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CANONICAL COSINE TRANSFORM AND THEIR OPTIMISTIC RESULTS

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Abstract. This paper is concerned with the different properties of Canonical Cosine transform. Such as linearity, Time Reverse, Shifting Property, differentiation, parity, Modulation theorem is also proved. Some important result regarding with the Kernel of canonical cosine transform is also proved.

AMS Classification: 42A38.

Keywords: Canonical transform; Generalized function; Fourier trans-form; Canonical cosine transform; Modulation theorem.

1. Introduction

Among all the integral transform Fourier transform is one of the well known integral transform. Since its introduction by Fourier in early 1800's. The entitites in the characteristic matrix are complex had been discussed by Torre [13]. He had defined as special case of complex linear canonical transform are Laplace transform and Bargmann transform.

In 1970 Moshinsky and Quesne [4] introduced a linear canonical transform which is more general integral transform. In 1980 by using eigen value

and eigen functions Namias [5] introduced the fractional Fourier transform
.

Almeida [1, 2] had introduced a fractional Fourier transform and proved many of its properties. Zayed [14] had discussed about convolution and product concering that transform. Bhosale and Choudhary [3] had studies it as a number of application of fractional Fourier transform in image processing, signal processing filtering and optics, tempered distribution etc are studied. Ozaktas and Medlovic [6] had explained the Optical implimentation of that transform. Pie and Ding [8, 9] defined linear canonical transform. canonical cosine transform had defined by S.B.Chavhan [9, 10, 11, 12] as follows.

$$\begin{aligned} CSTg(t)(s) &= \frac{1}{\sqrt{2\pi iq}} e^{\frac{i}{2}\left(\frac{m}{q}\right)s^2} \int_{-\infty}^{\infty} \cos\left(\frac{s}{q}t\right) e^{\frac{i}{2}\left(\frac{p}{q}t^2\right)} g(t)dt \quad \text{for } q \neq 0 \\ &= \sqrt{m} e^{\frac{i}{2}rms^2} g(m.s) \quad \text{for } q = 0 \end{aligned}$$

Terminology and notations of this paper is same as per Zemanian [15, 16]. The sturcture of this paper is organized as follows section two introduces the definition of canonical cosine transform on the space of generalized function and states the property of the kernel of the canonical cosine transform. Section three represents modulation theorem and section four describes some properties of canonical cosine transform are proved. Some opration transform in terms of parameter are discussed in section five. Lastly the conclusion is described.

2. Generalized Canonical Cosine Transform:

Definition 2.1. *: The canonical sine transform of generalized function is defined as*

$$\{CCTg(t)\}(s) = < g(t), K_{(p,q,r,m)}(t,s) >$$

Where kernel

$$K_{(p,q,r,m)}(t,s) = \frac{1}{\sqrt{2\pi iq}} e^{\frac{i}{2}\left(\frac{m}{q}\right)s^2} e^{\frac{i}{2}\left(\frac{p}{q}\right)t^2} \cos\left(\frac{s}{q}t\right) \tag{1}$$

Clearly Kernel $k_{\{p,q,r,m\}}(t,s) \in E$ and $k_{\{p,q,r,m\}}(t,s) \in E(R^n)$ the kernel (1) satisfies the following properties.

Properties of kernel:

