

Cosmological dynamics of accelerating model in $f(Q)$ gravity with latest observational data : Case studies

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Abstract

In this brief report we have presented two logical approaches to the work of Bhardwaj et al. [1] having an issue of non-retrieval of GR from the $f(Q)$ gravity theory. Basically, we have dealt with two distinct schemes to resolve the issue via the following two approaches: (i) when the $f(Q)$ gravity follows the functional relation $f(Q) = -Q$, and (ii) when one considers the late time acceleration of the universe. It has been shown that under any one of the logical approaches one can resolve the issue of the coupling parameter (or constant) which was apparently not always getting null or vanishing.

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1. Introduction

In their recent article Bhardwaj and Garg [1] have employed $f(Q)$ gravity [2, 3, 4, 5, 6, 7, 8, 9, 10] to present a cosmological model which is compatible with Λ CDM in connection to one of the novel platforms under the modified or alternative gravity theories [11, 12, 13, 14, 15].

Now, the action for ' $f(Q)$ gravity' can be provided as [3]

$$S = \int \left[\mathcal{L}_M - \frac{1}{2} f(Q) \right] \sqrt{-g} d^4x, \quad (1)$$

where $\mathcal{L}_M$ is the matter Lagrangian density, g is the determinant of the metric $g_{\mu\nu}$ and $f(Q)$ is a general function of Q with its functional form as

$$f(Q) = Q + nQ^m, \quad (2)$$

where m and n are two functional parameters.

Also, spatially flat FLRW line element can be provided in the Cartesian coordinates as

$$ds^2 = -dt^2 + a^2(t)[dx^2 + dy^2 + dz^2], \quad (3)$$

where $a(t)$ denotes the measures of the cosmic expansion and is known as scale factor.

Now, the energy-momentum tensor can be expressed by $T_{ij} = (\rho + p)u_i u_j + p g_{ij}$, with ρ as the energy density and p as the pressure, whereas $\omega = \frac{p}{\rho}$ represents the equation of state for the ideal fluid.

Now, for an isotropic and homogeneous universe equations of energy density and cosmic pressure are expressed as

$$\rho = -3H^2 - n(m - \frac{1}{2})6^m H^{2m}, \quad (4)$$

$$p = 6^{m-1}(2m - 1)n(3H^2 + 2H\dot{m})(H^2)^{m-1} + 3H^2 + 2\dot{H}. \quad (5)$$

Now, under the usual demand, the general relativity formalism can be retrieved from any modified gravity by taking the coupling term as zero. However, from the above Eqs. (4) and (5), one can notice that here the situation is not so easy as we do not get back GR status rather by putting $n = 0$ and $Q = 0$ in Eq. (2), we get from Eq. (4) the following result:

$$3H^2 = -\rho. \quad (6)$$

2. Validation of the model in the context of GR

2.1 Scheme 1 - STEGR approach:

The result presented by authors [1] in their work is that the $f(Q)$ model considered is not reducible to GR. However, though apparently this is an odd case but this is not holistically the true case as this is completely originated from the following logical fact. One can easily retrieve the usual Friedmann equations of GR for the case $f(Q) = -Q$ (Symmetric Teleparallel Equivalent of General Relativity, i.e. the STEGR case) and can be easily verified from the article "coincident general relativity" [3]. However, there are two points must be noted in the previous article which are as follows: (i) that the sign in the action must be

positive to produce the presented Friedmann equations in $f(Q)$ gravity with the assumed metric [5], and (ii) that since $f(Q) = -Q$ is the GR case, one must choose the form $f(Q) = -Q + nQ^m$, instead of $f(Q) = Q + nQ^m$ [16]. However, since it is well known that the parameters are free in the model $f(Q) = Q + nQ^m$, so one can easily retrieve the usual Friedmann equations of GR for the choice $m = 1$ and $n = -2$ which gives the STEGR case $f(Q) = -Q$. As a result, it is easily verifiable that the considered model [1] is properly reduces to the GR limit.

2.2 Scheme 2 - Late-time acceleration approach:

One may examine the situation which is in conformity with GR and not at all unexpected. The left-hand side of Eq. (6) being absolutely a positive quantity we must have a positive counter-part in the right-hand side also and this is only possible for a negative energy density. To comply with the demand, we may invoke a negative energy density which is not at all unavailable in the modern-day literature [17, 18, 19, 20].

Let us try to connect the negative energy density with GR approved late-time acceleration of the universe. Now, the Einstein's erstwhile cosmological constant in its dynamical feature, i.e. $\Lambda = p$, takes the role of dark energy (DE). Under this DE condition, we get $p = -\rho$ as $p + \rho = 0$ and eventually $\Lambda = p = -\rho$. In the cosmological context positive Λ is related to the repulsive pressure. This repulsive pressure 'p' via the DE connected Λ has been considered as the responsible physical agent for the late-time accelerating scenario of the universe. In this context it is to be noted that Λ has been theoretically connected by Zel'dovich [21, 22] with the zero-point energy related phenomena of quantum fluctuation and experimentally established the connection by Casimir via Casimir effect [23, 24, 25].

3. Conclusion

In the present brief report which is in continuation to the previous work of Bhardwaj and Garg [1]) having an issue of non-retrieval of GR from the $f(Q)$ gravity case. Here, we have provided two schemes to resolve the issue via the following two approaches, viz. (i) when the $f(Q)$ gravity follows the functional relation $f(Q) = -Q$ (the Symmetric Teleparallel General Relativity, i.e. STEGR case), and (ii) when one considers the late time acceleration of the universe.

It has been established convincingly that under any one of the logical approaches we can resolve the issue of the coupling parameter (or constant) which was apparently not always getting null or vanishing. This interesting result does not contradict the basic foundation of GR rather we can get some significant physical aspects of GR which in essence appreciable in connection to the theoretical as well as observational point of views.

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