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# A geometric study of Vaidya metric

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**Abstract**

Vaidya's spacetime is the simplest non-static generalization of Schwarzschild's solution to Einstein's field equation. Vaidya metric represents radiating universe.With the aid of differential forms, various geometrical quantities related to Vaidya metric are obtained. We obtained the electrical parts of the Weyl tensor, it is found that they are related to the radial vector. We explored kinematic parameters in the terms of Ricci rotation coefficients. Also, we investigate behavior of general observer quantities at naked singularity $r = 0$.

***Subject Classification (2020):*** *83Cxx.*

***Keywords:*** *Vaidya metric, Differential forms, NP formalism, General observer quantities, Naked singularity.*

## 1. Introduction

The analysis of analogy of gravitational field and electromagnetic field requires computations of electric-magnetic parts of Weyl tensor as Weyl tensor describes gravitation in vacuum. Also, general observer quantities give insight about some kinematic parameters and they are also to be computed. The computations involve non-linear combinations of second order derivatives of metric tensor. This task is very complicated and time consuming and prone to human error when done manually. To overcome these difficulties computer aided algebraic computations is used. Hasmani et. al. have prepared generic programs based on

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Mathematica for such computations (See [4, 1]). In these programs computations are based on tensors.

Alternatively, an analytic tool of differential forms is also available [13]. For the Gödel Universe, Hasmani and Katkar [2] investigated the electric part of the Weyl tensor using the differential form technique. This paper is aimed at exhibiting the power of differential forms by computing electric-magnetic part of Weyl tensor and kinematic parameters for Vaidya metric. Thereby understanding nature of singularity $r=0$ in Vaidya metric, which is known to present a naked singularity [10, 12, 9].

## 2. NP Formalism in Relation to Differential Forms

The Newmann-Penrose formalism is a tetrad formalism with special choice of the basis vectors. This formalism was proposed by Newmann and Penrose [7]. The Newmann-Penrose tetrad is composed of real and complex null vector fields represented as,

$$e_{(\alpha)i} = (l_i, n_i, m_i, \overline{m}_i). \tag{2.1}$$

Here the Greek letters $\alpha, \beta, ...$ indicate the tetrad indices while the Latin indices denote the tensor indices, all these indices run from 1 to 4, this terminology will be followed throughout the paper. Also, Einstein's summation convention is used. The Newmann and Penrose complex null vector fields $l_i, n_i, m_i, \overline{m}_i$ satisfy the conditions,

$$l_i n^i = 1 = -m_i \overline{m}^i, \tag{2.2}$$

and

$$\begin{aligned} l_i l^i = n_i n^i = m_i m^i = \overline{m}_i \overline{m}^i = 0, \\ m_i l^i = \overline{m}_i l^i = m_i n^i = \overline{m}_i n^i = 0. \end{aligned} \tag{2.3}$$

The differential forms, a significant tool for modern mathematics, are employed to reduce computational complexity. In differential forms, the roles of the 40 Christoffel symbols, which have no invariant meaning under changes in co-ordinates in the four-dimensional spacetime of theory of relativity, are played by only 6 components of connection 1-form. In this paper, the terminology and notations used are those used by Israel [13]. The four basis 1-forms corresponding to four basis vectors of the dual tetrad $e^{(\alpha)}$ are specified as follows:

$$\theta^\alpha = e^{(\alpha)}{}_i dx^i. \tag{2.4}$$

The Cartan's first equation of structure is given by,

$$d\theta^{\alpha} = -\omega^{\alpha}{}_{\beta} \wedge \theta^{\beta}. \tag{2.5}$$

The $\omega^{\alpha}{}_{\beta}$ represents anti-symmetric connection 1-forms and is defined as follows:

$$\omega^{\alpha}{}_{\beta} = \gamma^{\alpha}{}_{\beta\delta}\theta^{\delta}. \tag{2.6}$$

