

Travel time distribution estimation on arterial networks under the stationary conditions

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Abstract

Travel time estimation is a challenging task due to the lots of influential factors. The most pre-existing travel time literature assumes travel time follows a specified distribution. Their goal is to find the best fit distribution of travel time based on the observed travel times. Obviously, the role of some influential factors such as intersection signal times, demand variations, supply variations (changing network topology) etc., are neglected. In this paper, the uncertainty of time and its variations which are consequences of numerous stochastic elements such as uncertain travel demand, uncertain driver behaviors, and random arrival time is modelled by calculating the probability density function of travel time on each arterial link. Also, the concept of queuing theory is used to take the stochastic features and queue formation behind intersection signals into account;

Additionally, unlike previous model-based methods, the queue of vehicles is modeled as a horizontal queue, and its effects on the travel time are accurately calculated. Also this method is particularly useful in scenarios where, due to changes in the network, the previous data may not

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be highly reliable or in special situations which there is only an estimate of supply and demand such as, the closure of a street or changes in traffic signal timing or large sporting events or emergency situations etc. Eventually, the proposed model is validated and examined by comparison with micro-simulation experiments.

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1. Introduction

Congestion and traffic problems are frustrating daily phenomena in urban areas. Long travel times, air pollutions and noisy areas are the most undesirable consequences of traffic congestion. Transportation of passengers, commodities, and also supply services through Congested road networks, requires a lot of time and high cost. Billions of dollars and thousands of hours can be saved by finding an appropriate estimation of travel time and reducing it. Urban transportation network users, especially drivers and passengers, need to know some preliminary information about their travel time to make the optimal decisions in choosing their routes, suitable departure times, and the transportation mode. Road Network managers and researchers are seeking a proper travel time estimation as a measure of transportation effectiveness to improve the transportation network level of service. Travel time is a main objective of traffic studies and a crucial factor for users, drivers, urban network managers, traffic engineers, and researchers [1]. As travel time is highly affected by congestion, knowing the source and effect of network congestion plays an essential role in travel time estimation. Excessive travel demand, supply service limit, interruption in service due to weather conditions, and traffic accidents are some sources of urban road congestion. According to the viewpoint of users, experts, and researchers, not only long travel times but variations and uncertainty of travel time are significant issues [2]. Usually, users and drivers choose a reliable route with fewer travel time variations [2]. In this regard, the methods which ignore stochastic features of travel times or focus on expected travel times have limited capabilities to compute variations of travel time or determine the travel time reliability [3], [4]. Travel time probability distribution-as an appropriate concept that covers most stochastic features- is one of the topics of interest for transportation field researchers. Some features of travel time probability distributions are as follows:

- Investigating the uncertainty of the travel time.
- Giving some appropriate information on journeys time.
- Finding the probability of arriving or not arriving on time.
- The ability to determine the reliability of travel time.

In the following, the strengths and weaknesses of the existing methods and the research gaps of travel time literature are discussed.

2. Literature review

The current literature on urban travel times estimation can be divided into two main categories. Model-based and data-based approaches. Each one has its advantages and disadvantages. Model-based approaches use traffic volumes, network topology and road capacities, and proper theories of traffic behaviors, such as the shock wave theory of traffic flow [5], [6], and queuing theory to derive the travel time [7]. Data-based methods consider the previous or observed travel times to predict future travel times [8], [9]. For example, Bertsimas d. et al. estimated the origin-destination travel times using observed taxi travel data [10]. Most data-based approaches assume a certain probability distribution or the stochastic process of travel time [11], [12], [13]. So, these approaches are entirely dependent on previous observed data and are unable to consider the variations of demand and supply in networks.

Uncertainty and variations in travel time make people confused about the time amount they should spend on. It is also challenging to decide on the appropriate route, departure time, etc. Stochastic demand and service, different behaviors of drivers, and traffic signals are some of the primary sources of daily travel times uncertainty [1], [14]. During different hours of the day, the demand pattern has wide variations, and consequently, the variation ranges of travel time are wide [6], [15]. Also, each arterial link service rate variations (by traffic signals) cause uncertainty in travel time [4]. The weather conditions, bus maneuvers at the stations and pedestrian crossings, etc., are the other sources of uncertainty in travel time along the arterials.

Shockwave theory is applied to formulate the traffic flow and consequently provides the ability to estimate travel time along the urban roads in most model-based approaches [5], [6], [16]. The shockwave theory is a proper approach

Table 1
Recent data-based and model based approaches of travel time estimation

Model	Travel time Data-based	Model-based	Horizontal queue	Responsive to demand supply variations	Data gathering method
Bertsimas et al., [10]	✓				Travel time GPS data
Zheng et al., [6]		✓		✓	Supply-demand data
Qin et al., [9]	✓				RFID data
Ghandeharioun & Kouvelas, [18]	✓				GPS data
Li et al., [19]	✓				Travel time data using internet based devices
This paper		✓	✓	✓	Supply-demand data

estimating travel times on freeways. The Cell Transmission Model (CTM) [17] is a practical tool to study numerical shockwave theory and obtain the formula of freeway travel time. But this approach lacks the ability to consider stochastic properties of traffic flow, especially in arterial roads [5].