$$k_{\{p,q,r,m\}}(t,s) \neq k_{p,q,r,m}(s,t) \quad \text{if } p \neq d$$

$$k_{\{p,q,r,m\}}(t,s) = k_{p,q,r,m}(s,t) \quad \text{if } p = m$$

$$k_{\{p,q,r,m\}}(t,s) = k_{p,-q,-r,m}(t,s)$$

$$k_{\{p,q,r,m\}}(-t,s) = k_{p,q,r,m}(t,-s)$$

Proposition 2.2. *$K_r(t,s) \in E$,where*
$K_{(r)}(t,s) = \frac{1}{\sqrt{2\pi iq}} e^{\frac{i}{2}\left(\frac{m}{q}\right)s^2} e^{\frac{i}{2}\left(\frac{p}{q}\right)t^2} cos\left(\frac{s}{q}\right)t$
for showing $K_r(t,s) \in E$ *we prove* $\gamma_{E,K} K_r(t,s) < \infty$,
i.e.to show $sup|D_t^n K_r(t,s)| < \infty$
Proof:- We know that,
$K_{(r)}(t,s) = \frac{1}{\sqrt{2\pi iq}} e^{\frac{i}{2}\left(\frac{m}{q}\right)s^2} e^{\frac{i}{2}\left(\frac{p}{q}\right)t^2} cos\left(\frac{s}{q}t\right)$
$D_t^n\left(K_r(t,s)\right) = D_t^n\left[C_1 e^{\frac{i}{2}\left(\frac{p}{q}\right)t^2} cos\left(\frac{s}{q}\right)t\right]$
Let,

$$g = e^{\frac{i}{2}\left(\frac{p}{q}\right)t^2} cos\left(\frac{s}{q}\right)t$$

$$g' = e^{\frac{i}{2}\left(\frac{p}{q}\right)t^2}\left[-sin\left(\frac{s}{q}\right)t\left(\frac{s}{q}\right)\right] + \left[cos\left(\frac{s}{q}\right)te^{\frac{i}{2}\left(\frac{p}{q}\right)t^2} i\left(\frac{p}{q}\right)t\right]$$

$$g' = e^{\frac{i}{2}\left(\frac{p}{q}\right)t^2}\left[cos\left(\frac{s}{q}\right)ti\left(\frac{p}{q}\right)t - sin\left(\frac{s}{q}\right)t\left(\frac{s}{q}\right)\right]$$

$$\begin{aligned} g'' = e^{\frac{i}{2}\left(\frac{p}{q}\right)t^2}&\left[cos\left(\frac{s}{q}\right)ti\left(\frac{p}{q}\right) - sin\left(\frac{s}{q}\right)t\left(\frac{p}{q}\right)t.i\left(\frac{s}{q}\right)t - cos\left(\frac{s}{q}\right)t\left(\frac{s^2}{q^2}\right)\right] \\ &+ \left[cos\left(\frac{s}{q}\right)ti\left(\frac{p}{q}\right)t - sin\left(\frac{s}{q}\right)t\left(\frac{s}{q}\right)\right].e^{\frac{i}{2}\left(\frac{p}{q}\right)t^2}.i\left(\frac{p}{q}\right)t \end{aligned}$$

$$\begin{aligned} g'' &= e^{\frac{i}{2}\left(\frac{p}{q}\right)t^2}\left\{cos\left(\frac{s}{q}\right)ti\left(\frac{p}{q}\right) - 2\left[i\left(\frac{p}{q}\right)tsin\left(\frac{s}{q}\right)t\left(\frac{s}{q}\right)\right]\right. \\ &\left. -cos\left(\frac{s}{q}\right)t\left(\frac{s^2}{q^2}\right) - cos\left(\frac{s}{q}\right)t\left(\frac{p^2}{q^2}\right)t^2\right\} \end{aligned}$$

And so on,

$$G^n = e^{\frac{i}{2}(\frac{p}{q})t^2}\left[sin\left(\frac{s}{q}\right)tC_1(s) + cos\left(\frac{s}{q}\right)tC_2(s)\right]$$

Where, $C_1(s)$ *and* $C_2(s)$ *are funcions of s,*

$$\therefore |g^n| \leq \left|e^{\frac{i}{2}(\frac{p}{q})t^2}\right|\left[\left|sin\left(\frac{s}{q}\right)tC_1(s)\right| + \left|cos\left(\frac{s}{q}\right)tC_2(s)\right|\right]$$

$\therefore |f^n(t)| < \infty$ *and* $sup|D_t^n K_r(t,s)| < \infty$*, Hence* $K_r(t,s) \in E(R^n)$