Here, $\gamma^{\alpha}{}_{\beta\delta}$ indicates the Ricci rotation coefficients. In terms of Newmann-Penrose spin coefficients, the non-zero components of anti-symmetric connection 1-forms $\omega^{\alpha}{}_{\beta}$ can be written as [5]

$$\begin{aligned}
\omega^{1}{}_{1} &= (\varepsilon+\bar{\varepsilon})\theta^{1} + (\gamma+\bar{\gamma})\theta^{2} + (\bar{\alpha}+\beta)\theta^{3} + (\alpha+\bar{\beta})\theta^{4},\\
\omega^{1}{}_{3} &= \bar{\pi}\theta^{1} + \bar{\nu}\theta^{2} + \bar{\lambda}\theta^{3} + \bar{\mu}\theta^{4},\\
\omega^{1}{}_{4} &= \pi\theta^{1} + \nu\theta^{2} + \mu\theta^{3} + \lambda\theta^{4},\\
\omega^{2}{}_{3} &= -\kappa\theta^{1} - \tau\theta^{2} - \sigma\theta^{3} - \rho\theta^{4},\\
\omega^{2}{}_{4} &= -\bar{\kappa}\theta^{1} - \bar{\tau}\theta^{2} - \bar{\rho}\theta^{3} - \bar{\sigma}\theta^{4},\\
\omega^{3}{}_{3} &= (\varepsilon-\bar{\varepsilon})\theta^{1} + (\gamma-\bar{\gamma})\theta^{2} + (\beta-\bar{\alpha})\theta^{3} + (\alpha-\bar{\beta})\theta^{4}.
\end{aligned} \tag{2.7}$$

Using equations (2.7) in Cartan's first equation of structure (2.5) we get

$$\begin{aligned}
d\theta^{1} &= (\gamma+\bar{\gamma})\theta^{1}\wedge\theta^{2} + (\bar{\alpha}+\beta-\bar{\pi})\theta^{1}\wedge\theta^{3} + (\alpha+\bar{\beta}-\pi)\theta^{1}\wedge\theta^{4}\\
&\quad -\bar{\nu}\theta^{2}\wedge\theta^{3} - \nu\theta^{2}\wedge\theta^{4} + (\bar{\mu}-\mu)\theta^{3}\wedge\theta^{4},\\
d\theta^{2} &= (\varepsilon+\bar{\varepsilon})\theta^{1}\wedge\theta^{2} + (\tau-\bar{\alpha}-\beta)\theta^{2}\wedge\theta^{3} + (\bar{\tau}-\alpha-\bar{\beta})\theta^{2}\wedge\theta^{4}\\
&\quad +\kappa\theta^{1}\wedge\theta^{3} + \bar{\kappa}\theta^{1}\wedge\theta^{4} + (\bar{\rho}-\rho)\theta^{3}\wedge\theta^{4},\\
d\theta^{3} &= (\bar{\tau}+\pi)\theta^{2}\wedge\theta^{1} + (\bar{\rho}+\varepsilon-\bar{\varepsilon})\theta^{3}\wedge\theta^{1} + \bar{\sigma}\theta^{4}\wedge\theta^{1}\\
&\quad +[\mu-(\gamma-\bar{\gamma})]\theta^{2}\wedge\theta^{3} + \lambda\theta^{2}\wedge\theta^{4} + (\alpha-\bar{\beta})\theta^{3}\wedge\theta^{4},\\
d\theta^{4} &= (\tau+\bar{\pi})\theta^{2}\wedge\theta^{1} + (\rho+\bar{\varepsilon}-\varepsilon)\theta^{4}\wedge\theta^{1} + \sigma\theta^{3}\wedge\theta^{1}\\
&\quad +[\bar{\mu}-(\bar{\gamma}-\gamma)]\theta^{2}\wedge\theta^{4} + \bar{\lambda}\theta^{2}\wedge\theta^{3} + (\bar{\alpha}-\beta)\theta^{4}\wedge\theta^{3}.
\end{aligned} \tag{2.8}$$

## 3. Vaidya spacetime using Differential Forms

The geometry of Vaidya spacetime is described by the metric [11]

$$ds^{2} = 2drdu + \left(1 - \frac{2m[u]}{r}\right)du^{2} - r^{2}d\theta^{2} - r^{2}sin\theta^{2}d\phi^{2}, \tag{3.1}$$

where $m[u]$ is an arbitrary function of retarded time $u(=t-r)$. The coordinates $(u,r,\theta,\phi)$ are labelled as $x^{i}$ $(i=1,2,3,4)$. This line element

was obtained by Vaidya in 1943, to represent the exterior field of spherically symmetric radiating star, whose mass at retarded time $u$ is denoted $m[u]$. The non-vanishing components of the metric tensor $g_{ij}$ are given by,

$$g_{11} = 1 - \frac{2m[u]}{r}, g_{12} = 1 = g_{21}, g_{33} = -r^2, g_{44} = -r^2 \sin^2\theta \tag{3.2}$$

and hence the contravariant components $g^{ij}$ of the metric tensor are obtained as

$$g^{22} = -\left(1 - \frac{2m[u]}{r}\right), g^{12} = 1 = g^{21}, g^{33} = -\frac{1}{r^2}, g^{44} = -\frac{\csc^2\theta}{r^2}. \tag{3.3}$$