There is another category of model-based approaches which derives from queueing theory, and is utilized to get travel time on urban roads [20]. The properties and benefits of queueing theory in comparison with other methods are introduced and described by Van Woensel et al. Some of the advantages of using queueing theory in travel time estimation are the ability to perform some optimization and analysis related to networks transportation e.g., vehicle routing, sensitivity analysis, scenario analysis [7]. In addition, the excellent capability of queueing theory is to consider stochastic properties of traffic flow, especially in the formation of queues behind the stop line [21], [22]. Also, as a suitable application, queueing theory is applied to optimize traffic signals to have the most reliable travel time [23]. Eventually, according to the potential of queueing theory and its applications in recent research, more development of queue-based models to capture queue length, urban road blocking, the interaction between consecutive queues, and their effect on travel time will be beneficial.

Due to the absence of travel time data in certain cases or its unreliability caused by changes in network topology, traffic signal timing, or travel demand, it is essential to use fundamental model-based approaches in such situations. Developing a model that effectively represents travel time uncertainty and considers the effects of changes in network supply (network topology), signal timing, and travel demand can be beneficial. Additionally, employing a method capable of modeling vehicle queues horizontally is of great importance.

3. Main framework

Different hours of day and night have different traffic volumes. Network users may experience the same travel times in different volumes of traffic. This happens because of overlapping travel time distributions in different traffic states. Therefore, according to researchers' findings, only the traffic state factor is not enough to estimate proper travel time [24]. Travel time as a stochastic phenomenon depends on several random factors such as demand and service rate, weather conditions, driver's behaviors, traffic state, etc. In the current literature, to the best of our knowledge, neither different stochastic factors of travel time nor their interdependencies are considered in travel time estimations. Also in the most recent research, the queue behind the stop line is considered vertical, and consequently, the effect of queue length on free-running travel time is ignored [6] and often, the free-running travel time is considered as a deterministic variable while it is dependent on queue length behind the stop line of the link. In this context, traffic state and also other items affecting travel time, such as signal times, stochastic demand, stochastic service, horizontal queues (the queue behind the stop line is considered horizontal) etc., are considered to estimate the travel time correctly. In downtown areas where intersections are

closely spaced, the interactions of adjacent links during peak hours have a significant impact and cause queue blocking, reduction in upstream queues service rate, and downstream arrival rate. In this study, the interactions of adjacent links and their consequences are considered in travel time estimation. Transportation networks usually are studied in either dynamic (transient) or stationary conditions [5],[25]. In this study, travel time is calculated and analyzed in the network stationary condition.

In this study travel time of a vehicle along a link includes three components, free-running travel time, stochastic delay, and uniform delay (figure 1). Free-running travel time is a part of journey along the link that a vehicle travels without delay or loss time. Stochastic delay is a component of delay related to uncertainty in arrival and departure rate in link. Uniform delay is a part of delay a vehicle experience due to traffic signals. These stochastic quantities; free-running travel time, uniform delay, and stochastic delay are dependent random variables and affect each other. More precisely, the uniform delay which is created by stochastic delay due to the reduction of the storage capacity of the link, because of the presence of a uniform queue, (the queue behind the stop line is formed due to presence of signal traffic) Therefore, the free-running travel distance is changed due to both the uniform and the stochastic queues.

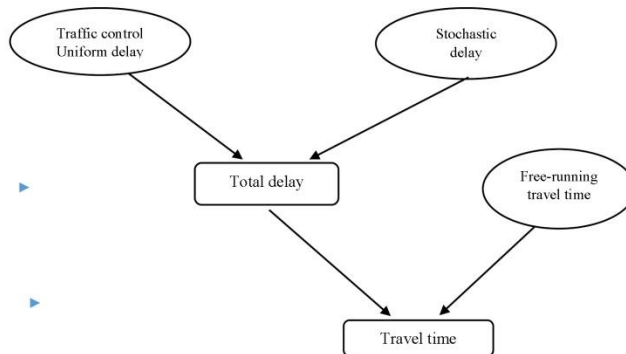


Figure 1
Components of arterial link travel time

In the following, the mathematical methodology of these components and their interdependence will be discussed.

3.1 Uniform Delay

As stated before, uniform delay -resulting from signal control- is a significant factor in travel time. To simplify the calculation of uniform delay function, often researchers assume the arrival and departure times (headways) are constant. In Figure 2, representing the saturation flow and delay at an intersection, q and s indicate the rates of arriving and saturation flows. These lines, with slopes proportional to q and s , reflect the volume of arriving and

departing vehicles to the intersection over time. The triangular area, bounded by these lines and the time axis, captures the total uniform delay experienced by vehicles due to queuing and flow dynamics. This representation, based on [26], allows for a visual interpretation of intersection delays, emphasizing the impact of flow imbalance on delay accumulation.

