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ANISOTROPIC MASSIVE SCALAR FIELD MODEL WITH DOMAIN WALLS IN $f(R,T)$ GRAVITY

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Abstract. The Bianchi type-V universe with massive scalar fields and domain walls examined within $f(R,T)$ theory of gravity described by Harko et al. (Phys. Rev. D. 84, 024020, 2011). Solving the field equations of the theory using a power law relation between the scalar field and average scale factor of the model we have presented an anisotropic massive scalar field model in this modified theory of gravity. In light of the recent scenario of the universe expanding at an accelerated rate and cosmological observations, we computed the cosmological parameters like energy density, pressure, deceleration parameter (q), statefinder parameters (r,s), and r-q plane of our model and discussed their physical significance.

AMS Classification: 83F05, 83D05.

Keywords: Bianchi type $-$ V universe; Anisotropic model; Massive scalar field; *f(R,T)* gravity; Domain walls.

1. Introduction

Through their cosmological studies, Riess et al. [1] and Perlmutter et al. [2] postulated that the universe is expanding at an accelerated rate. Dark energy, a mysterious form of unknown force with positive energy density and negative pressure, is thought to be the cause of this. Additionally, it has been suggested that dark energy, which is thought to make up more than 70% of our universe, exerts a powerful negative pressure and accelerates cosmic expansion. Additionally, gravitational matter makes up 30% of the universe's mass, although the majority of this so-called dark matter is non-baryonic. Dark matter and dark energy make up the cosmic fluid that makes up our universe, and they have been evolving on their own. Cosmic acceleration, however, continues to be a cosmological riddle.

In order to explain late time acceleration and the existence of dark energy components like quintessence, phantom, tachyon, and Chaplygin gas in the universe, general relativity is being modified more and more (Padmanabhan [3]; Caldwell [5]; Bagla et al. [6]). Scalar-tensor theories of gravity and f(R,T) gravity (Harko et al. [7]) are two of the general relativity modifications that are thought to be more widely used to explain late-time acceleration and dark energy. In the literature [**?**], a nice overview of modified theories of gravitation is provided. Here, we concentrate on the gravitational f(R,T) theory. The trace of the stress-energy tensor and the Ricci scalar can be used to calculate the gravitational Lagrangian, according to this modified theory of gravity. The reconstruction of gravity-based cosmological models has become a more important area of research. In the gravity formulation of Harko et al. [7], the gravitational Lagrangian is given as an arbitrary function of the Ricci scalar and the trace of energy-momentum tensor. The Hilbert-Einstein action principle is used to derive the field equations. Gravitation is caused by a change in the matter stress-energy tensor with respect to the metric. The Hilbert-Einstein type action principle was used to generate the field equations for this theory, which are provided by

$$S = \frac{1}{16\pi}\left[\int \left(L_m + f\left(R,T\right)\right)\sqrt{-g}\; d^4x\right] \tag{1}$$

where L_m is matter Lagrangian density. The stress-energy tensor of the matter is defined

$$T_{ij} = -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}L_m)}{\delta g_{ij}} \tag{2}$$

and its trace by $T = g^{ij}T_{ij}$ respectively. Numerous theoretical models for $f(R,T)$ gravity differ depending on the matter source corresponding to each option$f(R,T)$. Now assuming

$$f(R,T) = R + 2f(T) \tag{3}$$

after changing the gravitational field's action S, we arrive at the field equations for this theory as

$$R_{ij} - \frac{1}{2}g_{ij}R = 8\pi T_{ij} + 2f^{'}(T)\,T_{ij} + [2pf^{'}(T) + f(T)]g_{ij} \tag{4}$$

where $f^{'}(T)$ indicates differentiation with respect to T.

