

GROUP THEORETIC TRANSFORMATION TECHNIQUE FOR MHD VISCOUS FLUID WITH HEAT TRANSFER OVER A NON-LINEAR POROUS SURFACE

PANKAJ SONAWANE¹ AND M.G. TIMOL²

¹DEPARTMENT OF MATHEMATICS, DHANAJI NANA MAHAVIDYALAYA, FAIZPUR,
DIST JALGAON, MAHARASHTRA (INDIA)

²DEPARTMENT OF MATHEMATICS, VEER NARMAD SOUTH GUJARAT UNIVERSITY,
SURAT, GUJARAT, (INDIA)

E-MAIL: ¹SONAWANEPANKAJ311@GMAIL.COM

(Received: 8 June 2020, Accepted: 12 January 2021)

ABSTRACT. Using group theoretic transformation technique, class of similarity solution of heat transfer for MHD boundary layer flow of viscous fluid over a non linear porous surface. This similarity equation which is highly nonlinear ordinary differential equations which is solved numerically for particular fluids so called power-law fluids. The results obtain are found in good agreement with those available in literature.

AMS Classification: 76M60.

Keywords: Group-theoretic transformation method; MHD; Boundary layer flow; Power-law fluid.

1. INTRODUCTION

The process of heat transfer or heat and mass transfer together occurs simultaneously in a moving fluid and plays an important role in the design of chemical processing equipment, nuclear reactors, and formation and dispersion of fog. Dass

et al.[4] considered the mass transfer effects on flow past an impulsively started infinite isothermal vertical plate with constant mass flux. Muthucumaraswamy et al.[8] presented an exact solution to the problem of flow past an impulsively started infinite vertical plate in the presence of uniform heat and mass flux at the plate using Laplace transform technique.

The MHD boundary layer flow of an electrically conducting fluid is encountered in many problems in geophysics, astrophysics, engineering applications and other industrial areas. Engineers apply MHD principle in the design of heat exchangers, pumps, in space vehicle propulsion, thermal protection, control and re-entry and in creating novel power generating systems. However, hydromagnetic flow and heat transfer problems have become more important industrially. In many metallurgical processes involve the cooling of many continuous strips or filaments by drawing them through an electrically conducting fluid subject to a magnetic field, the rate of cooling can be controlled.

Lai and Kulacki [7] investigated the coupled heat and mass transfer by mixed convection from a vertical plate in a saturated porous medium. The porous media heat transfer problems have numerous thermal engineering applications such as geothermal energy recovery, crude oil extraction, thermal insulation, ground water pollution, oil extraction, thermal energy storage, thermal insulations and flow through filtering devices. Excellent reviews on this topic are provided in the literature by Nield and Bejan [9], Vafai [11], Ingham and Pop [6] and Vadasz [12]. Recently, Cheng and Lin [2] examined the melting effect on mixed convective heat transfer from a permeable over a continuous Surface embedded in a liquid saturated porous medium with aiding and opposing external flows. Sharma and Hazarika [10] studies the effects of variable viscosity and thermal conductivity on heat and mass transfer flow along a vertical plate in the presence of a magnetic field. Hassanien et al [5] presented a work on flow and heat transfer in power law fluid over a stretching porous surface with variable surface temperature. Abel and Mahesha [1] considered the effects of buoyancy and variable thermal conductivity in a power law fluid past on a vertical stretching sheet in the presence of non-uniform heat source. Darji and Timol [3] studied on similarity solution of heat transfer for MHD viscous power-law fluid over a non-linear porous surface.

So motivated by these, we represent in this paper, class of similarity solution for

MHD viscous fluid with heat transfer over a nonlinear porous surface by using linear group theoretic transformation technique.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

Consider a two-dimensional flow of an incompressible viscous fluid past a porous non-linear surface at $y=0$. The dimensionless equations of continuity, momentum, heat transfer as follow,

Continuity

$$(1) \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Momentum Equation

$$(2) \quad u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = n \left(\frac{\partial u}{\partial y} \right)^{n-1} \frac{\partial^2 u}{\partial y^2} - Su$$

Energy Equation

$$(3) \quad u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} + \lambda \left(\frac{\partial u}{\partial y} \right)^2$$

Where $S = \frac{\sigma B^2(x)}{\rho}$ is the magnetic field strength, $Pr = \frac{\rho C_p v}{\alpha}$ is the Prandtl number

and $\lambda = \frac{\mu U^2}{L^2}$ is the flow parameter.

