

A unified modified triple integral transform and its applications

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Abstract

The objective of this paper is to establish a generalization of the triple Laplace transform, and the other triple integral transforms available in the class of Laplace transform called the new modified triple integral transform. In this paper we have proved all the fundamental properties like linearity property, first shifting property, second shifting property, change of scale property, derivative property, periodic property, etc., for the new modified triple integral transform. The applicability of this transform is demonstrated by using this new integral transform for solving partial differential equations and partial integro-differential equations.

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1. Introduction

Triple integral transform is the extension of single integral transform to a function of three variables. It is used as a powerful tool for solving partial differential equations arising in heat transfer, fluid mechanics, and electromagnetic wave propagation, thereby converting a partial differential

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equation involving an unknown function of three variable into an algebraic equation.

Triple Laplace Aboodh-Sumudu transform [2], triple Aboodh transform [4], triple Shehu transform [5], triple Laplace transform [6], triple Elzaki transform [7], triple integral transform (SEE- Sadik- Shehu) [8], Laplace–ARA– Sumudu transform [9], new general triple integral transform [10], a new modified integral transform [11], triple mixed integral transformation [12] etc., are some of the extensions of the Laplace transform that researchers have contributed.

General frameworks like the triple g-transformation [1] and the Gamar transform [10] unify multiple existing transforms under a single formulation. The selection of an appropriate triple transform critically depends on the governing equation's nature, boundary conditions, and the physical interpretation of the model. Recent developments also include conformable triple Laplace–Sumudu transforms [3] and the triple SEE Shehu Sadik transform [8], extending applicability to fractional differential equations and multidimensional problems.

Abdelilah Kamal H. Sedeeg, in 2023 introduces a new general triple integral transform [10] (GTT) of integrable function $g(x, y, z)$ with three variables $x, y, z > 0$ denoted by $\mathcal{T}_3\{g(x, y, z)\}$ known as Gamar transform is defined as,

$$\begin{aligned} \mathcal{T}_3\{g(x, y, z)\} &= p(r)q(s)u(v) \\ &\times \int_0^\infty \int_0^\infty \int_0^\infty e^{-\mathcal{W}(r)x - \psi(s)y - \phi(v)z} g(x, y, z) dx dy dz, \end{aligned}$$

provided that all integrals exists for some $\mathcal{W}(r), \psi(s)$ and $\phi(v)$, where $\mathcal{W}(r), \psi(s)$ and $\phi(v)$ are transformed functions of x, y and z respectively.

The objective of this work is to introduce a new integral transform which is the generalization of most of the triple integral transforms available in the literature called as new modified triple integral transform. This new integral transform is the extension of one dimensional new modified integral transform [11] to three dimensional new modified integral transform.

2. Preliminaries

In this section, we introduce a new integral transform, which we refer to as the “new modified triple integral transform”. This transform extends and generalizes existing triple integral transforms, offering greater flexibility and wider applicability in mathematical and engineering contexts. We will begin by defining the new modified triple integral transform and then explore its fundamental properties and potential applications.

Definition 1: [10] A function $f(x, y, z)$ is said to be of exponential order $b > 0, c > 0$ and $d > 0$ on $0 \leq x < \infty, 0 \leq y < \infty, 0 \leq z < \infty$, if there exists a positive constant κ such that,

$$|f(x, y, z)| \leq \kappa e^{bx+cy+dz}, \quad (1)$$

and we write

$$f(x, y, z) = O(e^{bx+cy+dz}) \text{ as } x \rightarrow \infty, y \rightarrow \infty, z \rightarrow \infty. \quad (2)$$

Definition 2: [11] Let the integrable function $\mathfrak{f}(t)$ defined for $t \geq 0$, then the new modified integral transform is given by,

$$\mathcal{T}_a\{\mathfrak{f}(t), t \rightarrow \mathfrak{p}\} = \mathfrak{F}(\mathfrak{p}) = \phi(\mathfrak{p}) \int_0^\infty \mathfrak{f}(t) a^{-\psi(\mathfrak{p})t} dt, \quad (3)$$

provided the integral exists for some $\psi(\mathfrak{p})$, where $a \in (1, \infty)$ and $\phi(\mathfrak{p}), \psi(\mathfrak{p})$ are the regular complex functions such that $\phi(\mathfrak{p}) \neq 0, \forall \mathfrak{p} \in \mathbb{C}$.

Definition 3: The new modified double integral transform of a function $\mathfrak{f}(\mathfrak{x}, \mathfrak{y})$ of two variables $\mathfrak{x}$ and $\mathfrak{y}$ defined in the first quadrant of $\mathfrak{x} - \mathfrak{y}$ plane is defined by the double integral of the form,

$$\begin{aligned} \mathcal{T}_{a_2}\{\mathfrak{f}(\mathfrak{x}, \mathfrak{y})\} &= \mathfrak{F}(\mathfrak{p}, \mathfrak{q}) = \mathcal{T}_a[\mathcal{T}_a\{\mathfrak{f}(\mathfrak{x}, \mathfrak{y}); \mathfrak{x} \rightarrow \mathfrak{p}\}; \mathfrak{y} \rightarrow \mathfrak{q}] \\ &= \phi_1(\mathfrak{p})\phi_2(\mathfrak{q}) \int_0^\infty \int_0^\infty \mathfrak{f}(\mathfrak{x}, \mathfrak{y}) a^{-(\psi_1(\mathfrak{p})\mathfrak{x} + \psi_2(\mathfrak{q})\mathfrak{y})} d\mathfrak{x} d\mathfrak{y}, \end{aligned} \quad (4)$$

provided the integral exists for some $\psi_1(\mathfrak{p})$ and $\psi_2(\mathfrak{q})$, where $a \in (1, \infty)$ and $\phi_1(\mathfrak{p}), \phi_2(\mathfrak{q}), \psi_1(\mathfrak{p}), \psi_2(\mathfrak{q})$ are the regular complex functions such that $\phi_1(\mathfrak{p}) \neq 0, \phi_2(\mathfrak{q}) \neq 0, \forall \mathfrak{p}, \mathfrak{q} \in \mathbb{C}$.

Definition 4: The new modified triple integral transform of an integrable function $\mathfrak{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z})$ of three variables $\mathfrak{x}, \mathfrak{y}$ and $\mathfrak{z}$ defined in the first quadrant of $\mathfrak{x} - \mathfrak{y} - \mathfrak{z}$ plane is defined by the triple integral of the form,

$$\begin{aligned} \mathcal{T}_{a_3}\{\mathfrak{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z})\} &= \mathfrak{F}(\mathfrak{p}, \mathfrak{q}, \mathfrak{r}) \\ &= \mathcal{T}_a[\mathcal{T}_a[\mathcal{T}_a\{\mathfrak{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z}); \mathfrak{x} \rightarrow \mathfrak{p}\}; \mathfrak{y} \rightarrow \mathfrak{q}]; \mathfrak{z} \rightarrow \mathfrak{r}] \\ &= \phi_1(\mathfrak{p})\phi_2(\mathfrak{q})\phi_3(\mathfrak{r}) \\ &\times \int_0^\infty \int_0^\infty \int_0^\infty \mathfrak{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z}) a^{-(\psi_1(\mathfrak{p})\mathfrak{x} + \psi_2(\mathfrak{q})\mathfrak{y} + \psi_3(\mathfrak{r})\mathfrak{z})} d\mathfrak{x} d\mathfrak{y} d\mathfrak{z}, \end{aligned} \quad (5)$$

provided the integral exists for some $\psi_1(\mathfrak{p}), \psi_2(\mathfrak{q})$ and $\psi_3(\mathfrak{r})$, where $a \in (1, \infty)$ and $\phi_1(\mathfrak{p}), \phi_2(\mathfrak{q}), \phi_3(\mathfrak{r}), \psi_1(\mathfrak{p}), \psi_2(\mathfrak{q}), \psi_3(\mathfrak{r})$ are regular complex functions such that $\phi_1(\mathfrak{p}) \neq 0, \phi_2(\mathfrak{q}) \neq 0, \phi_3(\mathfrak{r}) \neq 0, \forall \mathfrak{p}, \mathfrak{q}, \mathfrak{r} \in \mathbb{C}$.

Theorem 1: (Existence theorem). *If $\mathfrak{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z})$ is piecewise continuous on $\mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}_+$ and is $O(e^{b\mathfrak{x} + c\mathfrak{y} + d\mathfrak{z}})$ for some $b, c, d > 0$, then $\mathfrak{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z})$ has the new modified triple integral transform $\mathfrak{F}(\mathfrak{p}, \mathfrak{q}, \mathfrak{r})$ for all $\text{Re}(\psi_1(\mathfrak{p})) > \frac{b}{\log a}, \text{Re}(\psi_2(\mathfrak{q})) > \frac{c}{\log a}, \text{Re}(\psi_3(\mathfrak{r})) > \frac{d}{\log a}$.*

Proof: In order to demonstrate the new modified triple integral transform $\mathfrak{F}(\mathfrak{p}, \mathfrak{q}, \mathfrak{r})$ of $\mathfrak{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z})$ exists for all $\text{Re}(\psi_1(\mathfrak{p})) > \frac{b}{\log a}, \text{Re}(\psi_2(\mathfrak{q})) > \frac{c}{\log a}, \text{Re}(\psi_3(\mathfrak{r})) > \frac{d}{\log a}$, we must show that the integral defined in (5) converges. Hence, we consider,

$$\begin{aligned} &\|\mathfrak{F}(\mathfrak{p}, \mathfrak{q}, \mathfrak{r})\| \\ &\leq |\phi_1(\mathfrak{p})\phi_2(\mathfrak{q})\phi_3(\mathfrak{r})| \\ &\times \int_0^\infty \int_0^\infty \int_0^\infty |\mathfrak{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z})| e^{-(\psi_1(\mathfrak{p})\mathfrak{x} + \psi_2(\mathfrak{q})\mathfrak{y} + \psi_3(\mathfrak{r})\mathfrak{z})\log a} d\mathfrak{x} d\mathfrak{y} d\mathfrak{z} \\ &= |\phi_1(\mathfrak{p})\phi_2(\mathfrak{q})\phi_3(\mathfrak{r})| \end{aligned} \quad (6)$$

$$\times \int_0^\infty \int_0^\infty \int_0^\infty |\tilde{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z})| \\ \times e^{-((\operatorname{Re}\psi_1(\mathfrak{p}))\mathfrak{x}+(\operatorname{Re}\psi_2(\mathfrak{q}))\mathfrak{y}+(\operatorname{Re}\psi_3(\mathfrak{r}))\mathfrak{z})\log a} d\mathfrak{x} d\mathfrak{y} d\mathfrak{z}.$$

Let $\tilde{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z})$ be piecewise continuous on $\mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}_+$ and is of $O(e^{b\mathfrak{x}+c\mathfrak{y}+d\mathfrak{z}})$ for some $b, c, d > 0$, therefore there exists a positive constant κ such that,

$$|\tilde{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z})| \leq \kappa e^{b\mathfrak{x}+c\mathfrak{y}+d\mathfrak{z}}, \quad (7)$$