Scalar fields (SF) are essential in cosmology because they explain matter fields with spin-less quanta and symbolize gravitational fields. Two categories of scalar fields (SFs) are distinguished: massive SFs, which represent short-range interactions, and zero mass SFs, which characterize long-range interactions. This physical significance is what has drawn the interest of numerous researchers to examine SFs. Additionally, it is hypothesized that SFs contribute to the acceleration of the universe's expansion and help cosmologists resolve the horizon issue. There are multiple dynamical DE models in the literature that have been put up by different writers in both modified theories of gravitation and general relativity. We are particularly interested in Bianchi type spatially homogenous and anisotropic dark energy models since they are crucial for understanding the early genesis of our universe and describing the minute quantities of anisotropy there. Both in general relativity and in modified theories of gravity, a plethora of authors have looked into spatially homogeneous and anisotropic dark energy models [**?**] (we just name a few of them here). Additionally, in the context of particle physics, where cosmological models must take into account the presence of huge SFs and scalar-meson fields in both general relativity and modified theories of gravitation, it is helpful to create workable Bianchi type SF models.

Because the origin of structure in the universe is still one of the biggest cosmological mysteries, there has been a lot of interest in studying the large-scale structure of the cosmos. Phase shifts in the early cosmos produced a wide range of topological flaws. According to Kibble [27] and Mermin [28], the topological defects include cosmic string, domain barriers, textures, and monopoles. Domain barriers and cosmic threads are more significant among these three topological flaws than they are since they do not contradict observation. Vilenkin [29] highlighted the significance of cosmic strings and domain walls to galaxy formation and the cosmos' overall structure. Many academics have recently investigated domain wall cosmological models in $f(R,T)$ gravity. Biswal et al. [30] have researched Kaluza-Klein domain walls in $f(R,T)$ gravity. Shaikh and Wankhede [31] explored non-static plane-symmetric domain walls, and Bishi et al. [32] discussed domain walls and dark matter cosmological models in the modified theory of gravity. While Tiwari et al. [34] investigated the transit cosmological models with domain walls, Katore and Hatkar [33] analyzed the Bianchi type-III and Kantowski-Sachs domain wall cosmological models.

To accurately depict the large-scale structure of the modern universe, Friedmann-Robertson-Walker (FRW) models that are spatially homogeneous and isotropic are best suited. Recent empirical data have verified this. However, FRW models do not provide the accurate description of matter in the early cosmos. Therefore, it is thought that the models with the anisotropic background are appropriate to characterize the early cosmos. Thus, a key to understanding the potential anisotropy of dark energy and its impact on the evolution of the cosmos is the exploration of Bianchi models in modified theories of gravitation. The spatially homogeneous and anisotropic Bianchi models have been examined by a number of authors to get insight into the relativistic view of the early cosmos. In addition to Singh and Singh [35], Sharif and Jawad [36], and Singh and Singh [37], other researchers have looked into how cosmological models can be reconstructed when perfect fluid and SFs are present. Singh et al. [38] have studied the behavior of the cosmological constant and SF in the f(R,T) theory of gravity. Naidu [39] discussed the Bianchi type-modified holographic Ricci DE model in the considerable presence of SF. Bianchi type- DE models in the presence of scalar meson fields have been studied by Naidu et al. [40]. The Kantowski-Sachs DE model was investigated by Reddy et al. [41] in the presence of the scalar-meson field. Aditya et al. [42] looked into the Kaluza-Klein DE model in the Lyra manifold with enormous SFs. When there was a significant SF in gravity, Aditya and Reddy [43] described the Bianchi type-model. Daniel Raju et al. [44]–[46] have conducted numerous studies on anisotropic DE models with huge SF. An anisotropic cosmic model with a significant SF in f(R,T) gravity has been studied by Aditya [47]. Recently, several authors in the literature [48]–[51] have investigated different dark energy models with massive scalar fields in alternative theories of gravitation.

In the context of the f(R,T) theory of gravity, the aforementioned discussion and investigations have prompted us to take into consideration the enormous SF model with domain walls in Bianchi type-Vspace-time. The following is the paper's outline: In section 2, we developed field equations in the presence of large SF and domain walls using a Bianchi type-V metric. The cosmological parameters of the model and their characteristics are covered in Sect. 3. Some findings are presented in the final portion.