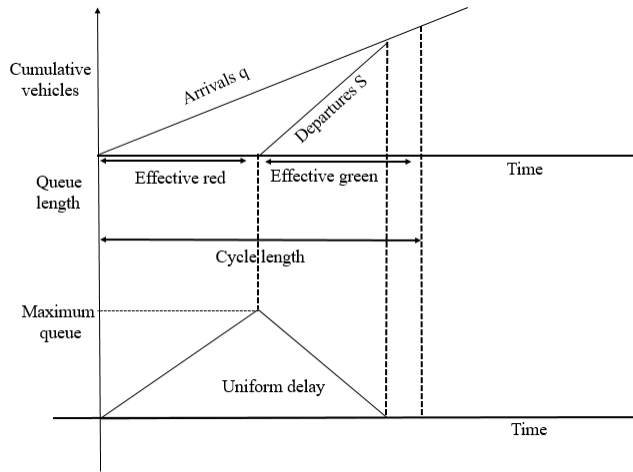


Figure 2
The overall uniform delay

The bottom triangle in Figure 2 shows the total uniform delay of all vehicles, but in this paper, we need to evaluate the amount of uniform delay experienced per vehicle. To obtain it, the arrival moment with respect to the beginning of the cycle time is crucial. The beginning of cycle time is considered the beginning of red time. As shown in figure (3.a), the uniform delay distribution depends on the flow and arrival moment with respect to the beginning of the cycle time.

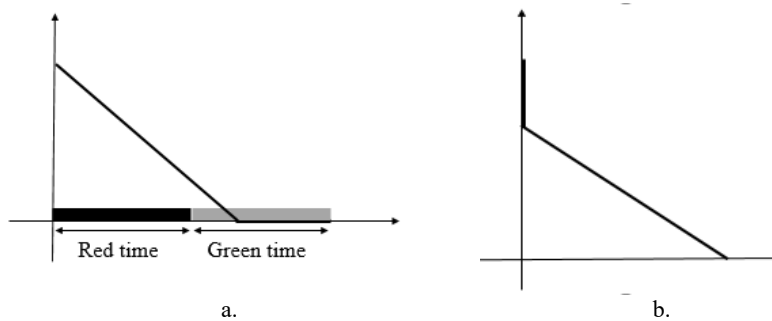


Figure 3
Uniform delay as a function of arrival moment (a),
The inverse relation of uniform delay (b).

In the first step, we compute the uniform delay experienced by every specific vehicle. Let t_a be the arrival moment of a vehicle in the queue behind the stop line in the case of under saturation traffic flow; the amount of uniform delay (t_u) is calculated in three cases according to vehicle arrival moment by formulas (1), (2), and (3).

$$t_u = D_u(t_a|n = qt_a) = t_r - t_a + \frac{qt_a+1}{s} \quad t_a \leq t_r \quad (1)$$

$$t_u = D_u\left(t_a|n = \frac{qt_a - s(t_a - t_r)}{s}\right) = t_r - t_a \times \left(1 - \frac{q}{s}\right) \quad t_r < t_a < \frac{t_r}{1 - \frac{q}{s}} \quad (2)$$

$$t_u = D_u(t_a|n = 0) = 0 \quad t_a > \frac{t_r}{1 - \frac{q}{s}} \quad (3)$$

Where t_r, s, n and q are red time, saturation flow, number of vehicles in queue, and the flow respectively. The formulas (1), (2), and (3) are constructed for three cases; arriving during red time, arriving during green time with presence of queue, and arriving during green time with no queue, respectively.

Usually, drivers are unaware of signal times, so the arrival moment of a vehicle according to the cycle time beginning, is random. To obtain the random distribution of uniform delay, the inverse of the delay function is applied [4], Figure 3. As seen in Figure 3.b, the inverse mapping is not a valid function. Hence, Zheng et al. applied the Dirac delta function with the following properties [4].

$$\begin{cases} \delta(x) = 0 & \text{if } x \neq 0 \\ \delta(x) = \infty & \text{if } x = 0 \\ \int_{-\infty}^{+\infty} \delta(x - x_0)f(x)dx = f(x_0) \end{cases} \quad (4)$$

Where $f(x)$ is an arbitrary continuous function. Then, the probability distribution function is defined as:

$$p(t_u) = \alpha\delta(t_u) + \beta, \quad 0 \leq t_u \leq t_r \quad (5)$$

Where $\alpha = 1 - \frac{t_r}{\text{cycle} \cdot (1 - \frac{q}{s})}$, $\beta = \frac{1}{\text{cycle} \cdot (1 - \frac{q}{s})}$ and Cycle is cycle length of traffic signal

3.2. Stochastic delay

Random arrivals, random departures and other random items in a link will increase the traffic queue and its resulted delay in each cycle. As said, Uniform delay results from signal control parameters and clearly its function during the typical cycle does not account for this additional delay. So the other component of the delay which relates to variations of traffic from one cycle to another cycle, stochastic delay, is used [4]. Also, stochastic delay includes the delay resulting from vehicles failing to discharge during the green times and so forming an initial queue at the beginning of the red time. The stochastic delay probability distribution on each link is calculated based on queue sojourn time (in queuing theory, queue spending-servicing time of a customer or vehicle is known as sojourn time). So each lane is modeled by $M/M/1/C_i$ where the inter-arrival time and service time are exponentially distributed and C_i is the upper

bound limit of queue length. In urban transportation systems, a network of interconnected queues is commonly encountered, where each queue influences others. To estimate parameters such as arrival rate, service rate, and the stochastic delay probability distribution, queuing theory is applied, specifically focusing on networks of queues with finite capacities. In cases where queues are constrained by an upper bound on queue length, the parameters like arrival rate, service rate, and traffic intensity for each queue may change due to potential of the downstream spillback or upstream blockage effects.