The boundary conditions for the problem are,

$$y = 0, \quad u = u_w(x), \quad v = v_w(x), \quad \theta = \theta_w(x)$$

$$(4) \quad y = \infty, \quad u = 0, \quad \theta = 0$$

Introducing stream function ψ such that,

$$(5) \quad u = \frac{\partial \psi^*}{\partial y^*} \quad v = -\frac{\partial \psi^*}{\partial x^*}$$

Equation of continuity (1) gets satisfied identically, equation (2-4) becomes

$$(6) \quad \frac{\partial \psi^*}{\partial y^*} \frac{\partial^2 \psi^*}{\partial x^* \partial y^*} - \frac{\partial \psi^*}{\partial x^*} \frac{\partial^2 \psi^*}{\partial y^{*2}} = n \left(\frac{\partial^2 \psi^*}{\partial y^{*2}} \right)^{n-1} \frac{\partial^3 \psi^*}{\partial y^{*3}} - S \frac{\partial \psi^*}{\partial y^*}$$

$$(7) \quad \frac{\partial \psi^*}{\partial y^*} \frac{\partial \theta^*}{\partial x^*} - \frac{\partial \psi^*}{\partial x^*} \frac{\partial \theta^*}{\partial y^*} = \frac{1}{Pr} \frac{\partial^2 \theta^*}{\partial y^{*2}} + \lambda \left(\frac{\partial^2 \psi^*}{\partial y^{*2}} \right)^2$$

With boundary conditions,

$$(8) \quad y = 0, \quad \frac{\partial \psi}{\partial y} = u_w(x), \quad \frac{\partial \psi}{\partial x} = v_w(x), \quad \theta = \theta_w(x)$$

$$(9) \quad y = \infty, \quad \frac{\partial \psi}{\partial y} = 0, \quad \theta^* = 0$$

3. SOLUTION OF THE PROBLEM

To reduce the above system of partial differential equations to a system of ordinary differential equations by introducing one parameter group of transformation given as

$$(10) \quad \begin{aligned} \bar{x}^* &= \gamma^{\xi_1} x^*, & \bar{y}^* &= \gamma^{\xi_2} y^*, & \bar{\psi}^* &= \gamma^{\xi_3} \psi^* \\ \bar{S}^* &= \gamma^{\xi_4} S^*, & \bar{\theta}^* &= \gamma^{\xi_5} \theta^* \end{aligned}$$

Where $\xi_1, \xi_2, \xi_3, \xi_4, \xi_5$ and γ are constants.

From equation (10) one obtains

$$(11) \quad \left(\frac{\bar{x}^*}{x^*} \right)^{\frac{1}{\xi_1}} = \left(\frac{\bar{y}^*}{y^*} \right)^{\frac{1}{\xi_2}} = \left(\frac{\bar{\psi}^*}{\psi^*} \right)^{\frac{1}{\xi_3}} = \left(\frac{\bar{S}^*}{S^*} \right)^{\frac{1}{\xi_4}} = \left(\frac{\bar{\theta}^*}{\theta^*} \right)^{\frac{1}{\xi_5}} = \gamma$$

Introducing the linear transformation, given by equation (10), into the Equations (6)-(7) result in

$$(12) \quad \begin{aligned} & \gamma^{\xi_1+2\xi_2-2\xi_3} \frac{\partial \bar{\psi}^*}{\partial \bar{y}^*} \frac{\partial^2 \bar{\psi}^*}{\partial \bar{x}^* \partial \bar{y}^*} - \gamma^{\xi_1+2\xi_2-2\xi_3} \frac{\partial \bar{\psi}^*}{\partial \bar{x}^*} \frac{\partial^2 \bar{\psi}^*}{\partial \bar{y}^{*2}} \\ &= n \left(\gamma^{2\xi_2-\xi_3} \frac{\partial^2 \bar{\psi}^*}{\partial \bar{y}^{*2}} \right)^{n-1} \gamma^{3\xi_2-\xi_3} \frac{\partial^3 \bar{\psi}^*}{\partial \bar{y}^{*3}} - S \gamma^{\xi_2-\xi_3} \frac{\partial \bar{\psi}^*}{\partial \bar{y}^*} \end{aligned}$$