Thus, equation (6) becomes,

$$\begin{aligned} & \|\mathfrak{F}(\mathfrak{p}, \mathfrak{q}, \mathfrak{r})\| \\ & \leq |\Phi_1(\mathfrak{p})\Phi_2(\mathfrak{q})\Phi_3(\mathfrak{r})| \int_0^\infty \int_0^\infty \int_0^\infty \kappa e^{b\mathfrak{x}+c\mathfrak{y}+d\mathfrak{z}} \\ & \times e^{-((\operatorname{Re}\psi_1(\mathfrak{p}))\mathfrak{x}+(\operatorname{Re}\psi_2(\mathfrak{q}))\mathfrak{y}+(\operatorname{Re}\psi_3(\mathfrak{r}))\mathfrak{z})\log a} d\mathfrak{x} d\mathfrak{y} d\mathfrak{z} \\ & = \kappa \frac{|\Phi_1(\mathfrak{p})|}{\operatorname{Re}\psi_1(\mathfrak{p})\log a - b} \cdot \frac{|\Phi_2(\mathfrak{q})|}{\operatorname{Re}\psi_2(\mathfrak{q})\log a - c} \cdot \frac{|\Phi_3(\mathfrak{r})|}{\operatorname{Re}\psi_3(\mathfrak{r})\log a - d}, \end{aligned} \quad (8)$$

provided that $\operatorname{Re}(\psi_1(\mathfrak{p})) > \frac{b}{\log a}$, $\operatorname{Re}(\psi_2(\mathfrak{q})) > \frac{c}{\log a}$, $\operatorname{Re}(\psi_3(\mathfrak{r})) > \frac{d}{\log a}$.

3. The Transforms of Some Typical Functions

1. Function: $\tilde{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z}) = 1$

Solution: By using definition of new modified triple integral transform, we get,

$$\begin{aligned} & \mathcal{T}_{a_3}\{1; \mathfrak{x} \rightarrow \mathfrak{p}, \mathfrak{y} \rightarrow \mathfrak{q}, \mathfrak{z} \rightarrow \mathfrak{r}\} \\ & = \Phi_1(\mathfrak{p})\Phi_2(\mathfrak{q})\Phi_3(\mathfrak{r}) \int_0^\infty \int_0^\infty \int_0^\infty 1 \cdot a^{-(\psi_1(\mathfrak{p})\mathfrak{x}+\psi_2(\mathfrak{q})\mathfrak{y}+\psi_3(\mathfrak{r})\mathfrak{z})} d\mathfrak{x} d\mathfrak{y} d\mathfrak{z} \\ & = \Phi_1(\mathfrak{p})\Phi_2(\mathfrak{q})\Phi_3(\mathfrak{r}) \int_0^\infty \int_0^\infty \int_0^\infty e^{-(\psi_1(\mathfrak{p})\mathfrak{x}+\psi_2(\mathfrak{q})\mathfrak{y}+\psi_3(\mathfrak{r})\mathfrak{z})\log a} d\mathfrak{x} d\mathfrak{y} d\mathfrak{z} \\ & = \Phi_1(\mathfrak{p})\Phi_2(\mathfrak{q})\Phi_3(\mathfrak{r}) \frac{e^{-\psi_1(\mathfrak{p})\mathfrak{x}\log a}}{-\psi_1(\mathfrak{p})\log a} \Big|_0^\infty \cdot \frac{e^{-\psi_2(\mathfrak{q})\mathfrak{y}\log a}}{-\psi_2(\mathfrak{q})\log a} \Big|_0^\infty \cdot \frac{e^{-\psi_3(\mathfrak{r})\mathfrak{z}\log a}}{-\psi_3(\mathfrak{r})\log a} \Big|_0^\infty. \end{aligned}$$

Thus, for $\operatorname{Re}(\psi_1(\mathfrak{p}))\log a > 0$ or $\operatorname{Re}(\psi_1(\mathfrak{p})) > 0$,
 $\operatorname{Re}(\psi_2(\mathfrak{q}))\log a > 0$ or $\operatorname{Re}(\psi_2(\mathfrak{q})) > 0$ and
 $\operatorname{Re}(\psi_3(\mathfrak{r}))\log a > 0$ or $\operatorname{Re}(\psi_3(\mathfrak{r})) > 0$ we have,

$$\mathcal{T}_{a_3}\{1; \mathfrak{x} \rightarrow \mathfrak{p}, \mathfrak{y} \rightarrow \mathfrak{q}, \mathfrak{z} \rightarrow \mathfrak{r}\} = \frac{\Phi_1(\mathfrak{p})\Phi_2(\mathfrak{q})\Phi_3(\mathfrak{r})}{\psi_1(\mathfrak{p})\psi_2(\mathfrak{q})\psi_3(\mathfrak{r})(\log a)^3}. \quad (9)$$

Similarly, we can derive the following transforms.

Table 1
Table of new modified triple integral transform

Function $\tilde{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z})$	$\mathfrak{F}(\mathfrak{p}, \mathfrak{q}, \mathfrak{r})$
$\mathfrak{x}\mathfrak{y}\mathfrak{z}$	$\frac{\Phi_1(\mathfrak{p})\Phi_2(\mathfrak{q})\Phi_3(\mathfrak{r})}{(\psi_1(\mathfrak{p})\psi_2(\mathfrak{q})\psi_3(\mathfrak{r}))^2(\log a)^6}$
	$\operatorname{Re}(\psi_1(\mathfrak{p})), \operatorname{Re}(\psi_2(\mathfrak{q})), \operatorname{Re}(\psi_3(\mathfrak{r})) > 0.$
$\cos(b\mathfrak{x} + c\mathfrak{y} + d\mathfrak{z})$	$\frac{\Phi_1(\mathfrak{p})\Phi_2(\mathfrak{q})\Phi_3(\mathfrak{r})(\psi_1(\mathfrak{p})\psi_2(\mathfrak{q})\psi_3(\mathfrak{r})(\log a)^3 - cd\psi_1(\mathfrak{p})\log a - bd\psi_2(\mathfrak{q})\log a - bc\psi_3(\mathfrak{r})\log a)}{((\psi_1(\mathfrak{p})\log a)^2 + b^2)((\psi_2(\mathfrak{q})\log a)^2 + c^2)((\psi_3(\mathfrak{r})\log a)^2 + d^2)}$
	$\operatorname{Re}(\psi_1(\mathfrak{p})) > 0, \operatorname{Re}(\psi_2(\mathfrak{q})) > 0, \operatorname{Re}(\psi_3(\mathfrak{r})) > 0.$
$\sin(b\mathfrak{x} + c\mathfrak{y} + d\mathfrak{z})$	$\frac{\Phi_1(\mathfrak{p})\Phi_2(\mathfrak{q})\Phi_3(\mathfrak{r})(b\psi_2(\mathfrak{q})\psi_3(\mathfrak{r})(\log a)^2 + c\psi_1(\mathfrak{p})\psi_3(\mathfrak{r})(\log a)^2 + d\psi_1(\mathfrak{p})\psi_2(\mathfrak{q})(\log a)^2 - bcd)}{((\psi_1(\mathfrak{p})\log a)^2 + b^2)((\psi_2(\mathfrak{q})\log a)^2 + c^2)((\psi_3(\mathfrak{r})\log a)^2 + d^2)}$
	$\operatorname{Re}(\psi_1(\mathfrak{p})) > 0, \operatorname{Re}(\psi_2(\mathfrak{q})) > 0, \operatorname{Re}(\psi_3(\mathfrak{r})) > 0.$

$\cosh(b\mathfrak{x} + c\mathfrak{y} + d\mathfrak{z})$	$\frac{\Phi_1(\mathfrak{p})\Phi_2(\mathfrak{q})\Phi_3(\mathfrak{r})(\psi_1(\mathfrak{p})\psi_2(\mathfrak{q})\psi_3(\mathfrak{r})(\log a)^3 + cd\psi_1(\mathfrak{p})\log a + bd\psi_2(\mathfrak{q})\log a + bc\psi_3(\mathfrak{r})\log a)}{((\psi_1(\mathfrak{p})\log a)^2 - b^2)((\psi_2(\mathfrak{q})\log a)^2 - c^2)((\psi_3(\mathfrak{r})\log a)^2 - d^2)},$ $\operatorname{Re}(\psi_1(\mathfrak{p})) > \frac{ b }{\log a}, \operatorname{Re}(\psi_2(\mathfrak{q})) > \frac{ c }{\log a}, \operatorname{Re}(\psi_3(\mathfrak{r})) > \frac{ d }{\log a}.$
$\sinh(b\mathfrak{x} + c\mathfrak{y} + d\mathfrak{z})$	$\frac{\Phi_1(\mathfrak{p})\Phi_2(\mathfrak{q})\Phi_3(\mathfrak{r})(b\psi_2(\mathfrak{q})\psi_3(\mathfrak{r})(\log a)^2 + c\psi_1(\mathfrak{p})\psi_3(\mathfrak{r})(\log a)^2 + d\psi_1(\mathfrak{p})\psi_2(\mathfrak{q})(\log a)^2 + bcd)}{((\psi_1(\mathfrak{p})\log a)^2 - b^2)((\psi_2(\mathfrak{q})\log a)^2 - c^2)((\psi_3(\mathfrak{r})\log a)^2 - d^2)},$ $\operatorname{Re}(\psi_1(\mathfrak{p})) > \frac{ b }{\log a}, \operatorname{Re}(\psi_2(\mathfrak{q})) > \frac{ c }{\log a}, \operatorname{Re}(\psi_3(\mathfrak{r})) > \frac{ d }{\log a}.$
$(\mathfrak{x}\mathfrak{y}\mathfrak{z})^n$	$\frac{(\pi!)^3 \Phi_1(\mathfrak{p})\Phi_2(\mathfrak{q})\Phi_3(\mathfrak{r})}{(\psi_1(\mathfrak{p})\psi_2(\mathfrak{q})\psi_3(\mathfrak{r}))^{n+1}(\log a)^{3n+3}},$ $\operatorname{Re}(\psi_1(\mathfrak{p})) > 0, \operatorname{Re}(\psi_2(\mathfrak{q})) > 0, \operatorname{Re}(\psi_3(\mathfrak{r})) > 0, n \in \mathbb{N}.$
$(\mathfrak{x}\mathfrak{y}\mathfrak{z})^b$	$\frac{(\Gamma(b+1))^3 \Phi_1(\mathfrak{p})\Phi_2(\mathfrak{q})\Phi_3(\mathfrak{r})}{(\psi_1(\mathfrak{p})\psi_2(\mathfrak{q})\psi_3(\mathfrak{r}))^{b+1}(\log a)^{3b+3}},$ $\operatorname{Re}(\psi_1(\mathfrak{p})) > 0, \operatorname{Re}(\psi_2(\mathfrak{q})) > 0, \operatorname{Re}(\psi_3(\mathfrak{r})) > 0 > 0, b > -1.$
$\mathfrak{x}^m \mathfrak{y}^n \mathfrak{z}^b$	$\frac{\Gamma(m+1)\Gamma(n+1)\Gamma(b+1)\Phi_1(\mathfrak{p})\Phi_2(\mathfrak{q})\Phi_3(\mathfrak{r})}{(\psi_1(\mathfrak{p})\log a)^{m+1}(\psi_2(\mathfrak{q})\log a)^{n+1}(\psi_3(\mathfrak{r})\log a)^{b+1}},$ $\operatorname{Re}(\psi_1(\mathfrak{p})) > 0, \operatorname{Re}(\psi_2(\mathfrak{q})) > 0, \operatorname{Re}(\psi_3(\mathfrak{r})) > 0, m, n, b > -1.$

4. Basic Operational Properties

Sometimes, it is a laborious process to compute new modified triple integral transform using definition 4, however the same can be computed with the help of some basic operational properties.