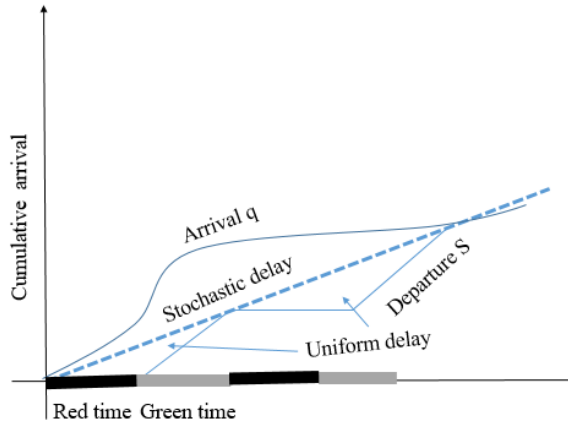


Figure 4
Stochastic delay

The marked areas in figure 4, show the amount of the uniform and the stochastic delays. In the next section, we will introduce and review the queuing theory. The notations and definitions are introduced as below [27].

$E\lambda_i$	External arrival rate to queue i ,
$T\lambda_i$	Total arrival rate to queue i ,
$Eff\lambda_i$	Effective arrival rate (excludes lost arrivals),
μ_i	Service rate of lane i ,
$Ub\mu_i$	Blocking rate service i (Unblocking rate),
$Eff\mu_i$	Effective service rate,
P_i	Probability of being blocked at queue i ,
T_{ij}	Transition rate from upstream queue i to downstream queue j (turning probability from queue i to queue j).
C_i	Storage capacity of i th link,
V_i	Number of vehicles in queue i ,
$P(V_i = C_i)$	Probability of queue i being full,

- D_i The set of downstream queues of queue i .
 ρ_i Traffic intensity of queue i

The finite capacity queueing networks model is held in the following system of equations [27],[25]

$$T\lambda_i = E\lambda_i + \frac{\sum_j T_{ji} T\lambda_j (1 - P(V_j = C_j))}{(1 - P(V_i = C_i))} \quad (6.a)$$

$$\frac{1}{Ub\mu_i} = \sum_{j \in D_i} \frac{T\lambda_j (1 - P(V_j = C_j))}{T\lambda_i (1 - P(V_i = C_i)) Eff\mu_j} \quad (6.b)$$

$$\frac{1}{Eff\mu_i} = \frac{1}{\mu_i} + P_i \frac{1}{Ub\mu_i} \quad (6.c)$$

$$P_i = \sum_j T_{ij} P(V_j = C_j), \quad (6.d)$$

$$P(V_i = C_i) = \frac{1 - \rho_i}{1 - \rho_i^{C_i+1}} \rho_i^{C_i}, \quad (6.e)$$

$$\rho_i = \frac{T\lambda_i}{Eff\mu_i} \quad (6.f)$$

The first equation defines the total arrival rate as a sum of the external arrival rate and the arrival rate from upstream queues. The term $1 - P(V_i = C_i)$, represents the probability queue i . is not full. Equation (6.b) indicates the service rate of blocking vehicles at queue i . In equation (6.c), the inverse of the service rate defines service time. So, it shows that the effective service time includes both ordinary service time and service time in the presence of blocking. Equation (6.d) represents blocking probability at queue i as a function of downstream queue spillback. The probability of queue i being full is illustrated in (6.e) (For more detail about (6.e), the interested readers are referenced to [28]). Equation (6.f) defines the traffic intensity of queue i . In the finite capacity queueing networks (so there is an upper bound for every queue length), the service rate is replaced by the effective service rate to capture the limits of capacity storage and queue downstream spillback phenomenon.

As mentioned before, travel time along the arterial is divided into two parts, queue waiting time and free-running journey time. The travel time estimation of queue in each link is formulated based on sojourn time . As we know $T\lambda_i$ is the arrival rate and this parameter is calculated by using the traffic flow, turning percentage of upstream links, and its external arrivals. Based on queueing theory, the queue length probability distribution of a $M/M/1/C_i$ system is obtained by the following formula [28].

$$P(V_i = n) = \frac{1 - \rho_i}{1 - \rho_i^{C_i+1}} \rho_i^n, \quad n = 1, 2, 3, \dots, C_i \quad (7)$$

Where ρ_i is the traffic intensity of i th queue and calculated as the ratio of the arrival rate to the departure rate. $P(V_i = n)$ is the probability of existing exactly n vehicles in the queue. Let S_i^m be the service time of m -th vehicle in queue i . The random variables S_i^m for $1 \leq m \leq n$, are independent and

exponentially distributed with mean of $\frac{1}{Eff\mu_i}$. To calculate the sojourn time distribution, first define the following:

$$t_s^i = \sum_{m=1}^n S_i^m \quad (8)$$

In the formula (8) the parameter t_s^i is sum of service times of all vehicles in queue i . t_s^i is sum of n independent exponential random variables. Therefore, it follows an Erlang- n distribution with mean $\frac{n}{Eff\mu_i}$ based on queueing theory [29].