The set of four basis 1-forms $\theta^\alpha$ are chosen as

$$\begin{aligned}
\theta^1 &= du, \\
\theta^2 &= dr + \frac{1}{2}\left(1 - \frac{2m}{r}\right)du, \\
\theta^3 &= \frac{r}{\sqrt{2}}(d\theta - i\sin\theta d\phi), \\
\theta^4 &= \frac{r}{\sqrt{2}}(d\theta + i\sin\theta d\phi).
\end{aligned} \tag{3.4}$$

Hence the metric (3.1) reduces to

$$ds^2 = 2\theta^1\theta^2 - 2\theta^3\theta^4. \tag{3.5}$$

The non-vanishing covariant components of the metric tensor, denoted by $\eta_{(\alpha)(\beta)}$, are provided for the metric form described in (3.5) as:

$$\eta_{(1)(2)} = 1 = \eta_{(2)(1)}, \eta_{(3)(4)} = -1 = \eta_{(4)(3)}. \tag{3.6}$$

Conventionally tetrad components are indicated by enclosing indices in parenthesis, for ease of reading we drop this notation.

The definition of 1-forms $\theta^\alpha = e^{(\alpha)}{}_i dx^i$ and the equations (3.4) lead to

$$\begin{aligned}
l_i &= \left(\frac{1}{2}\left(1 - \frac{2m}{r}\right), 1, 0, 0\right), \\
n_i &= (1, 0, 0, 0), \\
m_i &= \left(0, 0, -\frac{r}{\sqrt{2}}, -\frac{ir}{\sqrt{2}}\sin\theta\right), \\
\bar{m}_i &= \left(0, 0, -\frac{r}{\sqrt{2}}, \frac{ir}{\sqrt{2}}\sin\theta\right).
\end{aligned} \tag{3.7}$$

Using $g^{ij}$ given in (3.3) we get,

$$l^i = \left(1, \frac{-1}{2}\left(1 - \frac{2m}{r}\right), 0, 0\right),$$
$$n^i = (0, 1, 0, 0),$$
$$m^i = \left(0, 0, \frac{1}{\sqrt{2}r}, \frac{i}{\sqrt{2}r}\csc\theta\right),$$
$$\overline{m}^i = \left(0, 0, \frac{1}{\sqrt{2}r}, -\frac{i}{\sqrt{2}r}\csc\theta\right). \tag{3.8}$$

The basis 1-forms $\theta^\alpha$ have exterior derivatives that are as follows:

$$d\theta^1 = 0,$$
$$d\theta^2 = \frac{m}{r^2}\theta^2 \wedge \theta^1,$$
$$d\theta^3 = \frac{1}{r}\theta^2 \wedge \theta^3 - \frac{1}{2r}\left(1 - \frac{2m}{r}\right)\theta^1 \wedge \theta^3 - \frac{\cot\theta}{\sqrt{2}r}\theta^3 \wedge \theta^4,$$
$$d\theta^4 = \frac{1}{r}\theta^2 \wedge \theta^4 - \frac{1}{2r}\left(1 - \frac{2m}{r}\right)\theta^1 \wedge \theta^4 + \frac{\cot\theta}{\sqrt{2}r}\theta^3 \wedge \theta^4. \tag{3.9}$$

By comparing the respective coefficients in equations (2.8) and (3.9) and resolving them, we derive the following results:

$$\mu = \frac{1}{r}, \rho = \frac{1}{2r}\left(1 - \frac{2m}{r}\right),$$
$$\varepsilon = -\frac{m}{2r^2}, \alpha = -\frac{\cot\theta}{2\sqrt{2}r} = -\beta, \tag{3.10}$$

and the remaining spin coefficients vanish. These expressions are matching with the spin coefficients obtained by Hasmani and Panchal [3] using different method.