2. Metric and field equations

We consider the spatially homogeneous anisotropic Bianchi type-Vspace-time described by the following metric

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{-2\,x}dy^2 + C^2e^{2\,x}dz^2 \tag{5}$$

where$A, B$and C are functions of cosmic time t only. The volume (V) and average scale factor $(a(t))$ of Bianchi type-V space-time are specified as

$$V = \sqrt{-g} = ABC, \qquad a(t) = (ABC)^{\frac{1}{3}}. \tag{6}$$

The anisotropic parameter A_h is given by

$$A_h = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2, \tag{7}$$

where $H_1 = \frac{\dot{A}}{A}, H_2 = \frac{\dot{B}}{B}$, $H_3 = \frac{\dot{C}}{C}$ are directional Hubble's parameters and $H = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)$ is mean Hubble's parameter. Expansion scalar (θ) and shear scalar (σ^2) are defined as

$$\theta = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}, \tag{8}$$

$$\sigma^2 = \frac{1}{2}\left[\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2 - \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{\dot{C}}{C}\frac{\dot{A}}{A}\right]. \tag{9}$$

The deceleration parameter is given by

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1. \tag{10}$$

The energy-momentum tensor for this domain walls can be written as

$$T_{ij} = \rho\,(g_{ij} + w_i w_j) + p\, w_i w_j, \quad w^i w_j = -1, \tag{11}$$

where ρ is the energy density of the wall, p is the pressure in the direction normal to the plane of the wall and w_i is a unit space-like vector in the same direction (Rahaman and Bera [52]). Now in the co-moving coordinate system, we have from Eq. (11)

$$T_1^1 = T_2^2 = T_3^3 = \rho, \;\; T_0^0 = -p, \;\; T_j^i = 0, \; i \neq j \tag{12}$$

where the quantities ρ and p depend on t only. The energy-momentum tensor for the massive SF is

$$T_{ij}^{SF} = \phi_{,i}\phi_{,j} - \frac{1}{2}\left(\phi_{,k}\phi^{'k} - M^2\phi^2\right), \tag{13}$$

where ϕ is the massive SF and M is the mass of the SF. The massive SF satisfies the Klein-Gordon equation

$$g^{ij}\phi_{;ij} + M^2\phi = 0. \tag{14}$$

To derive the field equations of our model, we assume the particular choice of the function $f(R,T)$ (Harko et al. [7])

$$f(T) = \lambda T, \quad \lambda = constant. \tag{15}$$

Now, for the metric (5), using Eqs. (12)-(15), the explicit form of $f(R,T)$ gravity field equations (4), take the form:

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{1}{A^2} = 8\pi\left(\frac{1}{2}\left(\dot{\phi}^2 - M^2\phi^2\right) - \rho\right) - \lambda(5\rho + p + 2M^2\phi^2 - \dot{\phi}^2), \tag{16}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = 8\pi\left(\frac{1}{2}\left(\dot{\phi}^2 - M^2\phi^2\right) - \rho\right) - \lambda(5\rho + p + 2M^2\phi^2 - \dot{\phi}^2), \tag{17}$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{A}\dot{C}}{AC} - \frac{1}{A^2} = 8\pi\left(\frac{1}{2}\left(\dot{\phi}^2 - M^2\phi^2\right) - \rho\right) - \lambda(5\rho + p + 2M^2\phi^2 - \dot{\phi}^2) \tag{18}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{1}{A^2} = 8\pi\left(p - \frac{1}{2}\left(\dot{\phi}^2 + M^2\phi^2\right)\right) - \lambda(3\rho - p + 2M^2\phi^2 + \dot{\phi}^2), \tag{19}$$

$$2\frac{\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0 \tag{20}$$

and the Klein-Gordon equation (5) for the metric (1) takes the form

$$\ddot{\phi} + \dot{\phi}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + M^2\phi = 0, \tag{21}$$

where an overhead dot denotes differentiation with respect to cosmic time t. The geometrical and physical parameters to utilize in solving the $f(R,T)$ field equations for the Bianchi type-V space-time provided by Eq. (5) are given in Eqs. (6)-(10). From Eq. (20), we have $A^2 = BC$. Because of this relation, the field Eqs. (16)–(21) are a set of five independent equations with five unknowns A, B, ϕ, p and ρ. Hence, we assume the following physically plausible conditions to obtain a determinate solution:

- We may use the shear scalar σ^2 is proportional to scalar expansion θ (Collins et al. [53]) which leads to

$$B = C^k, \tag{22}$$

  where $k \neq 1$ is a constant that maintains the space-time's isotropic nature. Given that the Hubble expansion of the current universe is isotropic within 30% (Thorne [54]; Kantowski and Sachs [55]; Kristian and Sachs [56]), velocity-redshift measurements for extragalactic sources are possible. In addition, the red-shift study determined that the limit in our current galaxy is $\frac{\sigma}{H} \leq 0.3$.
- A power-law relation between the SF $\phi(t)$ and the average scale factor $a(t)$ of the model is given by (Johri and Sudharsan [57]; Johri and Desikan [58])

$$\phi \propto [a(t)]^n, \tag{23}$$

  where n is a power index. To reduce the mathematical complexity of the system, here, we consider the following relation between SF and metric potentials

$$\phi = \phi_0\ [a(t)]^n, \tag{24}$$

  which is, clearly, a consequence of Eq. (21). Several authors, in the literature, have considered the above relation to study anisotropic cosmological models with massive SFs (Singh and Rani [59]; Aditya et al. [60]; Naidu et al. [61, 62]; Naidu et al. [63]; Aditya et al. [64, 65]; Rao et al. [66, 67]).

Now from Eqs. (21), (22) and (24), we get the metric potentials

$$A = \left(\frac{c_1}{k_1} sinh\left(3k_1(t+c_2)\right) \right)^{\frac{1}{3}}, \quad B = \left(\frac{c_1}{k_1} sinh\left(3k_1(t+c_2)\right) \right)^{\frac{2k}{3(k+1)}}$$

$$C = \left(\frac{c_1}{k_1} sinh\left(3k_1(t+c_2)\right) \right)^{\frac{2}{3(k+1)}} \tag{25}$$

and SF $\varphi(t)$ as

$$\phi\left(t\right) = \phi_0 \left(\frac{c_1}{k_1} sinh\left(3k_1(t+c_2)\right) \right)^{\frac{n}{3}}, \tag{26}$$

where $k_1 = \left(-\frac{M^2 n}{3}\right)^{1/2}$, c_1 and c_2 are integrating constants. It is noteworthy that the solution obtained here is quite intriguing and physically feasible because Pradhan et al. [68] and Mishra et al. [69] proposed an average scale factor $a(t) = [sinh(t)]^{\frac{1}{n}}$ in combination with the research of dark energy models in the anisotropic backdrop. They found that it generates a few results that are plausible and match recent cosmic data. This kind of average scale factor has been employed by numerous researchers to examine various aspects of dark energy models in the literature (Amirhashchi et al. [70]; Mishra et al. [71]; Rao and Prasanthi [72]). This hyperbolic solution for scale factors (Eq. (25)) makes it interesting to explore the SF model.

Now the metric (5) with the help of Eq. (25) can be written as

$$\begin{aligned} ds^2 = -dt^2 &+ \left(\tfrac{c_1}{k_1} sinh\left(3k_1(t+c_2)\right) \right)^{\frac{2}{3}} dx^2 + \left(\tfrac{c_1}{k_1} sinh\left(3k_1(t+c_2)\right) \right)^{\frac{4k}{3(k+1)}} e^{-2\,x} dy^2 \\ &+ \left(\tfrac{c_1}{k_1} sinh\left(3k_1(t+c_2)\right) \right)^{\frac{4}{3(k+1)}} e^{2\,x} dz^2 \end{aligned} \tag{27}$$

Eq. (27) along with Eq. (26) shows Bianchi type-V universe with massive SFs and domain walls in $f(R,T)$ theory of gravity along with the following physical and cosmological parameters which are very crucial in the discussion of cosmology.