$$(13) \quad \begin{aligned} & \gamma^{\xi_1+\xi_2-\xi_3-\xi_5} \frac{\partial \bar{\psi}^*}{\partial \bar{y}^*} \frac{\partial \bar{\theta}^*}{\partial x^*} - \gamma^{\xi_1+\xi_2-\xi_3-\xi_5} \frac{\partial \bar{\psi}^*}{\partial \bar{x}^*} \frac{\partial \bar{\theta}^*}{\partial y^*} \\ &= \frac{1}{P_r} \gamma^{2\xi_2-\xi_5} \frac{\partial^2 \bar{\theta}^*}{\partial \bar{y}^{*2}} + \lambda \left(\gamma^{2\xi_2-\xi_3} \frac{\partial^2 \bar{\psi}^*}{\partial \bar{y}^{*2}} \right)^2. \end{aligned}$$

If one require that the differential equations are completely invariant to the proposed linear transformations, under the assumption that the exponent $\xi_1 \neq 0$, a ratio $\xi = \frac{\xi_2}{\xi_1}$. Thus one obtains the relations

$$(14) \quad \xi = \xi_1 = 3\xi_2 = \frac{3}{2}\xi_3 = -\frac{3}{2}\xi_4 = \frac{3}{2}\xi_5$$

Introducing equation(14) into equation(11) result in

$$(15) \quad \eta = \frac{y^*}{x^{\frac{2}{3}}}, \quad \psi^* = f(\eta)x^{\frac{2}{3}}, \quad \theta^* = G(\eta)x^{\frac{2}{3}}, \quad S = S_0x^{-\frac{2}{3}}$$

4. FORMULATION OF ORDINARY DIFFERENTIAL EQUATIONS

Transforming the similarity variables (15) into equations (6)-(7) result in

$$(16) \quad \frac{\partial \bar{\psi}^*}{\partial \bar{y}^*} = x^{\frac{1}{3}} f'(\eta), \quad \frac{\partial^2 \bar{\psi}^*}{\partial \bar{y}^{*2}} = f''(\eta), \quad \frac{\partial^2 \bar{\psi}^*}{\partial x^* \partial \bar{y}^*} = \frac{1}{3} x^{-\frac{2}{3}} f'(\eta) - \frac{1}{3} x^{-\frac{2}{3}} f''(\eta) \eta$$

$$\frac{\partial \bar{\psi}^*}{\partial x^*} = \frac{2}{3} x^{-\frac{1}{3}} f'(\eta) - \frac{1}{3} x^{-\frac{1}{3}} f''(\eta) \eta, \quad \frac{\partial^3 \bar{\psi}^*}{\partial \bar{y}^{*3}} = f'''(\eta) x^{-\frac{1}{3}}$$

$$\frac{\partial \bar{\theta}^*}{\partial x^*} = -\frac{1}{3} x^{\frac{2}{3}} G'(\eta) y x^{-\frac{4}{3}} + \frac{2}{3} x^{-\frac{1}{3}} G(\eta), \quad \frac{\partial \bar{\theta}^*}{\partial \bar{y}^*} = x^{\frac{1}{3}} G'(\eta), \quad \frac{\partial^2 \bar{\theta}^*}{\partial \bar{y}^{*2}} = G''(\eta)$$

Inserting these expression into equations (6)-(9) in to the following non-linear ordinary differential equations

$$(17) \quad n(f''(\eta))^{n-1} f'''(\eta) - \frac{1}{3} (f'(\eta))^2 + \frac{2}{3} f(\eta) f''(\eta) - S_0 f'(\eta) = 0$$

$$(18) \quad \frac{1}{Pr} g''(\eta) - \frac{2}{3} (f'(\eta) g(\eta) - f(\eta) g'(\eta)) + \frac{\mu}{L^2} (f''(\eta))^2 = 0$$

With the boundary condition

$$\eta = 0, \quad f = C_3, \quad f' = \epsilon, \quad g = C_1$$

$$(19) \quad \eta = \infty, \quad g = 0, \quad f' = 0$$

Where ϵ is shrinking/stretching parameter and C_3 is the wall mass transfer at the sheet.