Theorem 2: (Linearity Property). *If $\mathfrak{F}(\mathfrak{p}, \mathfrak{q}, \mathfrak{r})$ and $\mathcal{G}(\mathfrak{p}, \mathfrak{q}, \mathfrak{r})$ are the new modified triple integral transforms of $\mathfrak{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z})$ and $g(\mathfrak{x}, \mathfrak{y}, \mathfrak{z})$ respectively, then for any constants C_1 and C_2 ,*

$$\begin{aligned} & \mathcal{T}_{a_3}\{C_1\mathfrak{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z}) + C_2g(\mathfrak{x}, \mathfrak{y}, \mathfrak{z}); \mathfrak{x} \rightarrow \mathfrak{p}, \mathfrak{y} \rightarrow \mathfrak{q}, \mathfrak{z} \rightarrow \mathfrak{r}\} \\ &= C_1\mathfrak{F}(\mathfrak{p}, \mathfrak{q}, \mathfrak{r}) + C_2\mathcal{G}(\mathfrak{p}, \mathfrak{q}, \mathfrak{r}). \end{aligned}$$

Proof: By using definition of new modified triple integral transform and linearity property of integrals, we get,

$$\begin{aligned} & \mathcal{T}_{a_3}\{C_1\mathfrak{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z}) + C_2g(\mathfrak{x}, \mathfrak{y}, \mathfrak{z})\} \\ &= \Phi_1(\mathfrak{p})\Phi_2(\mathfrak{q})\Phi_3(\mathfrak{r}) \\ & \times \int_0^\infty \int_0^\infty \int_0^\infty [C_1\mathfrak{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z}) + C_2g(\mathfrak{x}, \mathfrak{y}, \mathfrak{z})] \\ & \times a^{-(\psi_1(\mathfrak{p})\mathfrak{x} + \psi_2(\mathfrak{q})\mathfrak{y} + \psi_3(\mathfrak{r})\mathfrak{z})} d\mathfrak{x} d\mathfrak{y} d\mathfrak{z} \\ &= C_1\Phi_1(\mathfrak{p})\Phi_2(\mathfrak{q})\Phi_3(\mathfrak{r}) \\ & \times \int_0^\infty \int_0^\infty \int_0^\infty \mathfrak{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z}) a^{-(\psi_1(\mathfrak{p})\mathfrak{x} + \psi_2(\mathfrak{q})\mathfrak{y} + \psi_3(\mathfrak{r})\mathfrak{z})} d\mathfrak{x} d\mathfrak{y} d\mathfrak{z} \\ & + C_2\Phi_1(\mathfrak{p})\Phi_2(\mathfrak{q})\Phi_3(\mathfrak{r}) \\ & \times \int_0^\infty \int_0^\infty \int_0^\infty g(\mathfrak{x}, \mathfrak{y}, \mathfrak{z}) a^{-(\psi_1(\mathfrak{p})\mathfrak{x} + \psi_2(\mathfrak{q})\mathfrak{y} + \psi_3(\mathfrak{r})\mathfrak{z})} d\mathfrak{x} d\mathfrak{y} d\mathfrak{z} \\ &= C_1\mathfrak{F}(\mathfrak{p}, \mathfrak{q}, \mathfrak{r}) + C_2\mathcal{G}(\mathfrak{p}, \mathfrak{q}, \mathfrak{r}). \end{aligned}$$

Theorem 3: (Change of Scale Property). *If $\mathfrak{F}(\mathfrak{p}, \mathfrak{q}, \mathfrak{r})$ is the new modified triple integral transform of $\mathfrak{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z})$, then*

$$\begin{aligned} & \mathcal{T}_{a_3}\{\mathfrak{f}(b\mathfrak{x}, c\mathfrak{y}, d\mathfrak{z}), \mathfrak{x} \rightarrow \mathfrak{p}, \mathfrak{y} \rightarrow \mathfrak{q}, \mathfrak{z} \rightarrow \mathfrak{r}\} \\ &= \frac{1}{bcd} \mathfrak{F}(\mathfrak{p}, \mathfrak{q}, \mathfrak{r}) \Big|_{\psi_1(\mathfrak{p}) \rightarrow \frac{\psi_1(\mathfrak{p})}{b}, \psi_2(\mathfrak{q}) \rightarrow \frac{\psi_2(\mathfrak{q})}{c}, \psi_3(\mathfrak{r}) \rightarrow \frac{\psi_3(\mathfrak{r})}{d}} \end{aligned}$$

where $b, c, d > 0$.

Proof: By using substitution $u = b\mathfrak{x}, v = c\mathfrak{y}, w = d\mathfrak{z}$, for $b, c, d > 0$, we get,

$$\begin{aligned} & \mathcal{T}_{a_3}\{\mathfrak{f}(b\mathfrak{x}, c\mathfrak{y}, d\mathfrak{z}); \mathfrak{x} \rightarrow \mathfrak{p}, \mathfrak{y} \rightarrow \mathfrak{q}, \mathfrak{z} \rightarrow \mathfrak{r}\} \\ &= \phi_1(\mathfrak{p})\phi_2(\mathfrak{q})\phi_3(\mathfrak{r}) \\ &\times \int_0^\infty \int_0^\infty \int_0^\infty \mathfrak{f}(b\mathfrak{x}, c\mathfrak{y}, d\mathfrak{z}) a^{-(\psi_1(\mathfrak{p})\mathfrak{x} + \psi_2(\mathfrak{q})\mathfrak{y} + \psi_3(\mathfrak{r})\mathfrak{z})} d\mathfrak{x} d\mathfrak{y} d\mathfrak{z} \\ &= \frac{1}{bcd} \phi_1(\mathfrak{p})\phi_2(\mathfrak{q})\phi_3(\mathfrak{r}) \\ &\times \int_0^\infty \int_0^\infty \int_0^\infty \mathfrak{f}(u, v, w) a^{-(\psi_1(\mathfrak{p})\frac{u}{b} + \psi_2(\mathfrak{q})\frac{v}{c} + \psi_3(\mathfrak{r})\frac{w}{d})} du dv dw \\ &= \frac{1}{bcd} \mathfrak{F}(\mathfrak{p}, \mathfrak{q}, \mathfrak{r}) \Big|_{\psi_1(\mathfrak{p}) \rightarrow \frac{\psi_1(\mathfrak{p})}{b}, \psi_2(\mathfrak{q}) \rightarrow \frac{\psi_2(\mathfrak{q})}{c}, \psi_3(\mathfrak{r}) \rightarrow \frac{\psi_3(\mathfrak{r})}{d}}, b, c, d > 0. \end{aligned}$$

Theorem 4: (First Shifting Property). *If $\mathfrak{F}(\mathfrak{p}, \mathfrak{q}, \mathfrak{r})$ is the new modified triple integral transform of $\mathfrak{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z})$, then*

$$\begin{aligned} & \mathcal{T}_{a_3}\{a^{-(b\mathfrak{x} + c\mathfrak{y} + d\mathfrak{z})}\mathfrak{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z}); \mathfrak{x} \rightarrow \mathfrak{p}, \mathfrak{y} \rightarrow \mathfrak{q}, \mathfrak{z} \rightarrow \mathfrak{r}\} \\ &= \mathfrak{F}(\mathfrak{p}, \mathfrak{q}, \mathfrak{r}) \Big|_{\psi_1(\mathfrak{p}) \rightarrow \psi_1(\mathfrak{p}) + b, \psi_2(\mathfrak{q}) \rightarrow \psi_2(\mathfrak{q}) + c, \psi_3(\mathfrak{r}) \rightarrow \psi_3(\mathfrak{r}) + d}. \end{aligned}$$

Proof: Using definition of new modified triple integral transform, we get,

$$\begin{aligned} & \mathcal{T}_{a_3}\{a^{-(b\mathfrak{x} + c\mathfrak{y} + d\mathfrak{z})}\mathfrak{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z}); \mathfrak{x} \rightarrow \mathfrak{p}, \mathfrak{y} \rightarrow \mathfrak{q}, \mathfrak{z} \rightarrow \mathfrak{r}\} \\ &= \phi_1(\mathfrak{p})\phi_2(\mathfrak{q})\phi_3(\mathfrak{r}) \\ &\times \int_0^\infty \int_0^\infty \int_0^\infty a^{-(b\mathfrak{x} + c\mathfrak{y} + d\mathfrak{z})}\mathfrak{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z}) a^{-(\psi_1(\mathfrak{p})\mathfrak{x} + \psi_2(\mathfrak{q})\mathfrak{y} + \psi_3(\mathfrak{r})\mathfrak{z})} d\mathfrak{x} d\mathfrak{y} d\mathfrak{z} \\ &= \phi_1(\mathfrak{p})\phi_2(\mathfrak{q})\phi_3(\mathfrak{r}) \\ &\times \int_0^\infty \int_0^\infty \int_0^\infty \mathfrak{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z}) a^{-[(\psi_1(\mathfrak{p}) + b)\mathfrak{x} + (\psi_2(\mathfrak{q}) + c)\mathfrak{y} + (\psi_3(\mathfrak{r}) + d)\mathfrak{z}]} d\mathfrak{x} d\mathfrak{y} d\mathfrak{z} \\ &= \mathfrak{F}(\mathfrak{p}, \mathfrak{q}, \mathfrak{r}) \Big|_{\psi_1(\mathfrak{p}) \rightarrow \psi_1(\mathfrak{p}) + b, \psi_2(\mathfrak{q}) \rightarrow \psi_2(\mathfrak{q}) + c, \psi_3(\mathfrak{r}) \rightarrow \psi_3(\mathfrak{r}) + d}. \end{aligned}$$

