 = k_1(x - a_1)$, $u_2 = k_2(y - a_2)$, $u_3 = k_3(z - a_3)$ and $u_4 = k_4(w - a_4)$. Therefore, system (13) leads to

$$\begin{aligned} \dot{x} &= \sigma_1 y - \sigma_2 x + \frac{aw}{1+bw} - k_1(x - a_1) \\ \dot{y} &= \beta x - \gamma y - xz - k_2(y - a_2) \\ \dot{z} &= xy - kz - k_3(z - a_3) \\ \dot{w} &= \eta x - \frac{cw}{1+bw} - k_4(w - a_4). \end{aligned} \tag{14}$$

Linearizing (14) about this equilibrium point, one gets

$$\begin{aligned} \dot{X} &= -(\sigma_2 + k_1)X + \sigma_1 Y + \frac{a}{(1+ba_4)^2} W \\ \dot{Y} &= (\beta - a_3)X - (\gamma + k_2)Y - a_1 Z \\ \dot{Z} &= a_2 X + a_1 Y - (k + k_3)Z \\ \dot{W} &= \eta X - (\frac{c}{(1+ba_4)^2} + k_4)W. \end{aligned} \tag{15}$$

Consider the unstable point $(a_1, a_2, a_3, a_4) = (0, 0, 0, 0)$ the system (15) leads to

$$\begin{aligned}
\dot{X} &= -(\sigma_2 + k_1)X + \sigma_1 Y + aW \\
\dot{Y} &= \beta X - (\gamma + k_2)Y \\
\dot{Z} &= -(k + k_3)Z \\
\dot{W} &= \eta X - (c + k_4)W.
\end{aligned} \tag{16}$$

To establish the asymptotic stability of the equilibrium solution $(0,0,0,0)$ of system(16), we employ the Lyapunov function method. Define the Lyapunov function for system (16) as follows:

$$V(X,Y,Z,W) = \frac{2X^2+Y^2+Z^2+W^2}{2}. \tag{17}$$

The function V satisfies the following properties:

- $V(0,0,0,0) = 0$,
- $V(X,Y,Z,W) > 0$ for (X,Y,Z,W) in a neighborhood of the origin. Hence, V is positive definite.

Moreover, the time derivative of V along the system trajectories is given by

$$\begin{aligned}
\frac{dV}{dt} &= -2(\sigma_2 + k_1)X^2 + 2\sigma_1 XY + 2aXW + \beta XY \\
&\quad -(\gamma + k_2)Y^2 - (k + k_3)Z^2 + \eta XW - (c + k_4)W^2, \\
&= -2\sigma_2 X^2 - \gamma Y^2 - (k + k_3)Z^2 - cW^2 \\
&\quad -(\sqrt{k_1}X - \sqrt{k_2}Y)^2 - (\sqrt{k_1}X - \sqrt{k_4}W)^2.
\end{aligned} \tag{18}$$

Therefore, the derivative $\frac{dV}{dt} < 0$ whenever $4k_1k_2 = (2\sigma_1 + \beta)^2$, $4k_1k_4 = (2a + \eta)^2$ and $k + k_3 > 0$ Figure 4 confirms that the linear feedback controller drives the state variables smoothly toward the equilibrium point, validating the theoretical stability results.

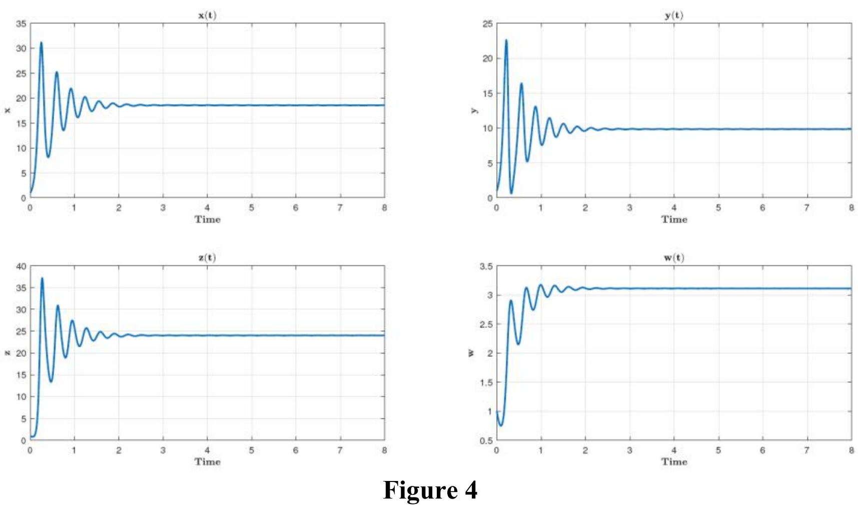

Figure 4
Time evolution of the supply chain model under linear feedback control.

6. Conclusion

In this paper, we investigated several control strategies for a newly proposed chaotic supply chain model, namely dislocated feedback control, enhancing feedback control, and linear feedback control. The controllers were designed using the Routh–Hurwitz stability criterion and Lyapunov-based stabilization theory. Both theoretical analysis and numerical simulations confirm that the proposed control schemes effectively stabilize the system and suppress chaotic behavior.

From a practical point of view, the proposed control strategies can help make supply chain operations more stable by reducing excessive fluctuations in demand and inventory and by lowering the operational risks linked to chaotic dynamics. Greater stability naturally improves predictability and supports more reliable decision-making. In this way, the proposed methods can offer useful guidance to managers and planners in designing supply chain policies that are efficient, resilient, and sustainable. Future research can extend this framework by including time delays, uncertainty effects, and parameter estimation based on real-world data.

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