3. Modulation Theorem for (CCT) Canonical Cosine Transform:-

Theorem 3.1. *If CCTg(t)(s) is canonical cosine transform of g(t),* $g(t) \in E^1(R^n)$ *then*

$$\{CCTcos(ut)g(t)\}(s) = \frac{e^{\frac{-i}{2}du^2q}}{2}\big[(CCTf(t))(s+qu)e^{-idus} + \{CCTg(t)\}(s-qu)e^{idus}\big]$$

Proof. By definition of canonical cosine transform

$$\{CCTg(t)\}(s) = \frac{1}{\sqrt{2\pi iq}}e^{\frac{i}{2}(\frac{m}{q})s^2}\int_{-\infty}^{\infty}\cos\left(\frac{s}{q}t\right)e^{\frac{i}{2}(\frac{p}{q})t^2}g(t)dt$$

$$\{CTC\cos(ut)g(t)\}(s) = \frac{1}{\sqrt{2\pi iq}}e^{\frac{i}{2}(\frac{m}{q})s^2}\int_{-\infty}^{\infty}\cos\left(\frac{s}{q}t\right)e^{\frac{i}{2}(\frac{p}{q})t^2}\cos(ut)f(t)dt$$

$$= \frac{1}{\sqrt{2\pi iq}}e^{\frac{i}{2}(\frac{m}{q})s^2}\int_{-\infty}^{\infty}\left[\frac{2\cos\left(\frac{s}{q}t\right)\cos ut}{2}\right]e^{\frac{i}{2}(\frac{p}{q})t^2}g(t)dt$$

$$= \frac{1}{2\sqrt{2\pi iq}}e^{\frac{i}{2}(\frac{m}{q})s^2}\int_{-\infty}^{\infty}\left[\cos\left(\frac{s}{q}t+ut\right) + \cos\left(\frac{s}{q}t-ut\right)\right]e^{\frac{i}{2}(\frac{p}{q})t^2}g(t)dt$$

$$= \frac{1}{2}\left[\frac{1}{\sqrt{2\pi iq}}e^{\frac{i}{2}(\frac{m}{q})s^2}\int_{-\infty}^{\infty}\cos\left(\frac{(s+uq)}{q}\right)te^{\frac{i}{2}(\frac{p}{q})t^2}f(t)dt\right]$$
$$+\frac{1}{2}\left[\frac{1}{\sqrt{2\pi iq}}e^{\frac{i}{2}(\frac{m}{q})s^2}\int_{-\infty}^{\infty}\cos\left(\frac{(s-uq)}{q}\right)te^{\frac{i}{2}(\frac{p}{q})t^2}g(t)dt\right]$$

$$= \frac{e^{\frac{-i}{2}(du^2q)}}{2}\Bigg\{\left[\frac{1}{\sqrt{2\pi iq}}e^{\frac{i}{2}(\frac{m}{q})(s+uq)^2}e^{-i(mus)}\int_{-\infty}^{\infty}\cos\left(\frac{(s+uq)}{q}\right)te^{\frac{i}{2}(\frac{p}{q})t^2}g(t)dt\right]$$
$$+ \left[\frac{1}{\sqrt{2\pi iq}}e^{\frac{i}{2}(\frac{m}{q})(s-uq)^2}e^{-i(mus)}\int_{-\infty}^{\infty}\cos\left(\frac{(s-uq)}{q}\right)te^{\frac{i}{2}(\frac{p}{q})t^2}g(t)dt\right]\Bigg\}$$

$$\{CTC\cos(ut)g(t)\}(s) = \frac{e^{\frac{-i}{2}(mu^2q)}}{2}\left[\{CCTg(t)\}(s+qu)e^{-imus} \right.$$
$$\left. +\{CCTg(t)\}(s-qu)e^{imus}\right]$$