Theorem 5: *If $\mathfrak{F}(\mathfrak{p}), \mathcal{G}(\mathfrak{p})$ and $\mathcal{H}(\mathfrak{r})$ are new modified integral transforms of $\mathfrak{f}(\mathfrak{x}), g(\mathfrak{y})$ and $h(\mathfrak{z})$ respectively then,*

$$\begin{aligned} & \mathcal{T}_{a_3}\{\mathfrak{f}(b\mathfrak{x})g(c\mathfrak{y})h(d\mathfrak{z}); \mathfrak{x} \rightarrow \mathfrak{p}, \mathfrak{y} \rightarrow \mathfrak{q}, \mathfrak{z} \rightarrow \mathfrak{r}\} \\ &= \frac{1}{b} \mathfrak{F}(\mathfrak{p}) \Big|_{\psi_1(\mathfrak{p}) \rightarrow \frac{\psi_1(\mathfrak{p})}{b}} \frac{1}{c} \mathcal{G}(\mathfrak{p}) \Big|_{\psi_2(\mathfrak{q}) \rightarrow \frac{\psi_2(\mathfrak{q})}{c}} \frac{1}{d} \mathcal{H}(\mathfrak{r}) \Big|_{\psi_3(\mathfrak{r}) \rightarrow \frac{\psi_3(\mathfrak{r})}{d}}, b, c, d > 0. \end{aligned}$$

Proof: By using definition of new modified triple integral transform and change of scale property of new modified integral transform [11], we get,

$$\begin{aligned} & \mathcal{T}_{a_3}\{\mathfrak{f}(b\mathfrak{x})g(c\mathfrak{y})h(d\mathfrak{z}); \mathfrak{x} \rightarrow \mathfrak{p}, \mathfrak{y} \rightarrow \mathfrak{q}, \mathfrak{z} \rightarrow \mathfrak{r}\} \\ &= \phi_1(\mathfrak{p})\phi_2(\mathfrak{q})\phi_3(\mathfrak{r}) \\ &\times \int_0^\infty \int_0^\infty \int_0^\infty \mathfrak{f}(b\mathfrak{x})g(c\mathfrak{y})h(d\mathfrak{z}) a^{-(\psi_1(\mathfrak{p})\mathfrak{x} + \psi_2(\mathfrak{q})\mathfrak{y} + \psi_3(\mathfrak{r})\mathfrak{z})} d\mathfrak{x} d\mathfrak{y} d\mathfrak{z} \\ &= \mathcal{T}_a\{\mathfrak{f}(b\mathfrak{x}), \mathfrak{x} \rightarrow \mathfrak{p}\} \cdot \mathcal{T}_a\{g(c\mathfrak{y}), \mathfrak{y} \rightarrow \mathfrak{q}\} \cdot \mathcal{T}_a\{h(d\mathfrak{z}), \mathfrak{z} \rightarrow \mathfrak{r}\} \end{aligned}$$

$$= \frac{1}{b} \mathfrak{F}(\mathfrak{p})|_{\psi_1(\mathfrak{p}) \rightarrow \frac{\psi_1(\mathfrak{p})}{b}} \frac{1}{c} \mathcal{G}(\mathfrak{p})|_{\psi_2(\mathfrak{q}) \rightarrow \frac{\psi_2(\mathfrak{q})}{c}} \frac{1}{d} \mathcal{H}(\mathfrak{r})|_{\psi_3(\mathfrak{r}) \rightarrow \frac{\psi_3(\mathfrak{r})}{d}}, b, c, d > 0.$$

Theorem 6: If $\mathfrak{F}(\mathfrak{p})$, $\mathcal{G}(\mathfrak{q})$ and $\mathcal{H}(\mathfrak{r})$ are new modified integral transforms of $\mathfrak{f}(\mathfrak{x})$, $g(\mathfrak{y})$ and $h(\mathfrak{z})$, then

$$\begin{aligned} (1) \quad \mathcal{T}_{a_3}\{\mathfrak{f}(\mathfrak{x}); \mathfrak{x} \rightarrow \mathfrak{p}, \mathfrak{y} \rightarrow \mathfrak{q}, \mathfrak{z} \rightarrow \mathfrak{r}\} &= \frac{\phi_2(\mathfrak{q})\phi_3(\mathfrak{r})}{\psi_2(\mathfrak{q})\psi_3(\mathfrak{r})(\log a)^2} \mathfrak{F}(\mathfrak{p}). \\ (2) \quad \mathcal{T}_{a_3}\{g(\mathfrak{y}); \mathfrak{x} \rightarrow \mathfrak{p}, \mathfrak{y} \rightarrow \mathfrak{q}, \mathfrak{z} \rightarrow \mathfrak{r}\} &= \frac{\phi_1(\mathfrak{p})\phi_3(\mathfrak{r})}{\psi_1(\mathfrak{p})\psi_3(\mathfrak{r})(\log a)^2} \mathcal{G}(\mathfrak{q}). \\ (3) \quad \mathcal{T}_{a_3}\{h(\mathfrak{z}); \mathfrak{x} \rightarrow \mathfrak{p}, \mathfrak{y} \rightarrow \mathfrak{q}, \mathfrak{z} \rightarrow \mathfrak{r}\} &= \frac{\phi_1(\mathfrak{p})\phi_2(\mathfrak{q})}{\psi_1(\mathfrak{p})\psi_2(\mathfrak{q})(\log a)^2} \mathcal{H}(\mathfrak{r}). \end{aligned}$$

Proof: By using definition of new modified triple integral transform, we get,

$$\begin{aligned} &\mathcal{T}_{a_3}\{\mathfrak{f}(\mathfrak{x}); \mathfrak{x} \rightarrow \mathfrak{p}, \mathfrak{y} \rightarrow \mathfrak{q}, \mathfrak{z} \rightarrow \mathfrak{r}\} \\ &= \phi_1(\mathfrak{p})\phi_2(\mathfrak{q})\phi_3(\mathfrak{r}) \\ &\times \int_0^\infty \int_0^\infty \int_0^\infty \mathfrak{f}(\mathfrak{x}) a^{-(\psi_1(\mathfrak{p})\mathfrak{x} + \psi_2(\mathfrak{q})\mathfrak{y} + \psi_3(\mathfrak{r})\mathfrak{z})} d\mathfrak{x} d\mathfrak{y} d\mathfrak{z} \\ &= \phi_1(\mathfrak{p}) \int_0^\infty \mathfrak{f}(\mathfrak{x}) a^{-\psi_1(\mathfrak{p})\mathfrak{x}} d\mathfrak{x} \times \phi_2(\mathfrak{q}) \int_0^\infty 1. a^{-\psi_2(\mathfrak{q})\mathfrak{y}} d\mathfrak{y} \\ &\times \phi_3(\mathfrak{r}) \int_0^\infty 1. a^{-\psi_3(\mathfrak{r})\mathfrak{z}} d\mathfrak{z} \\ &= \mathcal{T}_a\{\mathfrak{f}(\mathfrak{x}), \mathfrak{x} \rightarrow \mathfrak{p}\} \cdot \mathcal{T}_a\{1, \mathfrak{y} \rightarrow \mathfrak{q}\} \cdot \mathcal{T}_a\{1, \mathfrak{z} \rightarrow \mathfrak{r}\} \\ &= \frac{\phi_2(\mathfrak{q})\phi_3(\mathfrak{r})}{\psi_2(\mathfrak{q})\psi_3(\mathfrak{r})(\log a)^2} \mathfrak{F}(\mathfrak{p}). \end{aligned}$$

Similarly, we can prove the results (2) and (3).

Theorem 7: (Second Shifting Property). If $\mathfrak{F}(\mathfrak{p}, \mathfrak{q}, \mathfrak{r})$ is new modified triple integral transform of $\mathfrak{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z})$, then

$$\begin{aligned} &\mathcal{T}_{a_3}\{\mathfrak{f}(\mathfrak{x} - \zeta, \mathfrak{y} - \eta, \mathfrak{z} - \gamma) H(\mathfrak{x} - \zeta, \mathfrak{y} - \eta, \mathfrak{z} - \gamma); \mathfrak{x} \rightarrow \mathfrak{p}, \mathfrak{y} \rightarrow \mathfrak{q}, \mathfrak{z} \rightarrow \mathfrak{r}\} \\ &= a^{-(\psi(\mathfrak{p})\zeta + \psi_2(\mathfrak{q})\eta + \psi_3(\mathfrak{r})\gamma)} \mathfrak{F}(\mathfrak{p}, \mathfrak{q}, \mathfrak{r}), \end{aligned}$$

where, $H(\mathfrak{x} - \zeta, \mathfrak{y} - \eta, \mathfrak{z} - \gamma)$ is the Heaviside unit step function defined by,

$$H(\mathfrak{x} - \zeta, \mathfrak{y} - \eta, \mathfrak{z} - \gamma) = \begin{cases} 1, & \mathfrak{x} \geq \zeta, \quad \mathfrak{y} \geq \eta, \quad \mathfrak{z} \geq \gamma \\ 0, & \mathfrak{x} < \zeta, \quad \mathfrak{y} < \eta, \quad \mathfrak{z} < \gamma. \end{cases} \quad (10)$$

Proof: By using definition of new modified triple integral transform, we get,

$$\begin{aligned} &\mathcal{T}_{a_3}\{\mathfrak{f}(\mathfrak{x} - \zeta, \mathfrak{y} - \eta, \mathfrak{z} - \gamma) H(\mathfrak{x} - \zeta, \mathfrak{y} - \eta, \mathfrak{z} - \gamma); \mathfrak{x} \rightarrow \mathfrak{p}, \mathfrak{y} \rightarrow \mathfrak{q}, \mathfrak{z} \rightarrow \mathfrak{r}\} \\ &= \phi_1(\mathfrak{p})\phi_2(\mathfrak{q})\phi_3(\mathfrak{r}) \\ &\times \int_\zeta^\infty \int_\eta^\infty \int_\gamma^\infty \mathfrak{f}(\mathfrak{x} - \zeta, \mathfrak{y} - \eta, \mathfrak{z} - \gamma) \\ &\times a^{-(\psi_1(\mathfrak{p})\mathfrak{x} + \psi_2(\mathfrak{q})\mathfrak{y} + \psi_3(\mathfrak{r})\mathfrak{z})} d\mathfrak{x} d\mathfrak{y} d\mathfrak{z}. \end{aligned} \quad (11)$$

By substituting $\mathfrak{x} - \zeta = l$, $\mathfrak{y} - \eta = m$ and $\mathfrak{z} - \gamma = n$ in equation (11), we get,

$$\begin{aligned} &\mathcal{T}_{a_3}\{\mathfrak{f}(\mathfrak{x} - \zeta, \mathfrak{y} - \eta, \mathfrak{z} - \gamma) H(\mathfrak{x} - \zeta, \mathfrak{y} - \eta, \mathfrak{z} - \gamma); \mathfrak{x} \rightarrow \mathfrak{p}, \mathfrak{y} \rightarrow \mathfrak{q}, \mathfrak{z} \rightarrow \mathfrak{r}\} \\ &= \phi_1(\mathfrak{p})\phi_2(\mathfrak{q})\phi_3(\mathfrak{r}) \\ &\times \int_0^\infty \int_0^\infty \int_0^\infty \mathfrak{f}(l, m, n) a^{-(\psi_1(\mathfrak{p})(l+\zeta) + \psi_2(\mathfrak{q})(m+\eta) + \psi_3(\mathfrak{r})(n+\gamma))} dl dm dn \end{aligned}$$

$$\begin{aligned}
&= \phi_1(p)\phi_2(q)\phi_3(r)a^{-(\psi_1(p)\zeta+\psi_2(q)\eta+\psi_3(r)\gamma)} \\
&\times \int_0^\infty \int_0^\infty \int_0^\infty \tilde{f}(l, m, n)a^{-(\psi_1(p)l+\psi_2(q)m+\psi_3(r)n)} dl dm dn \\
&= a^{-(\psi_1(p)\zeta+\psi_2(q)\eta+\psi_3(r)\gamma)} \mathfrak{F}(p, q, r).
\end{aligned}$$