The tetrad components of connection 1-forms are easily obtained by substituting these values in equations (2.7) as follows:

$$\omega^1{}_1 = -\omega^2{}_2 = -\frac{m}{r^2}\theta^1, \omega^3{}_3 = -\omega^4{}_4 = \frac{\cot\theta}{\sqrt{2}r}(\theta^3 - \theta^4),$$
$$\omega^1{}_3 = \omega^4{}_2 = \frac{1}{r}\theta^4, \omega^1{}_4 = \omega^3{}_2 = \frac{1}{r}\theta^3,$$
$$\omega^2{}_3 = \omega^4{}_1 = \frac{-1}{2r}\left(1 - \frac{2m}{r}\right)\theta^4, \omega^2{}_4 = \omega^3{}_1 = \frac{-1}{2r}\left(1 - \frac{2m}{r}\right)\theta^3. \tag{3.11}$$

The Cartan's second equations of structure is expressed as:

$$\Omega^{\alpha}{}_{\beta} = d\omega^{\alpha}{}_{\beta} + \omega^{\alpha}{}_{\gamma} \wedge \omega^{\gamma}{}_{\beta}, \tag{3.12}$$

using these and then following necessary algebraic procedure we get

$$\Omega^{1}{}_{1} = -\Omega^{2}{}_{2} = \frac{2m}{r^3}\theta^2 \wedge \theta^1, \qquad \Omega^{3}{}_{3} = -\Omega^{4}{}_{4} = \frac{2m}{r^3}\theta^3 \wedge \theta^4,$$

$$\Omega^{1}{}_{3} = \Omega^{4}{}_{2} = \frac{-m}{r^3}\theta^1 \wedge \theta^4, \qquad \Omega^{1}{}_{4} = \Omega^{3}{}_{2} = \frac{-m}{r^3}\theta^1 \wedge \theta^3,$$

$$\Omega^{2}{}_{3} = \Omega^{4}{}_{1} = \frac{-m}{r^3}\theta^2 \wedge \theta^4 + \frac{m'}{r^2}\theta^1 \wedge \theta^4,$$

$$\Omega^{2}{}_{4} = \Omega^{3}{}_{1} = \frac{-m}{r^3}\theta^2 \wedge \theta^3 + \frac{m'}{r^2}\theta^1 \wedge \theta^3. \tag{3.13}$$

The curvature 2-forms and Riemann-Christoffel tensor are related by

$$\Omega^{\alpha}{}_{\beta} = \frac{1}{2}R^{\alpha}{}_{\beta\gamma\delta}\theta^{\gamma} \wedge \theta^{\delta}. \tag{3.14}$$

Comparing the coefficients of the corresponding basis 2-forms in equations (3.13) and using (3.14) we obtain non-zero independent Riemann-Christoffel tensor in tetrad frame as

$$R^{1}{}_{112} = -R^{2}{}_{212} = -R^{3}{}_{334} = R^{4}{}_{434} = \frac{-2m}{r^3},$$

$$R^{1}{}_{314} = R^{1}{}_{413} = R^{2}{}_{324} = R^{2}{}_{423} = R^{3}{}_{213} = R^{3}{}_{123} = R^{4}{}_{214} = R^{4}{}_{124} = \frac{-m}{r^3},$$

$$R^{2}{}_{314} = R^{2}{}_{413} = R^{3}{}_{113} = R^{4}{}_{114} = \frac{m'}{r^2}, \tag{3.15}$$

and hence non-zero tetrad components of the Riemannian tensor are,

$$R_{1212} = R_{3434} = \frac{2m}{r^3},$$

$$R_{2314} = R_{2413} = \frac{-m}{r^3}, \tag{3.16}$$

$$R_{1314} = \frac{m'}{r^2}.$$

Further, the only non-zero tetrad components of the Ricci tensor is,

$$R_{11} = \frac{-2m'}{r^2}. \tag{3.17}$$

Since $\eta^{11}$ is zero, the Ricci scalar $R$ vanishes. The tetrad components of the Weyl tensor are given by

$$C_{\alpha\beta\gamma\delta} = R_{\alpha\beta\gamma\delta} + \frac{1}{2}(\eta_{\alpha\gamma}R_{\beta\delta} + \eta_{\beta\delta}R_{\alpha\gamma} - \eta_{\beta\gamma}R_{\alpha\delta} - \eta_{\alpha\delta}R_{\beta\gamma}) + \frac{1}{6}(\eta_{\alpha\gamma}\eta_{\beta\delta} - \eta_{\alpha\delta}\eta_{\beta\gamma})R, \tag{3.18}$$

with cyclic property $C_{abcd} + C_{acdb} + C_{adbc} = 0$. Thus, the non-vanishing tetrad components of the Weyl tensor are

$$C_{1212} = \frac{2m}{r^3} = C_{3434},$$
$$C_{1324} = \frac{-m}{r^3} = C_{1423}. \tag{3.19}$$