3. Physical Discussion

Spatial volume $(V(t))$ and average scale factor $(a(t))$ are given by

$$V(t) = \frac{c_1}{k_1} sinh\left(3k_1(t+c_2)\right), \quad a\left(t\right) = \left(\frac{c_1}{k_1} sinh\left(3k_1(t+c_2)\right) \right)^{\frac{1}{3}}. \tag{28}$$

The mean Hubble parameter ($H(t)$) and scalar expansion ($\theta(t)$) are obtained as

(29) $$H\left(t\right) = k_1 \, \cot h\left(3k_1(t+c_2)\right), \quad \theta = 3k_1 \, \cot h\left(3k_1(t+c_2)\right).$$

The shear scalar ($\sigma(t)^2$) and average anisotropic parameter (A_h) is

(30)
$$\sigma^2 = \frac{3(k-1)^2 k_1 \left(coth\left(3k_1(t+c_2)\right)\right)^2}{2(k+1)^2}, \quad A_h = \frac{2}{3}\left(\frac{1+{k_1}^2+2k^2-k_1k-k}{(k+1)^2}\right).$$

Now using Eqs. (23) and (24) in the field equations (16)-(18) we get the energy density ρ and the pressure p as

(31)
$$\begin{aligned}\rho = {} & \tfrac{-1}{(16\pi+\lambda)(8\pi+\lambda)+6\lambda^2}\Big[3\lambda\Big[\tfrac{36{k_1}^2}{(3k+3)\tanh^2(3k_1(t+c_2))}\left(\tfrac{2k^2}{3k+3}-k+\tfrac{2}{3k+3}\right)+\tfrac{36{k_1}^2(k+1)}{3k+3} \\ & +2\left(\tfrac{-2{c_1}^2\cosh^2(3k_1(t+c_2))}{\left(\frac{c_1\sinh(3k_1(t+c_2))}{k_1}\right)^{4/3}}+\tfrac{3c_1k_1\sinh(3k_1(t+c_2))}{\left(\frac{c_1\sinh(3k_1(t+c_2))}{k_1}\right)^{1/3}}\right)\Big] \\ & -(16\pi+7\lambda)\left(\tfrac{6{k_1}^2}{3k+3}\coth^2(3k_1(t+c_2))\left(k+1+\tfrac{6k}{3k+3}\right)-\tfrac{3m}{\left(\frac{c_1\sinh(3k_1(t+c_2))}{k_1}\right)^{2/3}}\right) \\ & -3\lambda\left(\tfrac{c_1\sinh(3k_1(t+c_2))}{k_1}\right)^{2n/3}{\phi_0}^2(16\pi+10\lambda)\left(n^2\coth^2(3k_1(t+c_2)){k_1}^2((4\pi+\lambda)\right. \\ & \left.+3(8\pi+2\lambda)\lambda)-((4\pi+2\lambda)-3(8\pi+4\lambda)\lambda)M^2\right)\Big]\end{aligned}$$

Scalar field: For different values of a parameter n, the behavior of an SF in terms of redshift is shown in Figure 1. Over the course of the universe's development, the SF has diminished. As can be seen, the kinetic energy increases as the SF lowers. Additionally, as the parameter n value grows, the SF lowers. The parameter clearly plays a big part in how SF has evolved. We plotted the other cosmological parameters for various values of to explore the impact of the SF on the evolution of those parameters. In their discussion of anisotropic cosmological models with a significant SF, Raju et al. [45, 46] discovered that the SF is a decreasing function of cosmic time. Bianchi type-IX dark energy model with a huge scalar meson field in Lyra geometry has been investigated by Aditya et al. [65]. Numerous Bianchi type cosmological models with enormous SF were examined by Rao et al. [66, 67], and they discovered that the SF is diminishing in function. According to the description above, the study of our SF is pretty comparable to the SF models that were previously mentioned. Thus, our model's behavior for SF is highly consistent with recent research and observations.

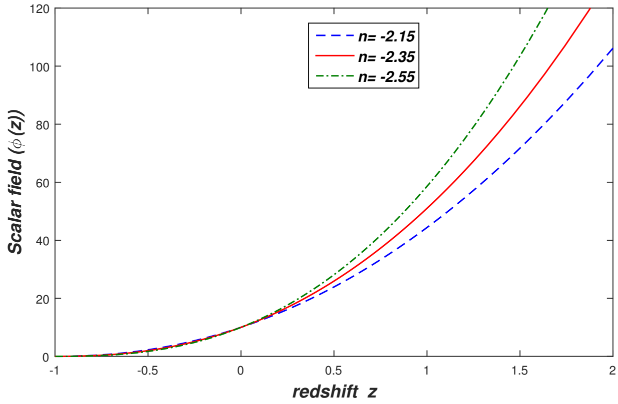

FIGURE 1. Plot of SF versus redshift for $c_2 = 0.05$, $M = 1.2$, $c_1 = 0.2$, $\phi_0 = 10$ and $k = 0.95$.