Theorem 8: (Derivative Property). *If $\tilde{f}(x, y, z)$ is piecewise continuous on $\mathbb{R}_+^3$, and is of $O(e^{bx+cy+dz})$ for some $b, c, d > 0$, then*

$$\begin{aligned}
(1) \quad &\mathcal{T}_{a_3}\{\tilde{f}_x; x \rightarrow p, y \rightarrow q, z \rightarrow r\} \\
&= \psi_1(p)\log a \mathcal{T}_{a_3}\{\tilde{f}(x, y, z)\} - \phi_1(p) \mathcal{T}_{a_2}\{\tilde{f}(0, y, z)\}.
\end{aligned}$$

$$\begin{aligned}
(2) \quad &\mathcal{T}_{a_3}\{\tilde{f}_y; x \rightarrow p, y \rightarrow q, z \rightarrow r\} \\
&= \psi_2(q)\log a \mathcal{T}_{a_3}\{\tilde{f}(x, y, z)\} - \phi_2(q) \mathcal{T}_{a_2}\{\tilde{f}(x, 0, z)\}.
\end{aligned}$$

$$\begin{aligned}
(3) \quad &\mathcal{T}_{a_3}\{\tilde{f}_{xx}; x \rightarrow p, y \rightarrow q, z \rightarrow r\} \\
&= (\psi_1(p)\log a)^2 \mathcal{T}_{a_3}\{\tilde{f}(x, y, z)\} - \phi_1(p) \mathcal{T}_{a_2}\{\tilde{f}_x(0, y, z)\} \\
&\quad - \phi_1(p)\psi_1(p)\log a \mathcal{T}_{a_2}\{\tilde{f}(0, y, z)\}.
\end{aligned}$$

$$\begin{aligned}
(4) \quad &\mathcal{T}_{a_3}\{\tilde{f}_{yy}; x \rightarrow p, y \rightarrow q, z \rightarrow r\} \\
&= (\psi_2(q)\log a)^2 \mathcal{T}_{a_3}\{\tilde{f}(x, y, z)\} - \phi_2(q) \mathcal{T}_{a_2}\{\tilde{f}_y(x, 0, z)\} \\
&\quad - \phi_2(q)\psi_2(q)\log a \mathcal{T}_{a_2}\{\tilde{f}(x, 0, z)\}.
\end{aligned}$$

$$\begin{aligned}
(5) \quad &\mathcal{T}_{a_3}\{\tilde{f}_{zz}; x \rightarrow p, y \rightarrow q, z \rightarrow r\} \\
&= (\psi_3(r)\log a)^2 \mathcal{T}_{a_3}\{\tilde{f}(x, y, z)\} - \phi_3(r) \mathcal{T}_{a_2}\{\tilde{f}_z(x, y, 0)\} \\
&\quad - \phi_3(r)\psi_3(r)\log a \mathcal{T}_{a_2}\{\tilde{f}(x, y, 0)\}.
\end{aligned}$$

$$\begin{aligned}
(6) \quad &\mathcal{T}_{a_3}\{\tilde{f}_{xxx}; x \rightarrow p, y \rightarrow q, z \rightarrow r\} \\
&= (\psi_1(p)\log a)^3 \mathcal{T}_{a_3}\{\tilde{f}(x, y, z)\} - \phi_1(p)\mathcal{T}_{a_2}\{f_{xx}(0, y, z)\} \\
&\quad - \phi_1(p)(\psi_1(p)\log a)^2 \mathcal{T}_{a_2}\{\tilde{f}(0, y, z)\} \\
&\quad - \phi_1(p)\psi_1(p)\log a \mathcal{T}_{a_2}\{f_x(0, y, z)\}.
\end{aligned}$$

$$\begin{aligned}
(7) \quad &\mathcal{T}_{a_3}\{\tilde{f}_{yyy}; x \rightarrow p, y \rightarrow q, z \rightarrow r\} \\
&= (\psi_2(q)\log a)^3 \mathcal{T}_{a_3}\{\tilde{f}(x, y, z)\} - \phi_2(q)\mathcal{T}_{a_2}\{f_{yy}(x, 0, z)\} \\
&\quad - \phi_2(q)(\psi_2(q)\log a)^2 \mathcal{T}_{a_2}\{\tilde{f}(x, 0, z)\} \\
&\quad - \phi_2(q)\psi_2(q)\log a \mathcal{T}_{a_2}\{f_y(x, 0, z)\}.
\end{aligned}$$

$$\begin{aligned}
(8) \quad &\mathcal{T}_{a_3}\{\tilde{f}_{zzz}; x \rightarrow p, y \rightarrow q, z \rightarrow r\} \\
&= (\psi_3(r)\log a)^3 \mathcal{T}_{a_3}\{\tilde{f}(x, y, z)\} - \phi_3(r)\mathcal{T}_{a_2}\{f_{zz}(x, y, 0)\} \\
&\quad - \phi_3(r)(\psi_3(r)\log a)^2 \mathcal{T}_{a_2}\{\tilde{f}(x, y, 0)\} \\
&\quad - \phi_3(r)\psi_3(r)\log a \mathcal{T}_{a_2}\{f_z(x, y, 0)\}.
\end{aligned}$$

$$\begin{aligned}
& \mathcal{T}_{a_3}\{\mathfrak{f}_{\mathfrak{x}\mathfrak{y}\mathfrak{z}}; \mathfrak{x} \rightarrow \mathfrak{p}, \mathfrak{y} \rightarrow \mathfrak{q}, \mathfrak{z} \rightarrow \mathfrak{r}\} \\
&= \psi_1(\mathfrak{p})\psi_2(\mathfrak{q})\psi_3(\mathfrak{r})(\log a)^3 \mathcal{T}_{a_3}\{\mathfrak{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z})\} \\
&\quad - \psi_1(\mathfrak{p})\psi_2(\mathfrak{q})\phi_3(\mathfrak{r})(\log a)^2 \mathcal{T}_{a_2}\{\mathfrak{f}(\mathfrak{x}, \mathfrak{y}, 0)\} \\
&\quad - \psi_1(\mathfrak{p})\psi_3(\mathfrak{r})\phi_2(\mathfrak{q})(\log a)^2 \mathcal{T}_{a_2}\{\mathfrak{f}(\mathfrak{x}, 0, \mathfrak{z})\} \\
(9) \quad & - \psi_2(\mathfrak{q})\psi_3(\mathfrak{r})\phi_1(\mathfrak{p})(\log a)^2 \mathcal{T}_{a_2}\{\mathfrak{f}(0, \mathfrak{y}, \mathfrak{z})\} \\
& + \psi_1(\mathfrak{p})\phi_2(\mathfrak{q})\phi_3(\mathfrak{r})\log a \mathcal{T}_a\{\mathfrak{f}(\mathfrak{x}, 0, 0)\} \\
& + \phi_1(\mathfrak{p})\phi_3(\mathfrak{r})\psi_2(\mathfrak{q})\log a \mathcal{T}_a\{\mathfrak{f}(0, \mathfrak{y}, 0)\} \\
& + \psi_3(\mathfrak{r})\phi_1(\mathfrak{p})\phi_2(\mathfrak{q})\log a \mathcal{T}_a\{\mathfrak{f}(0, 0, \mathfrak{z})\} \\
& - \phi_1(\mathfrak{p})\phi_2(\mathfrak{q})\phi_3(\mathfrak{r})\mathfrak{f}(0, 0, 0).
\end{aligned}$$

Proof: By using definition of new modified triple integral transform, we get,

$$\begin{aligned}
& \mathcal{T}_{a_3}\{\mathfrak{f}_{\mathfrak{x}}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z}); \mathfrak{x} \rightarrow \mathfrak{p}, \mathfrak{y} \rightarrow \mathfrak{q}, \mathfrak{z} \rightarrow \mathfrak{r}\} \\
&= \phi_1(\mathfrak{p})\phi_2(\mathfrak{q})\phi_3(\mathfrak{r}) \int_0^\infty \int_0^\infty \int_0^\infty \mathfrak{f}_{\mathfrak{x}}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z}) a^{-(\psi_1(\mathfrak{p})\mathfrak{x} + \psi_2(\mathfrak{q})\mathfrak{y} + \psi_3(\mathfrak{r})\mathfrak{z})} d\mathfrak{x} d\mathfrak{y} d\mathfrak{z} \\
&= \phi_1(\mathfrak{p})\phi_2(\mathfrak{q})\phi_3(\mathfrak{r}) \int_0^\infty \int_0^\infty \left(\int_0^\infty a^{-\psi_1(\mathfrak{p})\mathfrak{x}} \mathfrak{f}_{\mathfrak{x}}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z}) d\mathfrak{x} \right) a^{-(\psi_2(\mathfrak{q})\mathfrak{y} + \psi_3(\mathfrak{r})\mathfrak{z})} d\mathfrak{y} d\mathfrak{z} \\
&= \phi_1(\mathfrak{p})\phi_2(\mathfrak{q})\phi_3(\mathfrak{r}) \\
&\quad \times \int_0^\infty \int_0^\infty \{-\mathfrak{f}(0, \mathfrak{y}, \mathfrak{z}) + \psi_1(\mathfrak{p})(\log a) \int_0^\infty a^{-\psi_1(\mathfrak{p})\mathfrak{x}} \mathfrak{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z}) d\mathfrak{x}\} \\
&\quad \times a^{-(\psi_2(\mathfrak{q})\mathfrak{y} + \psi_3(\mathfrak{r})\mathfrak{z})} d\mathfrak{y} d\mathfrak{z} \\
&= -\phi_1(\mathfrak{p})\mathcal{T}_{a_2}\{\mathfrak{f}(0, \mathfrak{y}, \mathfrak{z})\} + \psi_1(\mathfrak{p})(\log a)\mathcal{T}_{a_3}\{\mathfrak{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z})\}.
\end{aligned}$$

On the same line we can prove the transform of remaining partial derivatives.