$$f_n(t) = \begin{cases} \frac{Eff\mu_i^n \times t^{n-1} e^{(-Eff\mu_i \times t)}}{(n-1)!} & t \geq 0 \\ 0 & t < 0 \end{cases} \quad (9)$$

So its cumulative distribution is [30]:

$$F_n(x) = P(T_n \leq t) = 1 - \sum_{k=1}^n \frac{(Eff\mu_i \times t)^{k-1}}{(k-1)!} e^{-Eff\mu_i t}, \quad t > 0 \quad (10)$$

This distribution yields:

$$P(t_s^i > t) = \sum_{k=1}^n \frac{(Eff\mu_i \times t)^{k-1}}{(n-1)!} e^{-Eff\mu_i t} \quad (11)$$

Since the sojourn time distribution is a continuous function, to obtain a discrete distribution, the referent interval on the horizontal time axis is divided into some subintervals $[t_j, t_{j+1}]$ for which the probability distribution of each subinterval is obtained from (12).

$$P(t_j \leq t_s^i < t_{j+1}) = P(t_s^i > t_j) - P(t_s^i > t_{j+1}). \quad (12)$$

3.3. Free-running travel time

Free-running travel time of i -th lane results from the division of link length by the average speed as below.

$$tf_i = \frac{L_i - (5 \times V_i)}{speed} \quad (13)$$

Where L_i represents the lane length and V_i shows the number of vehicles in the queue. This is a random variable, takes values $1, 2, \dots, k$ with the probabilities from (7), and 5 meters is typically part of the lane length occupied by a single vehicle. It is worth mentioning that free-running travel time is affected by different stochastic factors like pedestrians crossing, bus or taxi maneuvering at stations, weather conditions, and the length of the queue at the end of the link, etc. Most researchers have ignored the stochastic features of free-running travel time [6]. We also ignore them—except one: the effect of queue length at the end of the link on free-running length. Obviously, the queue length can alter the distance left of the free-running part and hence changes the free-running travel time. In this direction -capturing the influence of queue length on free-running travel time- the queue length is assumed to be horizontal, as typically observed on urban roads. Considering queues vertically, contradicts the reality of traffic on the real roads, especially on links connecting closely

spaced intersections. In this case, the vertical queues theory worsens the quality of the model.

3.4. A unified formula for stochastic delay and free-running travel time probability

Suppose a vehicle entered a link without uniform queue with n vehicles waiting in a stochastic queue with the probability (7). The available distance for free-running travel time is $L - 5n$ and the free-running travel time is:

$$tf = \frac{L-5n}{speed}.$$

In this case, the probability of the link travel time $\bar{t} = t_s + tf$ is:

$$p(t) = \sum_{tf} p(tf = t - t_s) p(t_s) \quad [6]$$

Therefore, we have:

$$\begin{aligned} p(t) &= \sum_{tf} p(tf = t - t_s) p(t_s) = \sum_{n=1}^{C_i} \sum_{k=1}^n \frac{(Eff\mu_i \times t)^{k-1}}{(n-1)!} e^{-Eff\mu_i t} \frac{1-\rho_i}{1-\rho_i^{C_i+1}} \rho_i^n \\ &= \frac{1-\rho_i}{1-\rho_i^{C_i+1}} \left(\sum_{n=1}^{C_i} \sum_{k=1}^n \frac{(Eff\mu_i \times t)^{k-1}}{(n-1)!} e^{-Eff\mu_i t} \times \rho_i^n - \right. \\ &\quad \left. \sum_{n=1}^{C_i} \sum_{k=1}^n \frac{(Eff\mu_i \times t)^{k-1}}{(n-1)!} e^{-Eff\mu_i t} \times \rho_i^n \right) \end{aligned} \quad (14)$$

3.5. Travel time distribution

Considering u as uniform delay time, s the stochastic delay, and tf as the free-running travel time, the probability of total travel time of a lane i is $t = u + s + tf$ and is calculated by:

$$p(t) = \int_0^t \int_0^s p(t-s)p(\tau-u)p(u) du ds = \int_u^t p(\bar{t} = t-u)p(u) du \quad (15)$$

(For the sake of simplicity in notation, the link index i is dropped.) Where $\tau = u + s$ and the integration area is depicted in figure 5. In formula (15), $p(\tau - u)$ is the conditional probability of the stochastic delay, $s = \tau - u$ conditional on u as realized uniform delay. Also, $p(t - \tau)$ is the probability of tf as free-running travel time conditional on realized τ , as sum of the uniform and stochastic delay.

$$p(t) = \sum_s \sum_u p(t-\tau)p(\tau-u)p(u) = \sum_u p(\bar{t} = t-u)p(u) \quad (16)$$

The three components of travel time in formulas (15) and (16) are interdependent. First, the uniform delay is calculated. Next, the stochastic delay is determined based on the given uniform delay. Finally, the free-flow travel time, influenced by queue length resulting from both uniform and stochastic delays, is computed. The algorithm below outlines the steps for calculating the travel time probability distribution using these components and formulas.