□

Theorem 3.2. *If $\{CCTg(t)\}(s)$ is canonical cosine transform of $g(t)$, $g(t) \in (R^n)$ Then*

$$\{CCT\sin(ut)g(t)\}(s) = \frac{e^{\frac{-i}{2}(mu^2q)}}{2}\left[\{CCTg(t)\}(s+qu)e^{imus} \right.$$
$$\left. -\{CSTg(t)\}(s-qu)e^{-imus}\right]$$

Proof. Using the of defintion canonical cosine transform

$$\{CCTg(t)\}(s) = \frac{1}{\sqrt{2\pi iq}}e^{\frac{i}{2}\left(\frac{m}{q}\right)s^2}\int_{-\infty}^{\infty}\cos\left(\frac{s}{q}t\right)e^{\frac{i}{2}\left(\frac{p}{q}\right)t^2}g(t)dt$$

$$\{CCT\sin(ut)g(t)\}(s) = \frac{1}{\sqrt{2\pi iq}}e^{\frac{i}{2}\left(\frac{m}{q}\right)s^2}\int_{-\infty}^{\infty}\cos\left(\frac{s}{q}t\right)e^{\frac{i}{2}\left(\frac{p}{q}\right)t^2}\sin(ut)g(t)dt$$

$$= \frac{1}{\sqrt{2\pi iq}}e^{\frac{i}{2}\left(\frac{m}{q}\right)s^2}\int_{-\infty}^{\infty}\left[\frac{2\cos\left(\frac{s}{q}t\right)\sin ut}{2}\right]e^{\frac{i}{2}\left(\frac{p}{q}\right)t^2}g(t)dt$$

$$= \frac{1}{\sqrt{2\pi ibq}}e^{\frac{i}{2}\left(\frac{m}{q}\right)s^2}\int_{-\infty}^{\infty}\left[\frac{\sin\left(\frac{s}{q}t+ut\right)-\sin\left(\frac{s}{q}t-ut\right)}{2}\right]e^{\frac{i}{2}\left(\frac{p}{q}\right)t^2}g(t)dt$$

$$= \frac{1}{2\sqrt{2\pi iq}}e^{\frac{i}{2}\left(\frac{m}{q}\right)s^2}\int_{-\infty}^{\infty}\left[\sin\left(\frac{s+uq}{q}\right)t-\sin\left(\frac{s-uq}{q}\right)t\right]e^{\frac{i}{2}\left(\frac{p}{q}\right)t^2}g(t)dt$$

$$= \frac{e^{\frac{-i}{2}(mu^2b)}}{2}\left\{\left[\frac{1}{\sqrt{2\pi iq}}e^{\frac{i}{2}\left(\frac{m}{q}\right)(s-uq)^2}e^{i(mus)}\int_{-\infty}^{\infty}\sin\left(\frac{(s+uq)}{q}\right)te^{\frac{i}{2}\left(\frac{p}{q}\right)t^2}g(t)dt\right]\right.$$
$$\left. - \left[\frac{1}{\sqrt{2\pi iq}}e^{\frac{i}{2}\left(\frac{m}{q}\right)(s+uq)^2}e^{-i(mus)}\int_{-\infty}^{\infty}\sin\left(\frac{(s-uq)}{q}\right)te^{\frac{i}{2}\left(\frac{p}{q}\right)t^2}g(t)dt\right]\right\}$$

$$\{CST\sin(ut)g(t)\}(s) = \frac{e^{\frac{-i}{2}(mu^2q)}}{2}\left[\{CCTf(t)\}(s+qu)e^{imus} \right.$$
$$\left. -\{CSTg(t)\}(s-qu)e^{-imus}\right]$$

□

4. Various Properties for Canonical Cosine Transform(CCT):-

4.1. **Linearity Property:** If A_1, A_2 are constant and g_1, g_2 are functions of then

$$\{CCT\left[A_1 g_1(t) + A_2 g_2(t)\right]\}(s) = A_1\{CCT g_1(t)\}(s) + A_2\{CCT g_2(t)\}(s)$$