Theorem 9: (Integration Property). *If $\mathfrak{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z})$ is piecewise continuous on $\mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}_+$, and is of $O(e^{b\mathfrak{x}+c\mathfrak{y}+d\mathfrak{z}})$ for some $b, c, d > 0$, then*

$$\begin{aligned}
& \mathcal{T}_{a_3}\{\int_0^{\mathfrak{x}} \int_0^{\mathfrak{y}} \int_0^{\mathfrak{z}} \mathfrak{f}(l, m, n) dl dm dn; \mathfrak{x} \rightarrow \mathfrak{p}, \mathfrak{y} \rightarrow \mathfrak{q}, \mathfrak{z} \rightarrow \mathfrak{r}\} \\
&= \frac{\mathfrak{F}(\mathfrak{p}, \mathfrak{q}, \mathfrak{r})}{\psi_1(\mathfrak{p})\psi_2(\mathfrak{q})\psi_3(\mathfrak{r})(\log a)^3},
\end{aligned}$$

for $Re(\psi_1(\mathfrak{p})) > 0, Re(\psi_2(\mathfrak{q})) > 0$, and $Re(\psi_3(\mathfrak{r})) > 0$, where $\mathfrak{F}(\mathfrak{p}, \mathfrak{q}, \mathfrak{r})$ is the new modified triple integral transform of $\mathfrak{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z})$.

Proof: Let us define a function

$$g(\mathfrak{x}, \mathfrak{y}, \mathfrak{z}) = \int_0^{\mathfrak{x}} \int_0^{\mathfrak{y}} \int_0^{\mathfrak{z}} \mathfrak{f}(l, m, n) dl dm dn,$$

which is a continuous function, since $\mathfrak{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z})$ is piecewise continuous; furthermore, $g(0, 0, 0) = 0$ and $g_{\mathfrak{x}\mathfrak{y}\mathfrak{z}}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z}) = \mathfrak{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z})$. Hence, g satisfies the conditions of differentiable property of transform, and therefore, it leads to,

$$\begin{aligned}
& \mathfrak{F}(\mathfrak{p}, \mathfrak{q}, \mathfrak{r}) \\
&= \mathcal{T}_{a_3}\{\mathfrak{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z}); \mathfrak{x} \rightarrow \mathfrak{p}, \mathfrak{y} \rightarrow \mathfrak{q}, \mathfrak{z} \rightarrow \mathfrak{r}\} \\
&= \mathcal{T}_{a_3}\{g_{\mathfrak{x}\mathfrak{y}\mathfrak{z}}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z}); \mathfrak{x} \rightarrow \mathfrak{p}, \mathfrak{y} \rightarrow \mathfrak{q}, \mathfrak{z} \rightarrow \mathfrak{r}\} \\
&= \psi_1(\mathfrak{p})\psi_2(\mathfrak{q})\psi_3(\mathfrak{r})(\log a)^3 \mathcal{T}_{a_3}\{g(\mathfrak{x}, \mathfrak{y}, \mathfrak{z}); \mathfrak{x} \rightarrow \mathfrak{p}, \mathfrak{y} \rightarrow \mathfrak{q}, \mathfrak{z} \rightarrow \mathfrak{r}\}
\end{aligned}$$

$$\begin{aligned}
& -\psi_1(p)\psi_2(q)\phi_3(r)(\log a)^2\mathcal{T}_{a_2}\{g(x, y, 0), x \rightarrow p, y \rightarrow q\} \\
& -\psi_2(q)\psi_3(r)\phi_1(p)(\log a)^2\mathcal{T}_{a_2}\{g(0, y, z), y \rightarrow q, z \rightarrow r\} \\
& -\psi_1(p)\psi_3(r)\phi_2(q)(\log a)^2\mathcal{T}_{a_2}\{g(x, 0, z), x \rightarrow p, z \rightarrow r\} \\
& +\psi_1(p)\phi_2(q)\phi_3(r)(\log a)\mathcal{T}_a\{g(x, 0, 0), x \rightarrow p\} \\
& +\psi_2(q)\phi_1(p)\phi_3(r)(\log a)\mathcal{T}_a\{g(0, y, 0), y \rightarrow q\} \\
& +\psi_3(r)\phi_1(p)\phi_2(q)(\log a)\mathcal{T}_a\{g(0, 0, z), z \rightarrow r\} \\
& -\phi_1(p)\phi_2(q)\phi_3(r)g(0, 0, 0) \\
& = \psi_1(p)\psi_2(q)\psi_3(r)(\log a)^3\mathcal{T}_{a_3}\{g(x, y, z); x \rightarrow p, y \rightarrow q, z \rightarrow r\} \\
& \mathcal{T}_{a_3}\left\{\int_0^x \int_0^y \int_0^z f(l, m, n) dl dm dn; x \rightarrow p, y \rightarrow q, z \rightarrow r\right\} \\
& = \frac{\mathfrak{F}(p, q, x)}{\psi_1(p)\psi_2(q)\psi_3(r)(\log a)^3},
\end{aligned}$$

for $\text{Re}(\psi_1(p)) > 0$, $\text{Re}(\psi_2(q)) > 0$, and $\text{Re}(\psi_3(r)) > 0$.

Theorem 10: (Multiplication by x property). *If $\mathfrak{F}(p, q, r)$ is the new modified triple integral transform of $f(x, y, z)$, then*

$$\mathcal{T}_{a_3}\{xf(x, y, z), x \rightarrow p, y \rightarrow q, z \rightarrow r\} = \frac{-\phi_1(p)}{\psi_1'(p) \log a} \frac{\partial}{\partial p} \left(\frac{\mathfrak{F}(p, q, x)}{\phi_1(p)} \right)$$

Proof: By using definition of new modified triple integral transform, we have,

$$\frac{\mathfrak{F}(p, q, x)}{\phi_1(p)\phi_2(q)\phi_3(r)} = \int_0^\infty \int_0^\infty \int_0^\infty f(x, y, z) a^{-(\psi_1(p)x + \psi_2(q)y + \psi_3(r)z)} dx dy dz.$$

Differentiating above equation partially with respect to p , we get,

$$\begin{aligned}
\frac{\partial}{\partial p} \left(\frac{\mathfrak{F}(p, q, x)}{\phi_1(p)\phi_2(q)\phi_3(r)} \right) &= -\psi_1'(p) \log a \int_0^\infty \int_0^\infty \int_0^\infty xf(x, y, z) \\
&\quad \times a^{-(\psi_1(p)x + \psi_2(q)y + \psi_3(r)z)} dx dy dz \\
&= -\psi_1'(p) \log a \frac{\mathcal{T}_{a_3}\{xf(x, y, z)\}}{\phi_1(p)\phi_2(q)\phi_3(r)}. \\
\therefore \mathcal{T}_{a_3}\{xf(x, y, z)\} &= \frac{\phi_1(p)\phi_2(q)\phi_3(r)}{-\psi_1'(p) \log a} \frac{\partial}{\partial p} \left(\frac{\mathfrak{F}(p, q, x)}{\phi_1(p)\phi_2(q)\phi_3(r)} \right) \\
\Rightarrow \mathcal{T}_{a_3}\{xf(x, y, z)\} &= \frac{-\phi_1(p)}{\psi_1'(p) \log a} \frac{\partial}{\partial p} \left(\frac{\mathfrak{F}(p, q, x)}{\phi_1(p)} \right)
\end{aligned}$$

Theorem 11: (Division by xyz property). *If $f(x, y, z)$ is piecewise continuous on $\mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}_+$, and is of $O(e^{bx+cy+dz})$ for some $b, c, d > 0$, then*

$$\begin{aligned}
& \mathcal{T}_{a_3} \left\{ \frac{f(x, y, z)}{xyz}; x \rightarrow p, y \rightarrow q, z \rightarrow r \right\} \\
&= \phi_1(p)\phi_2(q)\phi_3(r)(\log a)^3 \\
&\times \int_p^\infty \int_q^\infty \int_r^\infty \frac{\mathfrak{F}(p, q, x)}{\phi_1(p)\phi_2(q)\phi_3(r)} \psi_1'(p) \cdot \psi_2'(q) \psi_3'(r) dp dq dx.
\end{aligned}$$

Proof: We can write,

$$\frac{1}{xyz} = (\log a)^3 \int_p^\infty \int_q^\infty \int_r^\infty a^{-(\psi_1(p') - \psi_1(p))x} a^{-(\psi_2(q') - \psi_2(q))y}$$

$$\times a^{-(\psi_3(r')-\psi_3(\mathfrak{z}))\mathfrak{z}}\psi'_1(p')\psi'_2(q')\psi'_3(r') dp' dq' dr'. \quad (12)$$

By using definition of new modified triple integral transform, we get,

$$\begin{aligned} & \mathcal{T}_{a_3}\left\{\frac{\mathfrak{f}(\mathfrak{x},\mathfrak{y},\mathfrak{z})}{\mathfrak{x}\mathfrak{y}\mathfrak{z}}\right\} \\ &= \phi_1(p)\phi_2(q)\phi_3(r) \\ & \times \int_0^\infty \int_0^\infty \int_0^\infty \frac{\mathfrak{f}(\mathfrak{x},\mathfrak{y},\mathfrak{z})}{\mathfrak{x}\mathfrak{y}\mathfrak{z}} a^{-(\psi_1(p)\mathfrak{x}+\psi_2(q)\mathfrak{y}+\psi_3(r)\mathfrak{z})} d\mathfrak{x} d\mathfrak{y} d\mathfrak{z}. \end{aligned} \quad (13)$$

Now by using equation (12) in equation (13), we get,

$$\begin{aligned} & \mathcal{T}_{a_3}\left\{\frac{\mathfrak{f}(\mathfrak{x},\mathfrak{y},\mathfrak{z})}{\mathfrak{x}\mathfrak{y}\mathfrak{z}}\right\} \\ &= \phi_1(p)\phi_2(q)\phi_3(r) \int_0^\infty \int_0^\infty \int_0^\infty \mathfrak{f}(\mathfrak{x},\mathfrak{y},\mathfrak{z}) a^{-(\psi_1(p)\mathfrak{x}+\psi_2(q)\mathfrak{y}+\psi_3(r)\mathfrak{z})} (\log a)^3 \\ & \times \left\{ \int_p^\infty \int_q^\infty \int_r^\infty a^{-(\psi_1(p')-\psi_1(p))\mathfrak{x}} \cdot a^{-(\psi_2(q')-\psi_2(q))\mathfrak{y}} \cdot a^{-(\psi_3(r')-\psi_3(r))\mathfrak{z}} \right. \\ & \times \left. \psi'_1(p')\psi'_2(q')\psi'_3(r') dp' dq' dr' \right\} d\mathfrak{x} d\mathfrak{y} d\mathfrak{z}. \end{aligned}$$

By using Fubini's theorem,

$$\begin{aligned} & \mathcal{T}_{a_3}\left\{\frac{\mathfrak{f}(\mathfrak{x},\mathfrak{y},\mathfrak{z})}{\mathfrak{x}\mathfrak{y}\mathfrak{z}}\right\} \\ &= \phi_1(p)\phi_2(q)\phi_3(r)(\log a)^3 \int_p^\infty \int_q^\infty \int_r^\infty \psi'_1(p')\psi'_2(q')\psi'_3(r') \\ & \times \left\{ \int_0^\infty \int_0^\infty \int_0^\infty a^{-(\psi_1(p')\mathfrak{x}+\psi_2(q')\mathfrak{y}+\psi_3(r')\mathfrak{z})} \times \mathfrak{f}(\mathfrak{x},\mathfrak{y},\mathfrak{z}) d\mathfrak{x} d\mathfrak{y} d\mathfrak{z} \right\} dp' dq' dr' \\ & \Rightarrow \mathcal{T}_{a_3}\left\{\frac{\mathfrak{f}(\mathfrak{x},\mathfrak{y},\mathfrak{z})}{\mathfrak{x}\mathfrak{y}\mathfrak{z}}\right\} \\ &= \phi_1(p)\phi_2(q)\phi_3(r)(\log a)^3 \\ & \times \int_p^\infty \int_q^\infty \int_r^\infty \frac{\mathfrak{F}(p',q',r')}{\phi_1(p')\phi_2(q')\phi_3(r')} \psi'_1(p')\psi'_2(q')\psi'_3(r') dp' dq' dr' \end{aligned}$$