5. RESULTS AND DISCUSSION

The system of non-linear ordinary differential equations (17)-(18) with boundary conditions (19) has been solved numerically using MATLAB ode solver. The effects of various parameters, the power-law fluid index n , magnetic field number S_0 and the Prandtl number Pr , controlling the parameter $b_1 = 1, C_1 = 1$ and wall mass transfer $C_3 = 4$ on the velocity $f'(\eta)$ and the temperature field $g(\eta)$ are shown in Figures (1)-(8).

Figure 1 show that Comparison of the velocity $f'(\eta)$ between Newtonian and non-Newtonian fluids with $S_0 = 0$ and $P_r = 0.71$.

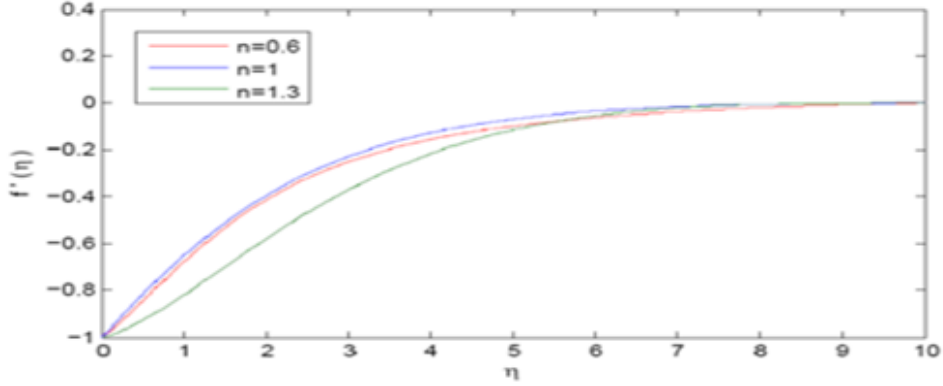


FIGURE 1. Comparison of the velocity $f'(\eta)$ between Newtonian and non-Newtonian

Figure 2 show that a comparison between hydrodynamic $S_0 = 0$ and $S_0 \neq 0$ on non-Newtonian pseudo-plastic fluid for $n = 0.6$, $P_r = 0.71$, $\epsilon = -1$ and $C_3 = 4$ in case linear shrinking sheet. Here we observe that if increase in magnetic field numbers S_0 then the velocity $f'(\eta)$ decrease and for higher value of $S_0 = 0, 0.1, 0.2, 0.3$ the boundary layer thickness decreases.

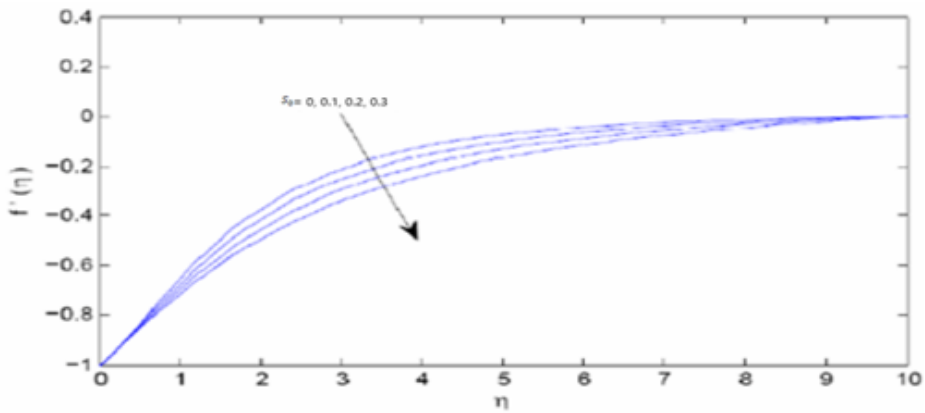


FIGURE 2. Influence of magnetic field on velocity Pseudo-plastic fluid over shrinking sheet

Also in figure 3 we observe that velocity and boundary layer thickness decreases by increasing magnetic field S_0 for stretching sheet $\epsilon = 1$ with Pseudo-plastic fluid $n = 0.6$, $P_r = 0.71$, and $C_3 = 4$.