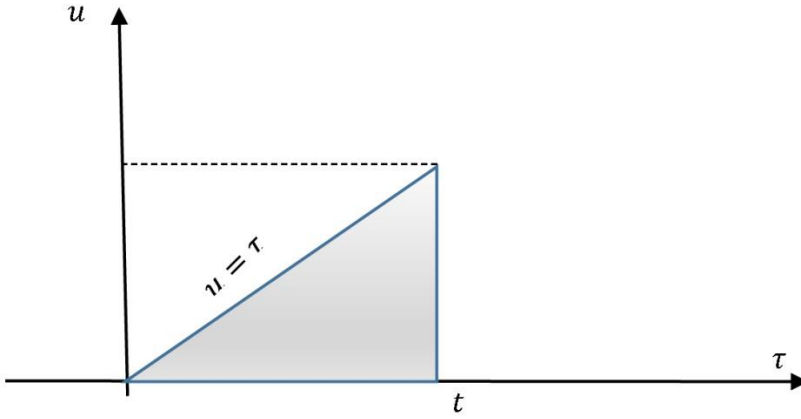


Figure 5
Integration area of integral formula (15)

Algorithm 1: The proposed algorithm

1. Determine the length of link, upstream and downstream links with $T\lambda_i, T_{ij}, L_i, \mu_i$.
2. Find the uniform delay distribution by formulas (1)-(3).
3. For every u as a uniform delay time with probability $p(u)$ do:
 - 3.1 Find the corresponding uniform queue length $l(u)$.
 - 3.2 Determine the remaining link length and obtain the storage capacity of the link: $c_i(u) = \frac{(l-L(u))}{5}$.
 - 3.3 Solve the system of equations (6) to obtain storage capacity $c_i(u)$.
 - 3.4 Obtain a discrete distribution of stochastic delay and corresponding stochastic queue length using equations (7), (10), and (11).
 - 3.5 For every stochastic delay s and corresponding queue length $l(u)$ with probability $p(s)$, do:
 - 3.5.1 Find free-running tf using $tf_i = \frac{L_i - l(s) - l(u)}{speed}$ with conditional probability 1.
 - 3.5.2 Determine the link travel time components (u, s, tf) with probability $p(t-s)p(\tau-u)p(u) = 1 \times p(s) \times p(u)$
4. Calculate travel time probability distribution using (16).

4. Illustrative example

In this section, a numerical example is introduced to show the capabilities of the proposed travel time model. The network topology and related data in detail are given in figure 6 and table 2 and 3 respectively.

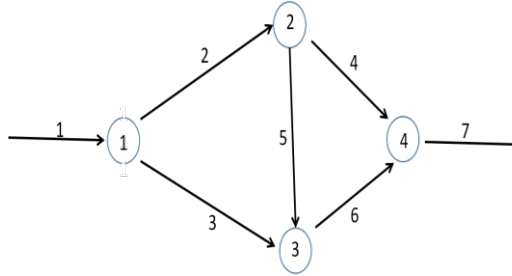


Figure 6
A test case network

Table 2
Characteristics of links in Figure 6

Link	length (meter)	Saturation flow (vehicle per hour)	Green time of the signal at the end of link (second)	speed (kilometers per hour)
1	500	3600	45	50
2	600	3600	45	50
3	550	3600	45	50
4	250	3600	45	50
5	400	3600	45	50
6	250	3600	45	50
7	600	3600	45	50

To obtain the storage capacity of each lane, the lane length is divided into 5 (meters), which is the typical occupation length of a single vehicle in the queue. So, the storage capacity of each lane is $\frac{\text{lane length}}{5}$. The external arrival rate and mean service rate are given in table 1. The parameter service rate μ_i is closely related to saturation flow¹ and the green time duration and is obtained from the following formula.

$$\mu_i = \frac{g_i}{\text{cycle}_i} \text{Sat}_i \quad (17)$$

Where $g_i \leftarrow \text{cycle}_i$, and Sat_i are the green time duration, cycle length, and the saturation flow of lane i .

The saturation flow of a lane (with no left or right turning) is considered to be 1850 vehicles per hour [31]. To evaluate the saturation flow of lanes with nonzero left or right turning percentages, Synchro 8 is used [32], [33]. In this example, the turning percentage of link 2 to links 5 and 4 are 50% and 50%, respectively. The turning percentage of links 3 and 5 to link 6, both are 100%.

¹ The maximum number of vehicles that could leave a lane per hour in existence of vehicle queue

Table 3
External arrival, service rate, and storage capacity of the lane

	1	2	3	4	5	6
$E\lambda_i$	900	450	450	0	0	0
μ_i	1800	1800	1800	1800	1800	1800
c_i	1000	120	110	50	80	50

To solve the system of nonlinear equations (6), MATLAB routine solve was applied. Since the fsolve algorithm depends on initial guesses, different initial points were examined to obtain the solution. To get a certain positive result, each variable is substituted by the square of the new variable. The solution to this example is shown in table 4. The links probability distributions are obtained from (7) and (8), and are shown in figure 7.

Table 4
Blocking rate service, efficient service rate, probability of being full, and traffic intensity

	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$	$i = 6$
$Ub\mu_i$	900	427	450	900	450	900
$Eff\mu_i$	1600	560	450	900	450	900
$P(V_i = C_i)$	0.58	0.49	0.59	0.00	0.57	0.50
ρ_i	0.56	0.65	0.85	0.91	0.75	0.93

To further examine the accuracy and capability of the proposed approach, AIMSUN Micro-simulation experiments are conducted, and as in figure 7 is shown, the results and validity of the proposed approach are confirmed.