4.2. **Time Reverse:** If $\{CCTg(t)\}$ is canonical cosine transform of$g(t)$, $g(t) \in E'(R^n)$ then

$$\{CCTg(-t)\}(s) = \{CCTg(t)\}(s)$$

4.3. **Shifting Property:** If $\{CCT(g(t+\tau))\}(s)$ indicates the generalized canonical cosine transform of $g(t)$ and τ is any real number then,

$$\{CCT\left(g(t+\tau)\right)\}(s) = e^{\frac{i}{2}\left(\frac{p}{q}\right)t^2}\{\sin\left(\frac{p}{q}\right)\tau\left[CCTg(t)e^{-it\tau\left(\frac{p}{q}\right)}\right](s)$$
$$+\cos\left(\frac{s}{q}\right)\tau\left[CSTg(t)e^{-it\tau\left(\frac{p}{q}\right)}\right](s)\}$$

4.4. **Parity:** If $\{CCTg(t)\}$ is canonical cosine transform of $g(t)$, $g(t) \in E'(R^n)$ then

$$\{CCTg(-t)\}(s) = \{CCTg(t)\}(-s)$$

4.5. **Scaling Property:** If $\{CCTg(t)\}(s)$ indicates the generalized canonical cosine transform of g(t),then

$$\{CCTg(kt)\}(s) = \frac{1}{k}e^{\left(1-\frac{1}{k}\right)\frac{i}{2}\frac{m}{qk}s^2}\left[CCTg(t)e^{\left(\frac{1}{k}-1\right)\frac{i}{2}\frac{p}{qk}t^2}\right](s)$$

4.6. **Differential Property:** If $\{CCTg(t)\}(s)$ indicates the generalized canonical cosine transform of $g(t)$,then

$$\{CCT[g'(t)]\}(s) = \{\left(\frac{s}{q}\right)[CSTg(t)](s) + \left(\frac{p}{q}\right)[CSTg(t)](s)\}$$

5. Operation Transform in Terms of Parameter:

In this section we represent the operational formulae for differential canonical cosine transform with respective s and canonical cosine transform with shifted parameter S. If CCTg(t)(s) indicates the canonical cosine transform of g(t)in terms of parameter S, then

$$\{CCTg(t)\}(s+\beta) = e^{\frac{i}{2}\left(\frac{m}{q}\right)(2s\beta+s^2)}\left[(CCT\cos\left(\frac{\beta}{q}t\right)g(t)\right](s) + e^{\frac{i}{2}\left(\frac{m}{q}\right)(2s\beta+s^2)}\left[(CST\sin\left(\frac{\beta}{q}t\right)g(t)\right](\beta)$$

Proof. We have,

$$\{CCTg(t)\}(s) = \frac{1}{\sqrt{2\pi iq}}e^{\frac{i}{2}\left(\frac{m}{q}\right)s^2}\int_{-\infty}^{\infty}\cos\left(\frac{s}{q}t\right)e^{\frac{i}{2}\left(\frac{p}{q}\right)t^2}g(t)dt$$

$$\{CCTg(t)\}(s+\beta)\} = \frac{1}{\sqrt{2\pi iq}}e^{\frac{i}{2}\left(\frac{m}{q}\right)s+\beta^2}\int_{-\infty}^{\infty}\cos\frac{(s+\beta)}{q}te^{\frac{i}{2}\left(\frac{p}{q}\right)t^2}g(t)dt$$

$$= \frac{1}{\sqrt{2\pi iq}}e^{\frac{i}{2}\left(\frac{m}{q}\right)(s+\beta)^2}\int_{-\infty}^{\infty}\left[\cos\left(\frac{s}{q}t\right).\cos\left(\frac{\beta}{q}t\right)\sin\left(\frac{s}{q}t\right)\sin\left(\frac{\beta}{q}t\right)\right]e^{\frac{i}{2}\left(\frac{p}{q}\right)t^2}g(t)dt$$