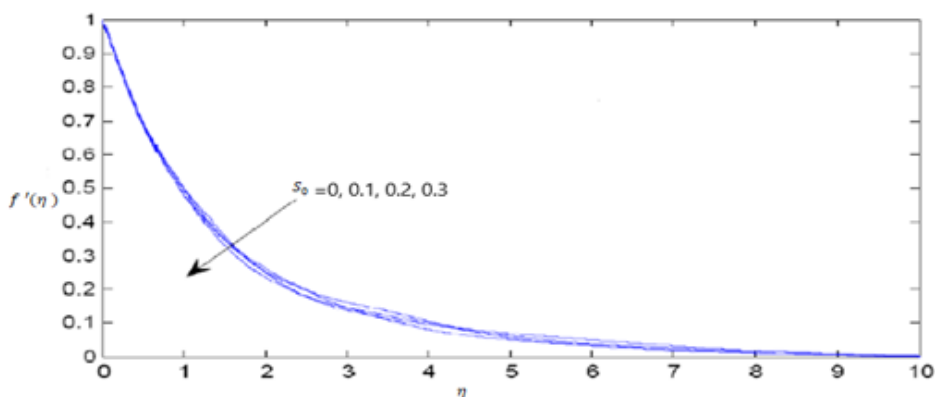


FIGURE 3. Influence of magnetic field on velocity of Pseudo-plastic fluid over stretching sheet

Figure 4 show that the effect of Prandtl number on velocity of pseudo-plastic fluid in presence of magnetic field in case of shrinking sheet. Here $n = 0.6$, $S_0 = 0.1$, $\epsilon = -1$, $C_4 = 4$ observe that as Prandtl number increase the velocity of fluid and the momentum boundary layer thickness decrease.

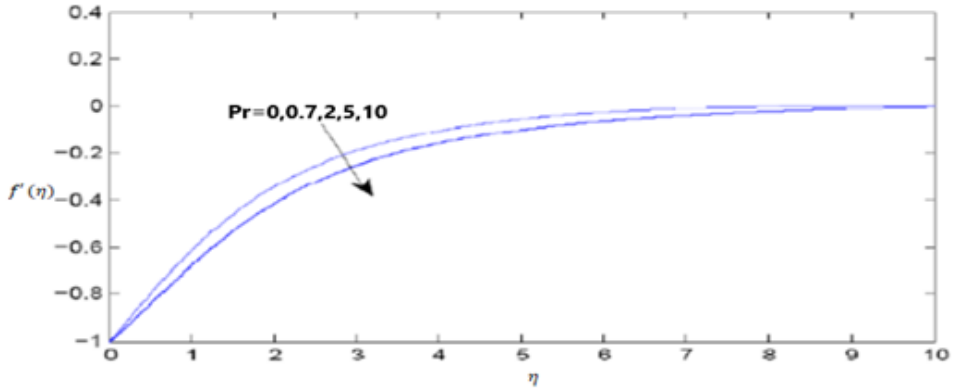


FIGURE 4. Effect of Prandtl number on velocity of MHD pseudo-plastic fluid for shrinking sheet.

Figure 5 present the change in temperature $g(\eta)$ for both Newtonian and non-Newtonian fluids with $\epsilon = -1$, $S_0 = 0.1$ and fixed $P_r = 0.71$.

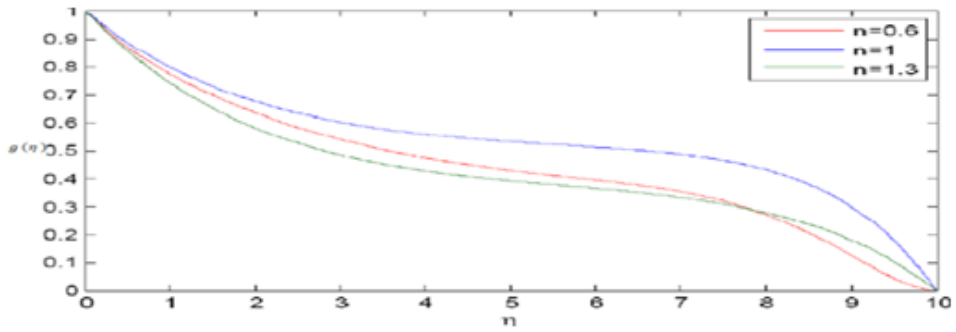


FIGURE 5. Comparison of temperature $g(\eta)$ for MHD Newtonian and Non-Newtonian fluids.