In the Newmann-Penrose formalism, the Weyl tensor is expressed as five complex scalars $\Psi_0$, $\Psi_1$, $\Psi_2$, $\Psi_3$, $\Psi_4$, when projected onto the null tetrad. In the case of Vaidya metric only non-vanishing Weyl scalar is,

$$\Psi_2 = -C_{1342} = -C_{hijk}l^h m^i \overline{m}^j n^k = \frac{-m}{r^3}.$$

Since only non-vanishing complex scalar is $\Psi_2$, the Vaidya metric (10) is of Petrov type D [see, [7]]. It is also known that gravitational field in Petrov type D spacetime is of Coulomb type.

### 3.1 *Electric-Magnetic Parts of Weyl Tensor*

In the tetrad frame, the electric and magnetic parts of Weyl tensor $C_{\alpha\beta\gamma\delta}$ are denoted by $E_{\alpha\beta}$ and $H_{\alpha\beta}$ respectively with respect to an observer with velocity $u^\alpha$ are defined by,

$$E_{\alpha\gamma} = C_{\alpha\beta\gamma\delta}u^\beta u^\delta, \tag{3.20}$$
$$H_{\alpha\gamma} = C^\star_{\alpha\beta\gamma\delta}u^\beta u^\delta, \tag{3.21}$$

where $C^\star_{\alpha\beta\gamma\delta}$ is dual of $C_{\alpha\beta\gamma\delta}$ defined by

$$C^\star_{\alpha\beta\gamma\delta} = \frac{1}{2}\varepsilon_{\alpha\beta\zeta\iota}\eta^{\zeta\zeta_1}\eta^{\iota\iota_1}C_{\gamma\delta\zeta_1\iota_1}, \tag{3.22}$$

and $\varepsilon_{\alpha\beta\zeta\iota}$ denoting Levi-Civita symbols.

Here, $u^\alpha$ is tetrad components of unit time-like vector $u^i$ defined as,

$$u^\alpha = u^i e^{(\alpha)}{}_i, \tag{3.23}$$

where unit time-like vector $u^i$ is taken as

$$u^i = \frac{1}{\sqrt{2}}(l^i + n^i),$$

so from (3.8) we write,

$$u^i = \frac{1}{\sqrt{2}}\left(1, \frac{1}{2} + \frac{m}{r}, 0, 0\right), \tag{3.24}$$

to be velocity vector, then we will obtain tetrad components of unit time-like vector as,

$$u^\alpha = \frac{1}{\sqrt{2}}(1, 1, 0, 0). \tag{3.25}$$

The non-vanishing components of electric part of Weyl tensor with respect to the velocity vector (3.25) are,

$$E_{11} = E_{22} = -E_{12} = -E_{34} = \frac{m}{r^3}. \tag{3.26}$$

By equation (3.22) non-vanishing indepedent coponents of dual of Weyl tensor are,

$$C^\star_{1234} = -\frac{2m}{r^3} \text{ and } C^\star_{1324} = -\frac{m}{r^3} = -C^\star_{1423}. \tag{3.27}$$

It can be easily seen that all components of magnetic parts are vanishing.

Hence, the Vaidya metric is purely electric.

## 4. Kinematic Parameters for Vaidya metric

In 1971 Ellis [8] introduced general observer quantities for a given spacetime.The velocity of the observer is a unit time-like vector $u_i$. Hasmani and Panchal [4] developed a Mathematica program to compute these quantities for the Gödel universe. Modifying that program we computed the general observer quantities for Vaidya spacetime. The table 1 shows general expressions of the quantities and their computed values for Vaidya metric.