Energy density and Pressure: We look into how domain wall pressure and energy density change with respect to redshift for different values of n. The energy density and pressure of domain walls are both decreasing functions and positive throughout the evolution of the cosmos, as can be shown in Figs. 2 and 3. The energy criteria of the model are met since both the energy density and pressure have been positive throughout the history of the universe. Figures 2 and 3 show that the energy density and pressure of domain barriers decrease with value n. As a result, it should be noted that the energy density and pressure of domain barriers rise as the SF increases.

Deceleration parameter: The deceleration parameter $(q(t))$ is a crucial tool for defining the behaviour of the model. When $q > 0$ the universe decelerates whereas when *q=0* the model expands steadily. If $-1 \leq q < 0$, the cosmological model with accelerated expansion. For our model, the deceleration parameter is obtained as

$$q(t) = 2 - 3\left(\tan h(3k_1(t + c_2))\right)^2 . \tag{32}$$

Figure 4 shows a plot of the deceleration parameter's behavior. The deceleration parameter of our model initially changes throughout the region $q > 0$ and gradually achieves the value $q < 0$, as seen in Fig. 4. This proves that the cosmos in our model exhibits a transition between its early decelerating phase and its current accelerating phase. Additionally, the universe changed from a decelerating to an

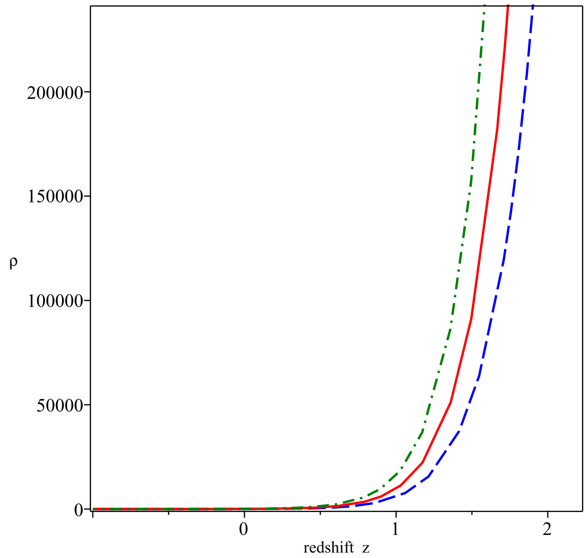

FIGURE 2. Plot of energy density versus redshift for $c_2 = 0.05$, $M = 1.2$, $c_1 = 0.2$, $\phi_0 = 10$, $\lambda = 8$and $k = 0.95$.

accelerating phase around 0.3 < z <0.8. This agrees well with cosmological observations (Muthukrishna and Parkinson [74]; Capozziello et al. [73]). The transition redshift (z_t) of the accelerating expansion is given by $0.3 < z_t < 0.8$, according to Capozziello et al. [73] study of cosmographic restrictions on the cosmic deceleration-acceleration transition redshift in the f(R) theory of gravity. Using SN-Ia and BAO, Muthukrishna and Parkinson [74] investigated cosmographic analysis of the transition to acceleration and derived constraints on the redshift of the transition from deceleration to acceleration for the various expansions. In the most conservative