$$\{CCTg(t)\}(s+\beta)\} = \frac{1}{2\sqrt{2\pi iq}}e^{\frac{i}{2}\left(\frac{m}{q}\right)(s+\beta)^2}\left[\int_{-\infty}^{\infty}\cos\left(\frac{s}{q}t\right)\cos\left(\frac{\beta}{q}t\right)e^{\frac{i}{2}\left(\frac{p}{q}\right)t^2}g(t)dt - \int_{-\infty}^{\infty}\sin\left(\frac{s}{q}t\right)\sin\left(\frac{\beta}{q}t\right)e^{\frac{i}{2}\left(\frac{p}{q}\right)t^2}g(t)dt\right]$$

$$= \frac{1}{2\sqrt{2\pi iq}}e^{\frac{i}{2}\left(\frac{m}{q}\right)(s)^2}e^{\frac{i}{2}\left(\frac{m}{q}\right)(2s\beta+\beta^2)}\left[\int_{-\infty}^{\infty}\cos\left(\frac{s}{q}t\right)\cos\left(\frac{\beta}{q}t\right)e^{\frac{i}{2}\left(\frac{p}{q}\right)t^2}g(t)dt - \int_{-\infty}^{\infty}\sin\left(\frac{s}{q}t\right)\sin\left(\frac{\beta}{q}t\right)e^{\frac{i}{2}\left(\frac{p}{q}\right)t^2}g(t)dt\right]$$

$$= \frac{1}{2\sqrt{2\pi iq}}e^{\frac{i}{2}\left(\frac{m}{q}\right)(s)^2}e^{\frac{i}{2}\left(\frac{m}{q}\right)(2s\beta)}e^{\frac{i}{2}\left(\frac{m}{q}\right)(\beta^2)}\left[\int_{-\infty}^{\infty}\cos\left(\frac{s}{q}t\right)\cos\left(\frac{\beta}{q}t\right)e^{\frac{i}{2}\left(\frac{p}{q}\right)t^2}g(t)dt - \int_{-\infty}^{\infty}\sin\left(\frac{s}{q}t\right)\sin\left(\frac{\beta}{q}t\right)e^{\frac{i}{2}\left(\frac{p}{q}\right)t^2}g(t)dt\right]$$

$$= \frac{1}{2\sqrt{2\pi iq}} e^{\frac{i}{2}\left(\frac{m}{q}\right)(s)^2} e^{i\left(\frac{m}{q}\right)(2s\beta+\beta^2)} \int_{-\infty}^{\infty} \cos\left(\frac{s}{q}t\right) \cos\left(\frac{\beta}{q}t\right) e^{\frac{i}{2}\left(\frac{p}{q}\right)t^2} g(t)dt$$
$$- \frac{1}{2\sqrt{2\pi iq}} e^{\frac{i}{2}\left(\frac{m}{q}\right)(\beta)^2} e^{i\left(\frac{m}{q}\right)(2s\beta+s^2)} \int_{-\infty}^{\infty} \sin\left(\frac{s}{q}t\right) \sin\left(\frac{\beta}{q}t\right) e^{\frac{i}{2}\left(\frac{p}{q}\right)t^2} g(t)dt$$

$$\{CCTg(t)\}(s+\beta) = e^{\frac{i}{2}\left(\frac{m}{q}\right)(2s\beta+s^2)} \left[(CCT \cos\left(\frac{\beta}{q}t\right) g(t) \right] (s)$$
$$+ e^{\frac{i}{2}\left(\frac{m}{q}\right)(2s\beta+s^2)} \left[(CST \sin\left(\frac{\beta}{q}t\right) g(t) \right] (\beta)$$

□

6. Conclusion

This work described a canonical cosine transform which is generalized transform from number of other transform. We described several of its properties like time reverse property, Modulation theorem and linearity property. The applications of canonical cosine transform may be created and used to solve partial differential equation with variable coefficients and ordinary differential equation. This transform is an important tool in signal processing and many other branches of engineering.

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