**Table 1**

**General expressions of the quantities and their computed values for Vaidya metric**

| General observer quantities | General observer quantities for Vaidya metric |
|---|---|
| The projection tensor $h_{ij}$:<br>$h_{ij} = g_{ij} - u_i u_j$<br>and has the properties<br>$h_{ij} = h_{ji}, \quad h_i^k h_{kj} = h_{ij},$<br>$h_i^i = 3, \quad h_{ij} u^j = 0.$ | Projection tensor,<br>$h_{11} = \frac{-(r+2m)^2}{8r^2} \quad h_{12} = \frac{r+2m}{4r} = h_{21},$<br>$h_{22} = -\frac{1}{2}, \quad h_{33} = -r^2,$<br>$h_{44} = -r^2 \sin^2\theta.$ |
| The expansion scalar $\theta$:<br>$\theta = u^i_{;i}.$<br>where semicolon denotes the covariant derivative. | Expansion scalar,<br>$\theta = \frac{r+m}{\sqrt{2}r^2}.$ |
| The acceleration vector $a_i$:<br>$a_i = \dot{u}_i = u_{i;j} u^j$<br>and is such that<br>$a_i u^i = 0.$ | Acceleration,<br>$a_i = \left( \frac{mr+2m^2}{4r^3}, \frac{-m}{2r^2}, 0, 0 \right).$ |
| The symmetric shear tensor $\sigma_{ij}$: | Shear Tensor, |
| $\sigma_{ij} = h_i^k h_j^l u_{(k;l)} - \frac{1}{3}\theta h_{ij}$<br>and has the properties<br>$\sigma_{ij} u^j = 0, \quad \sigma_i^i = 0.$ | $\sigma_{11} = \frac{(r+2m)^2(r+4m)}{24\sqrt{2}r^4},$<br>$\sigma_{12} = \frac{-(r+2m)(r+4m)}{12\sqrt{2}r^3} = \sigma_{21},$<br>$\sigma_{22} = \frac{r+4m}{6\sqrt{2}r^2},$<br>$\sigma_{33} = \frac{-(r+4m)}{6\sqrt{2}},$<br>$\sigma_{44} = \frac{-(r+4m)\sin^2\theta}{6\sqrt{2}}.$ |
| The anti-symmetric vorticity $\omega_{ij}$:<br>$\omega_{ij} = h_i^k h_j^l u_{[k;l]}.$<br>This rotation tensor satisfies<br>$\omega_{ij} u^i = \omega_{ij} u^j = 0.$ | Vorticity<br>$\omega_{ij} = 0 \quad \forall\, i, j.$ |

The various general observer quantities are defined in the first column, whereas the second column represents computed general observer quantities for Vaidya metric in tensor form.

The tetrad components of these quantities are as follows,

(1) Projection,

$$h_{11} = h_{22} = -h_{12} = -h_{21} = \frac{-1}{2}, h_{34} = h_{43} = -1, h_{33} = h_{44} = 0. \tag{4.1}$$

(2) Expansion,

$$\theta = \frac{r+m}{\sqrt{2}r^2} = \frac{1}{\sqrt{2}}(\mu - 2\varepsilon). \tag{4.2}$$

(3) Acceleration,

$$a_\alpha = \frac{m}{2r^2}(1,-1,0,0) = \varepsilon(-1,1,0,0). \tag{4.3}$$

(4) Shear,

$$\sigma_{11} = \sigma_{22} = -\sigma_{12} = -\sigma_{21} = -\sigma_{34} = -\sigma_{43} = \frac{(r+4m)}{6\sqrt{2}r^2} = \frac{1}{\sqrt{2}}(\mu - 8\varepsilon). \tag{4.4}$$

(5) Vorticity

$$\omega_{ij} = 0 \forall i, j \tag{4.5}$$

Since vorticity identically vanishes the Vaidya spacetime is expanding with acceleration and it is non-rotating. The tetrad components of acceleration are inversely proportional to radial coordinate which are related to the Coulomb-type field, this is in tandem with Vaidya spacetime being of Petrov type D.

## 5. Conclusion

We obtained various geometric quantities for Vaidya spacetime using Cartan's first and second equations of structure. Also, electric and magnetic parts of Weyl tensors in the form of tetrad components have been obtained and proved that the Vaidya metric is of purely electric type. This supports the conjecture by McIntosh et.al. [6] which states that "The Weyl tensor to be purely electric or purely magnetic if and only if the Weyl tensor is of Petrov type D". We observed that in tetrad form general observer quantities reduce compared to tensor form and hence calculations are easy. Since we assume that there is an outflow of radiation which results in to $m[u]$ to be decreasing function and also it is non-negative definite function which means that $\theta > 0$ and hence there is expansion. Also, it is noted that the

tetrad form of general observer quantities can be written as Ricci rotation coefficients (except projection tensor). Various kinematic quantities (Eq. (4.1)-(4.5)) diverges at naked singularity $r = 0$ except tetrad components of projection tensor and vorticity.

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