Definition 5: Let $\mathfrak{f}(\mathfrak{x},\mathfrak{y},\mathfrak{z})$ and $g(\mathfrak{x},\mathfrak{y},\mathfrak{z})$ be integrable functions of three variables, then the triple convolution of $\mathfrak{f}(\mathfrak{x},\mathfrak{y},\mathfrak{z})$ and $g(\mathfrak{x},\mathfrak{y},\mathfrak{z})$ is given by

$$\mathfrak{f}(\mathfrak{x},\mathfrak{y},\mathfrak{z}) * g(\mathfrak{x},\mathfrak{y},\mathfrak{z}) = \int_0^\mathfrak{x} \int_0^\mathfrak{y} \int_0^\mathfrak{z} \mathfrak{f}(\mathfrak{x}-\tau, \mathfrak{y}-\mu, \mathfrak{z}-\eta) g(\tau, \mu, \eta) d\tau d\mu d\eta,$$

where the symbol $*$ denotes the triple convolution with respect to $\mathfrak{x}, \mathfrak{y}$ and $\mathfrak{z}$.

Theorem 12: (Triple convolution theorem) *If new modified triple integral transforms of $\mathfrak{f}(\mathfrak{x},\mathfrak{y},\mathfrak{z})$ and $g(\mathfrak{x},\mathfrak{y},\mathfrak{z})$ be $\mathfrak{F}(p,q,r)$ and $\mathcal{G}(p,q,r)$ respectively, then*

$$\mathcal{T}_{a_3}\{\mathfrak{f}(\mathfrak{x},\mathfrak{y},\mathfrak{z}) * g(\mathfrak{x},\mathfrak{y},\mathfrak{z}); \mathfrak{x} \rightarrow p, \mathfrak{y} \rightarrow q, \mathfrak{z} \rightarrow r\} = \frac{\mathfrak{F}(p,q,r)\mathcal{G}(p,q,r)}{\phi_1(p)\phi_2(q)\phi_3(r)},$$

where the symbol $*$ denotes the triple convolution with respect to $\mathfrak{x}, \mathfrak{y}$ and $\mathfrak{z}$.

Proof: Let $H = \phi_1(p)\phi_2(q)\phi_3(r)$. By using definition of new modified triple integral transform and triple convolution theorem, we get,

$$\begin{aligned} & \mathcal{T}_{a_3}\{\mathfrak{f}(\mathfrak{x},\mathfrak{y},\mathfrak{z}) * g(\mathfrak{x},\mathfrak{y},\mathfrak{z}); \mathfrak{x} \rightarrow p, \mathfrak{y} \rightarrow q, \mathfrak{z} \rightarrow r\} \\ &= H \int_0^\infty \int_0^\infty \int_0^\infty \left\{ \int_0^\mathfrak{x} \int_0^\mathfrak{y} \int_0^\mathfrak{z} \mathfrak{f}(\mathfrak{x}-\tau, \mathfrak{y}-\mu, \mathfrak{z}-\eta) g(\tau, \mu, \eta) d\tau d\mu d\eta \right\} \end{aligned}$$

$$\begin{aligned}
& \times a^{-(\psi_1(p)x+\psi_2(q)y+\psi_3(r)z)} dx dy dz \\
& = H \int_0^\infty \int_0^\infty \int_0^\infty \left\{ \int_0^\infty \int_0^\infty \int_0^\infty \check{f}(x-\tau, y-\mu, z-\eta) \right. \\
& \times H(x-\tau, y-\mu, z-\eta) g(\tau, \mu, \eta) d\tau d\mu d\eta \} \\
& \times a^{-(\psi_1(p)x+\psi_2(q)y+\psi_3(r)z)} dx dy dz \\
& = H \int_0^\infty \int_0^\infty \int_0^\infty g(\tau, \mu, \eta) \left\{ \int_0^\infty \int_0^\infty \int_0^\infty \check{f}(x-\tau, y-\mu, z-\eta) \right. \\
& \times H(x-\tau, y-\mu, z-\eta) a^{-(\psi_1(p)x+\psi_2(q)y+\psi_3(r)z)} dx dy dz \} d\tau d\mu d\eta \\
& = \int_0^\infty \int_0^\infty \int_0^\infty g(\tau, \mu, \eta) a^{-(\psi_1(p)\tau+\psi_2(q)\mu+\psi_3(r)\eta)} \mathfrak{F}(p, q, r) d\tau d\mu d\eta \\
& = \frac{\mathfrak{F}(p, q, r) \mathcal{G}(p, q, r)}{\phi_1(p) \phi_2(q) \phi_3(r)}.
\end{aligned}$$

Theorem 13: (Periodic Property) *Let $\check{f}(x, y, z)$ be a periodic function of periods α, β and γ , where $\check{f}(x+\alpha, y+\beta, z+\gamma) = \check{f}(x, y, z)$ for all x, y and z , and $\mathfrak{F}(p, q, r)$ exists, then*

$$\begin{aligned}
& \mathcal{T}_{a_3} \{ \check{f}(x, y, z); x \rightarrow p, y \rightarrow q, z \rightarrow r \} \\
& = \frac{\phi_1(p) \phi_2(q) \phi_3(r)}{1 - a^{-(\psi_1(p)\alpha+\psi_2(q)\beta+\psi_3(r)\gamma)}} \int_0^\alpha \int_0^\beta \int_0^\gamma \check{f}(x, y, z) \\
& \times a^{-(\psi_1(p)x+\psi_2(q)y+\psi_3(r)z)} dx dy dz.
\end{aligned}$$

Proof: By using definition of new modified triple integral transform, we get,

$$\begin{aligned}
& \mathcal{T}_{a_3} \{ \check{f}(x, y, z); x \rightarrow p, y \rightarrow q, z \rightarrow r \} \\
& = \phi_1(p) \phi_2(q) \phi_3(r) \int_0^\infty \int_0^\infty \int_0^\infty \check{f}(x, y, z) \\
& \times a^{-(\psi_1(p)x+\psi_2(q)y+\psi_3(r)z)} dx dy dz \\
& = \phi_1(p) \phi_2(q) \phi_3(r) \left\{ \int_0^\alpha \int_0^\beta \int_0^\gamma \check{f}(x, y, z) \right. \\
& \times a^{-(\psi_1(p)x+\psi_2(q)y+\psi_3(r)z)} dx dy dz \\
& \left. + \int_\alpha^\infty \int_\beta^\infty \int_\gamma^\infty \check{f}(x, y, z) a^{-(\psi_1(p)x+\psi_2(q)y+\psi_3(r)z)} dx dy dz \right\}.
\end{aligned} \tag{14}$$

By using substitution $x = u + \alpha, y = v + \beta$ and $z = w + \gamma$ in the second integral of equation (14), we get,

$$\begin{aligned}
& \mathcal{T}_{a_3} \{ \check{f}(x, y, z); x \rightarrow p, y \rightarrow q, z \rightarrow r \} \\
& = \phi_1(p) \phi_2(q) \phi_3(r) \left\{ \int_0^\alpha \int_0^\beta \int_0^\gamma \check{f}(x, y, z) \right. \\
& \times a^{-(\psi_1(p)x+\psi_2(q)y+\psi_3(r)z)} dx dy dz \\
& \left. + a^{-(\psi_1(p)\alpha+\psi_2(q)\beta+\psi_3(r)\gamma)} \int_0^\infty \int_0^\infty \int_0^\infty \check{f}(u, v, w) \right. \\
& \times a^{-(\psi_1(p)u+\psi_2(q)v+\psi_3(r)w)} du dv dw \}
\end{aligned} \tag{15}$$

$$\begin{aligned}
& = \phi_1(p) \phi_2(q) \phi_3(r) \int_0^\alpha \int_0^\beta \int_0^\gamma \check{f}(x, y, z) \\
& \times a^{-(\psi_1(p)x+\psi_2(q)y+\psi_3(r)z)} dx dy dz \\
& + a^{-(\psi_1(p)\alpha+\psi_2(q)\beta+\psi_3(r)\gamma)} \mathcal{T}_{a_3} \{ \check{f}(x, y, z); x \rightarrow p, y \rightarrow q, z \rightarrow r \}.
\end{aligned} \tag{16}$$

Solving equation (16) for $\mathcal{T}_{a_3} \{ \check{f}(x, y, z); x \rightarrow p, y \rightarrow q, z \rightarrow r \}$, we get,

$$\begin{aligned} & \mathcal{T}_{a_3}\{\mathfrak{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z})\} \\ &= \frac{\phi_1(\mathfrak{p})\phi_2(\mathfrak{q})\phi_3(\mathfrak{r})}{1-a^{-(\psi_1(\mathfrak{p})\alpha+\psi_2(\mathfrak{q})\beta+\psi_3(\mathfrak{r})\gamma)}} \\ & \times \int_0^\alpha \int_0^\beta \int_0^\gamma \mathfrak{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z}) a^{-(\psi_1(\mathfrak{p})\mathfrak{x}+\psi_2(\mathfrak{q})\mathfrak{y}+\psi_3(\mathfrak{r})\mathfrak{z})} d\mathfrak{x} d\mathfrak{y} d\mathfrak{z}. \end{aligned}$$

5. Applications of New Modified Triple Integral Transform

Example 5.1: Consider the following Laplace Equation (Steady-state temperature distribution in a corner with prescribed heat fluxes),

$$f_{xx}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z}) + f_{yy}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z}) + f_{zz}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z}) = 0, (\mathfrak{x}, \mathfrak{y}, \mathfrak{z}) \in \mathbb{R}_+^3, \quad (17)$$

subject to the conditions,

$$\mathfrak{f}(0, \mathfrak{y}, \mathfrak{z}) = 0, f_x(0, \mathfrak{y}, \mathfrak{z}) = \sin \mathfrak{y} \sinh \sqrt{2}\mathfrak{z}, \quad (18)$$

$$\mathfrak{f}(\mathfrak{x}, 0, \mathfrak{z}) = 0, f_y(\mathfrak{x}, 0, \mathfrak{z}) = \sin \mathfrak{x} \sinh \sqrt{2}\mathfrak{z}, \quad (19)$$

$$\mathfrak{f}(\mathfrak{x}, \mathfrak{y}, 0) = 0, f_z(\mathfrak{x}, \mathfrak{y}, 0) = \sqrt{2} \sin \mathfrak{x} \sin \mathfrak{y}. \quad (20)$$