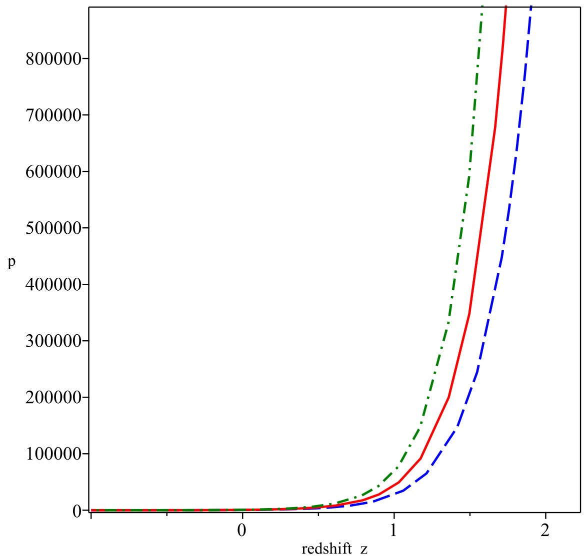

FIGURE 3. Plot of pressure versus redshift for$c_2 = 0.05$, $M = 1.2$, $c_1 = 0.2$, $\phi_0 = 10$, $\lambda = 8$and $k = 0.95$.

case, they found $z_{acc} > 0.14$ at 95% confidence. Additionally, the deceleration parameter and, consequently, the accelerated expansion of our cosmological model appear to be significantly impacted by the SF.

Statefinder parameters: Different DE models can be distinguished using the statefinder parameters r and s. Since they have a geometrical nature and are produced directly from the metric, the parameters and are model-dependent. Two new dimensionless parameters known as statefinders were introduced by Sahni et al. [75] with the

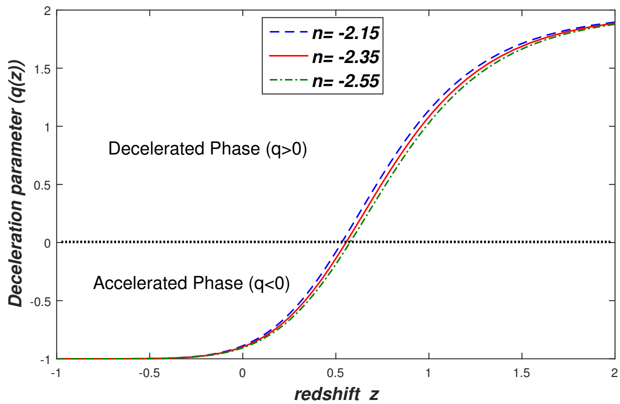

FIGURE 4. Plot of deceleration parameter versus redshift for $c_2 = 0.05$, $M = 1.2$, $c_1 = 0.2$, $\phi_0 = 10$ and $k = 0.95$.

following definitions:

$$r = \frac{\dddot{a}}{aH^3}, \qquad s = \frac{r-1}{3\left(q - \frac{1}{2}\right)}.$$

For our model, the above parameters are obtained as

$$r = 10 - \frac{18\,\left(tanh\left(3k_1(t+c_2)\right)\right)^2}{\left(sinh\left(3k_1(t+c_2)\right)\right)} + 9\left(tanh\left(3k_1(t+c_2)\right)\right)^2 \tag{33}$$

$$s = \left(1 - \frac{2\,\left(tanh\left(3k_1(t+c_2)\right)\right)^2}{\left(sinh\left(3k_1(t+c_2)\right)\right)} + \left(tanh\left(3k_1(t+c_2)\right)\right)^2\right) \times \left(\frac{1}{2} - \left(\tan h(3k_1(t+c_2))\right)^2\right)^{-1} \tag{34}$$

There is a limit for $(r, s) = (1, 1)$ CDM limit and for $(r, s) = (1, 0)$ ΛCDM limit. For $r > 1$ and $s < 0$, the model corresponds to the Chaplygin gas. For $r < 1$ and $s > 0$, the model corresponds to quintessence and phantom regions. It is very clear from Fig. 5 that our model becomes ΛCDM model at late times. Also, it can be seen that our model is similar to the Chaplygin gas and phantom model.

r-q plane: The values $(r, q) = (1, 0.5)$ and $(r, q) = (1, -1)$, respectively, represent the steady state (SS) and standard cold dark matter (SCDM) theories. The CDM model evolves along the dotted line (see Fig. 6) starting at a fixed location in the SCDM model until it reaches the fixed location in the SS model. The behavior of our model in the $r - q$ plane is depicted in Fig. 6. Our model eventually resembles