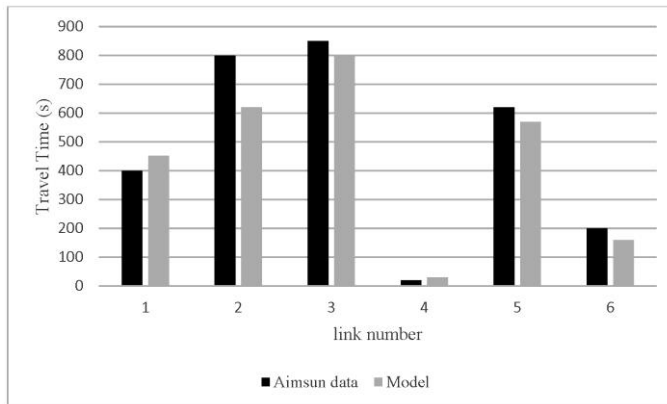


Figure 7
The comparison of the resulting mean travel time between AIMSUN Simulation output and the proposed model.

5. Case Study

In this part, an arterial street in Shiraz (Enghelab street) is selected as a case study, and the practical application of the proposed method is presented. Enghelab street includes 6 intersections and 14 links; each having two or three lanes. The main arterial contains two bounds, northbound (WB) and southbound (EB). The intersections are signalized except the second and the fourth, south-north (T-shaped intersections are un-signalized) figure 8. In this study, the traffic volume on the links of this arterial at the peak time (11 AM –1 PM) is considered, and travel time distributions are calculated (based on the proposed method).

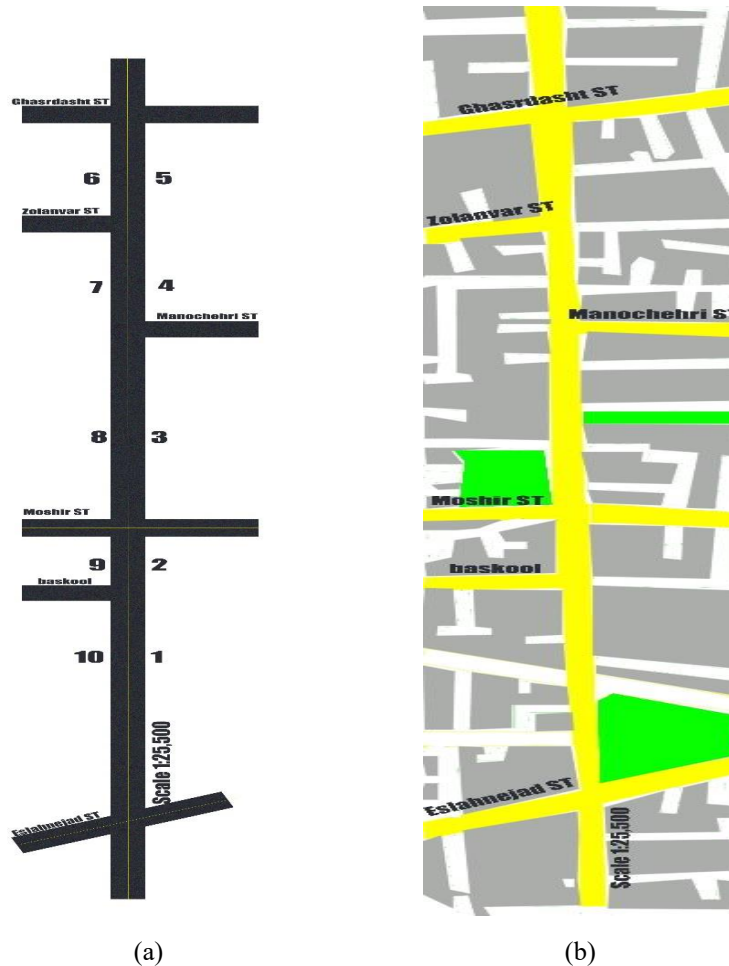


Figure 8
Study site; (a) The SUMO model of the arterial,
(b) The google map image of the arterial.

As the traffic flow is affected by many random factors and complex issues and it is not possible to model all its details, one of the methods to consider and examine it, is to use of simulation experiments. Micro-simulation has the ability to check the desired details, including the speed, acceleration and behavior of drivers in each of the vehicles separately. Micro-simulation experiments involve modeling individual vehicles and considering their interactions in a transportation system. Each vehicle's movement is simulated based on specific parameters, such as traffic signals, road geometry, and driver behavior. Micro-simulation collects detailed data on individual vehicle movements, allowing for a granular understanding of traffic flow and travel times. This method takes into account the dynamic nature of traffic, considering factors like congestion, lane changes, and interactions between vehicles. It is useful for capturing the complexity of real-world traffic conditions. Also, it can provide insights into the variability of travel times and help assess the impact of different variables on the transportation system. Micro-simulation experiments are generally considered more accurate as they account for individual vehicle interactions and real-world complexities. These experiments are computationally very expensive and time-consuming due to modeling of each vehicle individually, and therefore Micro-simulation experiments are usually used to check and estimate the validity of