In figure 6 we observe that both temperature and thermal boundary layer thickness decreases as increasing the values of magnetic number $S_0 = 0, 0.1, 0.2, 0.3$ with $\epsilon = -1$, $C_3 = 4$ and $P_r = 0.71$.

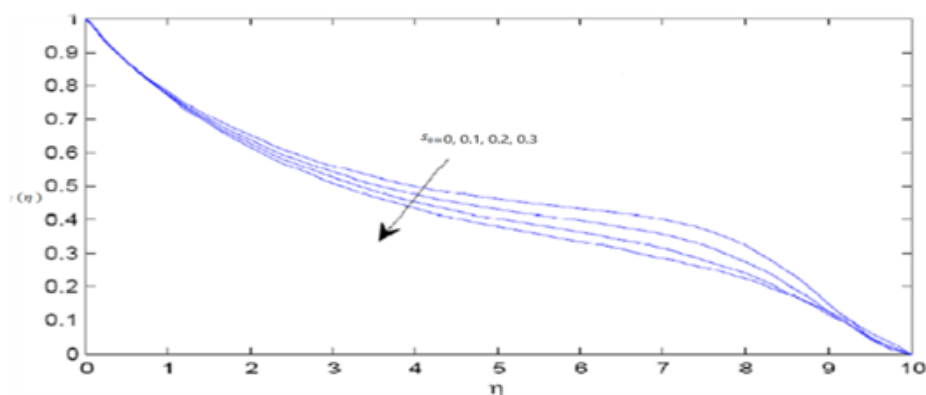


FIGURE 6. Influence of magnetic field on temperature of pseudo-plastic over shrinking sheet

In figure 7 show that if increase in magnetic strength $S_0 = 0, 0.1, 0.2, 0.3$ then decrease in the temperature $g(\eta)$ of non-Newtonian fluid is smaller for $\epsilon = 1$.

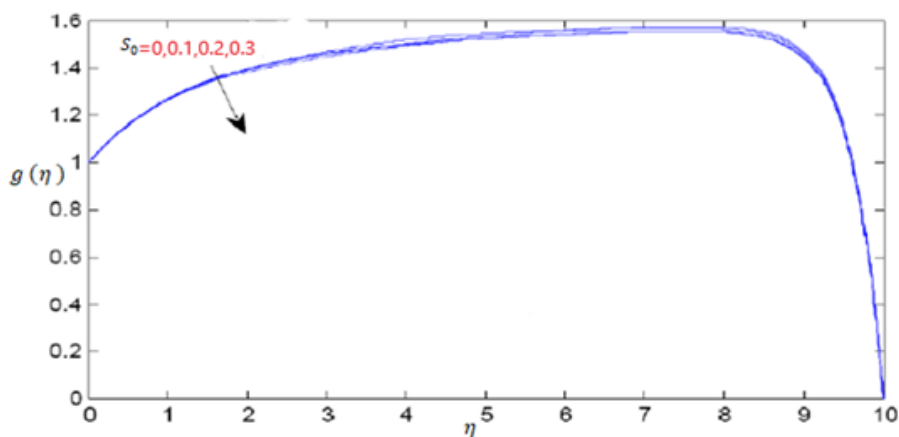


FIGURE 7. Effect of magnetic field on temperature of pseudo-plastic fluid over stretching sheet

In figure 8 we observe that both the temperature and thermal boundary layer thickness decreases by increasing the Prandtl number $Pr = 0, 0.7, 2, 5, 7$ with fixed value $m = 0.1, \epsilon = -1, b = 1, C_3 = 4$.