Solution: Taking new modified triple integral transform on both sides of equation (17), we get,

$$\begin{aligned} & (\psi_1(\mathfrak{p})\log a)^2 \mathcal{T}_{a_3}\{\mathfrak{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z})\} - \phi_1(\mathfrak{p})\mathcal{T}_{a_2}\{f_x(0, \mathfrak{y}, \mathfrak{z})\} \\ & - \phi_1(\mathfrak{p})\psi_1(\mathfrak{p})\log a \mathcal{T}_{a_2}\{\mathfrak{f}(0, \mathfrak{y}, \mathfrak{z})\} \\ & + (\psi_2(\mathfrak{q})\log a)^2 \mathcal{T}_{a_3}\{\mathfrak{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z})\} - \phi_2(\mathfrak{q})\mathcal{T}_{a_2}\{f_y(\mathfrak{x}, 0, \mathfrak{z})\} \\ & - \phi_2(\mathfrak{q})\psi_2(\mathfrak{q})\log a \mathcal{T}_{a_2}\{\mathfrak{f}(\mathfrak{x}, 0, \mathfrak{z})\} \\ & + (\psi_3(\mathfrak{r})\log a)^2 \mathcal{T}_{a_3}\{\mathfrak{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z})\} - \phi_3(\mathfrak{r})\mathcal{T}_{a_2}\{f_z(\mathfrak{x}, \mathfrak{y}, 0)\} \\ & - \phi_3(\mathfrak{r})\psi_3(\mathfrak{r})\log a \mathcal{T}_{a_2}\{\mathfrak{f}(\mathfrak{x}, \mathfrak{y}, 0)\} = 0. \end{aligned}$$

By using conditions mentioned in equation (18), (19) and (20) we get the solution of above equation as,

$$\begin{aligned} & \mathcal{T}_{a_3}\{\mathfrak{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z})\} \\ &= \frac{\sqrt{2}\phi_1(\mathfrak{p})\phi_2(\mathfrak{q})\phi_3(\mathfrak{r})}{((\psi_1(\mathfrak{p})\log a)^2+1)((\psi_2(\mathfrak{q})\log a)^2+1)((\psi_3(\mathfrak{r})\log a)^2-2)}, \end{aligned}$$

Now, by taking inverse new modified triple integral transform on both sides, we get,

$$\mathfrak{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z}) = \sqrt{2} \sin \mathfrak{y} \cdot \sin \mathfrak{x} \cdot \sinh \sqrt{2}\mathfrak{z}.$$

Example 5.2: Consider the following non-homogeneous wave equation,

$$2g_{zz}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z}) = g_{xx}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z}) + g_{yy}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z}) + 24\mathfrak{z}^2 + 4\mathfrak{y}, \quad (21)$$

for $(\mathfrak{x}, \mathfrak{y}, \mathfrak{z}) \in \mathbb{R}_+^3$, subject to the conditions,

$$g(0, \mathfrak{y}, \mathfrak{z}) = \mathfrak{z}^4 + \mathfrak{y}\mathfrak{z}^2, \quad g_x(0, \mathfrak{y}, \mathfrak{z}) = \sin \mathfrak{y} \sin \mathfrak{z}, \quad (22)$$

$$g(\mathfrak{x}, 0, \mathfrak{z}) = \mathfrak{z}^4, \quad g_y(\mathfrak{x}, 0, \mathfrak{z}) = \mathfrak{z}^2 + \sin \mathfrak{x} \sin \mathfrak{z}, \quad (23)$$

$$g(\mathfrak{x}, \mathfrak{y}, 0) = 0, \quad g_z(\mathfrak{x}, \mathfrak{y}, 0) = \sin \mathfrak{x} \sin \mathfrak{y}. \quad (24)$$

Solution: Taking new modified triple integral transform on both sides of equation (21), we get,

$$\begin{aligned}
& 2(\psi_3(r)\log a)^2 \mathcal{T}_{a_3}\{\mathfrak{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z})\} - \phi_3(r)\psi_3(r)\log a \mathcal{T}_{a_2}\{\mathfrak{f}(\mathfrak{x}, \mathfrak{y}, 0)\} \\
& - \phi_3(r)\mathcal{T}_{a_2}\{\mathfrak{f}_{\mathfrak{z}}(\mathfrak{x}, \mathfrak{y}, 0)\} \\
& = (\psi_1(p)\log a)^2 \mathcal{T}_{a_3}\{\mathfrak{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z})\} - \phi_1(p)\mathcal{T}_{a_2}\{f_{\mathfrak{x}}(0, \mathfrak{y}, \mathfrak{z})\} \\
& - \phi_1(p)\psi_1(p)\log a \mathcal{T}_{a_2}\{\mathfrak{f}(0, \mathfrak{y}, \mathfrak{z})\} \\
& + (\psi_2(q)\log a)^2 \mathcal{T}_{a_3}\{\mathfrak{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z})\} - \phi_2(q)\mathcal{T}_{a_2}\{f_{\mathfrak{y}}(\mathfrak{x}, 0, \mathfrak{z})\} \\
& - \phi_2(q)\psi_2(q)\log a \mathcal{T}_{a_2}\{\mathfrak{f}(\mathfrak{x}, 0, \mathfrak{z})\} \\
& + 24 \frac{\phi_1(p)\phi_2(q)}{\psi_1(p)\psi_2(q)(\log a)^2} \cdot \left(\frac{\phi_3(r)2!}{(\psi_3(r)\log a)^3} \right) \\
& + 4 \frac{\phi_1(p)\phi_3(r)}{\psi_1(p)\psi_3(r)(\log a)^2} \cdot \left(\frac{\phi_2(q)}{(\psi_2(q)\log a)^2} \right).
\end{aligned}$$

By using conditions mentioned in equation (22), (23) and (24) we get the solution of above equation as,

$$\begin{aligned}
& \mathcal{T}_{a_3}\{g(\mathfrak{x}, \mathfrak{y}, \mathfrak{z})\} \\
& = \frac{\phi_1(p)\phi_2(q)\phi_3(r)}{((\psi_1(p)\log a)^2+1)((\psi_2(q)\log a)^2+1)((\psi_3(r)\log a)^2+1)} \\
& + \frac{24\phi_1(p)\phi_2(q)\phi_3(r)}{(\log a)^2\psi_1(p)\psi_2(q)(\psi_3(r))^5(\log a)^5} \\
& + \frac{2\phi_1(p)\phi_2(q)\phi_3(r)}{(\log a)^2\psi_1(p)(\psi_2(q))^2(\psi_3(r))^3(\log a)^4}.
\end{aligned}$$

Now, by taking inverse new modified triple integral transform on both sides, we get,

$$g(\mathfrak{x}, \mathfrak{y}, \mathfrak{z}) = \sin \mathfrak{x} \sin \mathfrak{y} \sin \mathfrak{z} + \mathfrak{z}^4 + \mathfrak{y}\mathfrak{z}^2.$$

Example 5.3: Consider the following volterra partial integro differential equation,

$$\begin{aligned}
& f_{\mathfrak{x}}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z}) + f_{\mathfrak{y}}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z}) + f_{\mathfrak{z}}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z}) \\
& = -(\mathfrak{x}\mathfrak{y}\mathfrak{z})^2 + 4\mathfrak{x}\mathfrak{y} + 4\mathfrak{x}\mathfrak{z} + 4\mathfrak{y}\mathfrak{z} + 2 \\
& + \int_0^{\mathfrak{z}} \int_0^{\mathfrak{y}} \int_0^{\mathfrak{x}} \mathfrak{f}(a, b, c) da db dc.
\end{aligned} \tag{25}$$

subject to the conditions,

$$\mathfrak{f}(\mathfrak{x}, \mathfrak{y}, 0) = \mathfrak{f}(\mathfrak{x}, 0, \mathfrak{z}) = \mathfrak{f}(0, \mathfrak{y}, \mathfrak{z}) = 0. \tag{26}$$

Solution: Taking new modified triple integral transform on both sides of equation (25), we get,

$$\begin{aligned}
& \psi_1(p)\log a \mathcal{T}_{a_3}\{\mathfrak{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z})\} - \phi_1(p)\mathcal{T}_{a_2}\{\mathfrak{f}(0, \mathfrak{y}, \mathfrak{z})\} \\
& + \psi_2(q)\log a \mathcal{T}_{a_3}\{\mathfrak{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z})\} - \phi_2(q)\mathcal{T}_{a_2}\{\mathfrak{f}(\mathfrak{x}, 0, \mathfrak{z})\} \\
& + \psi_3(r)\log a \mathcal{T}_{a_3}\{\mathfrak{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z})\} - \phi_3(r)\mathcal{T}_{a_2}\{\mathfrak{f}(\mathfrak{x}, \mathfrak{y}, 0)\} \\
& = - \frac{\phi_1(p)2!}{(\psi_1(p)\log a)^3} \cdot \frac{\phi_2(q)2!}{(\psi_2(q)\log a)^3} \cdot \frac{\phi_3(r)2!}{(\psi_3(r)\log a)^3} \\
& + 4 \frac{\phi_1(p)}{(\psi_1(p)\log a)^2} \cdot \frac{\phi_2(q)}{(\psi_2(q)\log a)^2} \cdot \frac{\phi_3(r)}{(\psi_3(r)\log a)}
\end{aligned}$$

$$\begin{aligned}
& +4 \frac{\phi_1(p)}{(\psi_1(p)\log a)^2} \cdot \frac{\phi_2(q)}{(\psi_2(q)\log a)} \cdot \frac{\phi_3(r)}{(\psi_3(r)\log a)^2} \\
& +4 \frac{\phi_1(p)}{(\psi_1(p)\log a)} \cdot \frac{\phi_2(q)}{(\psi_2(q)\log a)^2} \cdot \frac{\phi_3(r)}{(\psi_3(r)\log a)^2} \\
& +2 \frac{\mathcal{T}_{a_3}\{\mathfrak{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z})\}}{\psi_1(p)\psi_2(q)\psi_3(r)(\log a)^3}
\end{aligned}$$

by using conditions mentioned in equation (26) we get the solution of above equation as,

$$\mathcal{T}_{a_3}\{\mathfrak{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z})\} = \frac{4\phi_1(p)\phi_2(q)\phi_3(r)}{(\psi_1(p)\log a)^2(\psi_2(q)\log a)^2(\psi_3(r)\log a)^2}$$

Now, by taking inverse new modified triple integral transform on both sides, we get,

$$\mathfrak{f}(\mathfrak{x}, \mathfrak{y}, \mathfrak{z}) = 4\mathfrak{x}\mathfrak{y}\mathfrak{z}.$$

6. Conclusion

In this article, a new modified triple integral transform is introduced. Existence theorem and properties such as linearity property, shifting property, convolution theorem, periodic property, etc., including triple integral transform of partial derivatives of a function are proved, and their applicability is tested by solving partial differential and partial integro differential equations.

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