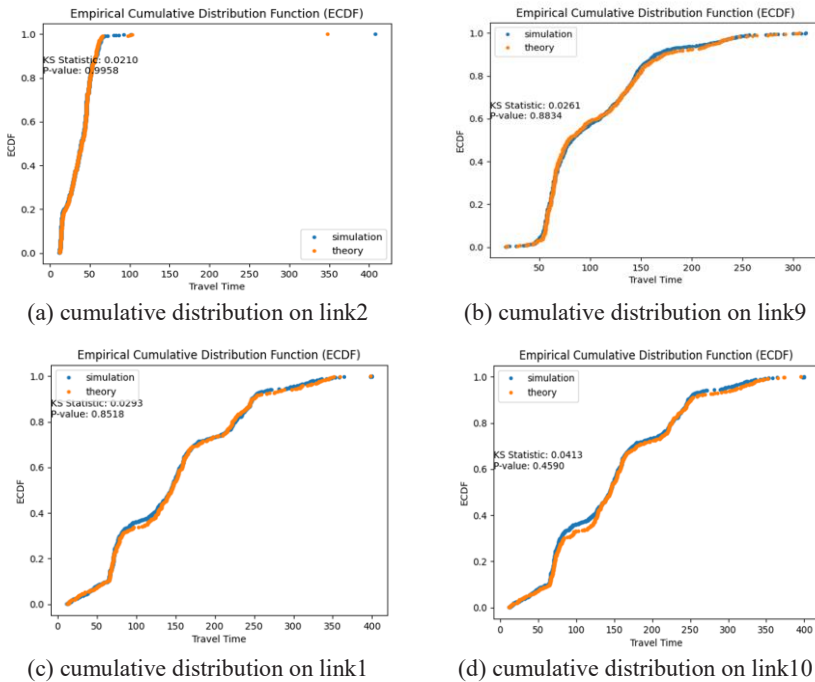


Figure 9

The comparison of simulation approach and the proposed approach

models, as we have done. Theoretical approaches, while providing valuable insights, may have limitations in capturing the nuances of traffic behavior. Theoretical approaches are often computationally less complicated and more economical, making them suitable for larger-scale applications.

To evaluate and examine the accuracy of the calculated travel time distribution, SUMO Micro-simulation experiments are conducted. To effectively compare of the obtained travel time between the proposed and simulation approach, the ECDF (Empirical Cumulative Distribution Function) at a specific time t is calculated as follows

$$F(t) = \frac{\text{number of travel time} \leq t}{\text{total number of travel time}} \quad (18)$$

The comparison of the travel time distribution of the SUMO micro-simulation approach and the proposed approach is shown in figure 9.

To evaluate the accuracy of the proposed model-based approach, first we computed the travel time of links based on Micro-simulation experiments (figure 9). To verify the accuracy of the model, we have conducted Kolmogorov–Smirnov test. The results of test for links 1, 2, 9 and 10 are shown in Table 5 as follows.

Table 5
p-values and KS-statistic in Kolmogorov Smirnov test

Link	KS-statistic	P-value
1	0.03	0.85
2	0.2	0.99
9	0.02	0.88
10	0.04	0.46

The P-values and KS-statistic across all links in the above table, shows the null hypothesis is accepted for all links and there is no significant difference between the simulated and theoretical distributions of travel times. Therefore, the results show the similarity of the distributions, and a close match between the empirical cumulative distribution function of the two data sets, the proposed model and simulation results.

In this approach the impact of signal offset has not taken into account. The focus of this paper is on the congested links where the intersection blocking is prevalent. To understand the effect of various amount of offset on travel time in uncongested network or link, it is possible to incorporate it the uniform delay formula.

6. Conclusion

In this paper, a new model-based travel time formulation for the congested arterial streets in the stationary condition is introduced. The resulting distributions consider both, the uncertainty of travel demand and intersection service time. The interactions of adjacent links are taken into account by considering the blocking and unblocking processes and using efficient service

time instead of service time. Another feature of the proposed travel time model is to find and compute the effect of intersection signal time (red and green duration) on travel time distribution in congested arterial networks. Also, the influence of three dependent significant factors; free-running travel time, uniform delay, and stochastic delay on travel time distribution is considered. One of the distinguishing features of this study, is modeling the queue of vehicles at the end of the links as a horizontal queue, which means, the changes in the length of the queue lead to changes in the distance of the free-running part of the link (what happens in reality). Based on our information, this issue has been studied and researched for the first time. Another prominent feature of this study, is to present a travel time prediction method based on supply and demand data, not experienced travel time data, which calculates travel time probability distribution. In other words, this method is particularly useful in scenarios where, due to changes in the network, the previous data may not be highly reliable or in special situations which there is only an estimate of supply and demand such as, the closure of a street or changes in traffic signal timing or large sporting events or emergency situations, etc.,.

7. Future works

Travel time is a crucial tool for all users of transportation networks, and dealing with the uncertainty of travel time is vital for experts and managers. In this regard, we developed a travel time model on links that covers the stochastic properties. The proposed model can be extended to a path consisting of several consecutive links and can be applied in modeling other transportation problems such as vehicle routing, network design, and reliable signal timing. Also, this model-based approach can be used both in the optimization of transportation problems, and doing some analysis such as sensitivity analysis and what-if analysis. Studying travel time on oversaturated links in both stationary and transient conditions is suggested.

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