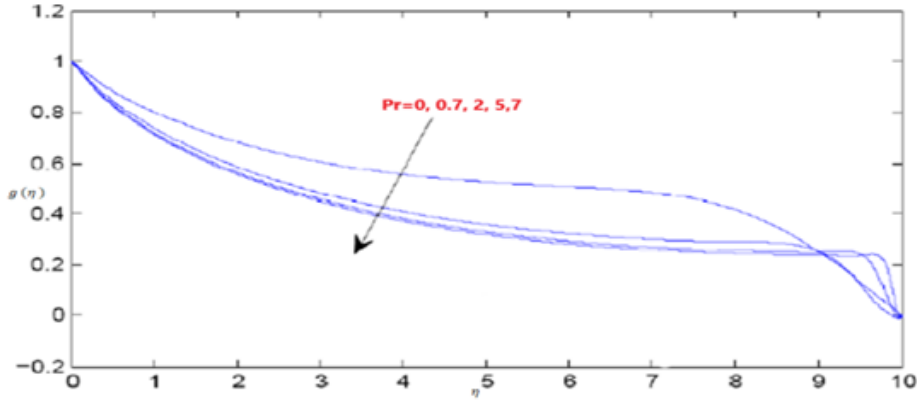


FIGURE 8. Effect of Prandtl number on temperature of pseudo-plastic fluid over shrinking sheet

6. CONCLUSION

Class of similarity solution for MHD boundary layer flow of viscous power law fluid with heat transfer over a non linear porous surface is studied by using group theoretic transformation technique. The governing partial differential equations are converted into ordinary differential equations. Numerical solutions are presented in a graphical form for power law fluid. If increasing the values of magnetic parameter S_0 then pseudo-plastic fluid the momentum boundary layer thickness decreases for both cases of shrinking and stretching sheets. If increase in Prandtl number Pr for power-law fluids over non-linear surface then both the temperature and thermal boundary layer thickness are decreased.

REFERENCES

- [1] Abel M. S. and Mahesha N., Effects of thermal buoyancy and variable thermal conductivity in a power law fluid past a vertical stretching sheet in the presence of non uniform heat source, Int. J. Nonlinear Mech., Vol. 44, pp. 1-12.(2009).

- [2] Cheng, W.T., Lin C.H., Melting effect on mixed convective heat transfer with aiding and opposing external flows from the vertical plate in a liquid saturated porous medium. *Int. J. Heat Mass Transfer* 50: 3026-3034, (2007).
- [3] Darji R.M., Timol M.G., Invariance analysis and similarity solution of heat transfer for MHD viscous power-law fluid over a non-linear porous surface, *Int. J. of Adv. in Appl. Math and Mech.* 1 (2): 116-132, (2013).
- [4] Dass U. N., Ray S. N., and Soundalgekar V. M., Mass transfer effects on flow past an impulsively started infinite vertical plate with constant mass flux an exact solution, *Heat and Mass Transfer*, vol. 31, no. 3, pp. 163167, (1996).
- [5] Hassanien I. A., Abdullah A. A. and Gorla R.S.R., Flow and heat transfer in a power law fluid over a non-isothermal stretching sheet, *Math. Comput. Model.* , Vol. 28, pp. 105-116,(1998).
- [6] Ingham D.B., Pop I., *Transport Phenomena in Porous Media III*. Elsevier, Oxford, (2005).
- [7] Lai F.C, Kulacki F.A., Non-Darcy mixed convection along a vertical wall in a saturated porous medium. *International J. Heat Mass Transfer*; 113: 252-255, (1991).
- [8] Muthucumaraswamy R., Ganesan P., and Soundalgekar V. M., Heat and mass transfer effects on flow past an impulsively started vertical plate, *Acta Mechanica*, vol. 146, no. 1-2, pp. 18, (2001).
- [9] Nield, D.A. and Bejan A., *Convection in Porous Media* (3rd Edition). Springer, New York, (2006).
- [10] Sarma U., Hazarika G. C., Effects of variable viscosity and thermal conductivity on heat and mass transfer flow along a vertical plate in the presence of a magnetic field, *Lat. Am. J. Phys. Educ.*, Vol. 5 (1), pp. 100-106,(2011).
- [11] Vafai K., *Handbook of Porous Media* (2nd Edition). Taylor and Francis, New York, (2005)
- [12] Vadasz P., *Emerging Topics in Heat and Mass Transfer in Porous Media*. Springer, New York, (2008).