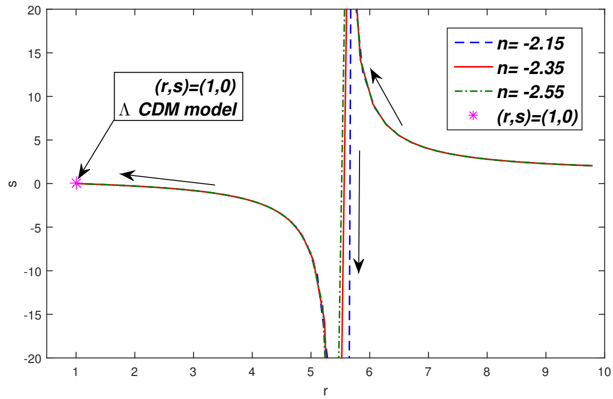

FIGURE 5. Plot of statefinder parameters for $c_2 = 0.05$, $M = 1.2$, $c_1 = 0.2$, $\phi_0 = 10$ and $k = 0.95$.

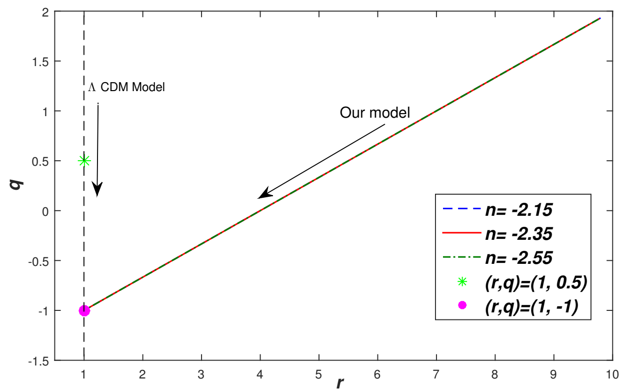

FIGURE 6. Plot of r-q plane for $c_2 = 0.05$, $M = 1.2$, $c_1 = 0.2$, $\phi_0 = 10$ and $k = 0.95$.

the SS model. The $r - q$ plane analysis of the anisotropic dark energy model in the presence of a massive scalar meson field in Lyra geometry and holographic Ricci dark energy with constant bulk viscosity in f(R,T) gravity, respectively, has been studied by Aditya et al. [65] and Singh and Kumar [76]. It has been noted that our model's $r - q$ trajectory exhibits strong similarities to the SF models that have been previously discussed in the literature (ref. Aditya et al. [65]; Singh and Kumar [76]).

4. Conclusion

In the context of $f(R,T)$ theory of gravity, we have examined the Binachi type-V massive SF cosmological model with domain walls in this research work. We presupposed a relationship between metric potentials and a power law between the SF and the average scale factor of the model in order to produce a deterministic model. We have acquired various cosmic parameters and provided the physical description of them. Some of the conclusions are as follows:

- While the volume of our model rises over time, physical parameters like the Hubble parameter (H(t)), expansion scalar (θ), and shear scalar (σ^2) have finite values at the starting epoch (at $t = 0$) and tend to have a constant value for sufficiently high values of time. Therefore, the universe is expanding quickly. Because the average anisotropy parameter is constant, the model is anisotropic.
- As the universe evolves, the SF is observed to be a diminishing function (Fig. 1). We may observe that the kinetic energy rises as the SF lowers. In their discussions of anisotropic dark energy cosmological models with enormous SF, Raju et al. [45, 46], Aditya et al. [65], and Rao et al. [66, 67] discovered that the SF is a decreasing function of cosmic time. This discussion leads to the conclusion that the behavior of our SF is pretty similar to the SF models mentioned above.
- According to Figs. 2 and 3, our model's energy density and pressure are positive and seem to be decreasing as the universe expands. The energy criteria are met since energy pressure and density have been increasing throughout the history of the universe. Additionally, when SF grows, so do energy density and domain wall pressure. As a result, the SF has a big impact on how energy density and domain wall pressure behave.
- The deceleration parameter ($q(t)$) starts from a positive region (q>0) and eventually approaches $q{=}{-1}$ at late times. That is our model begins in a decelerating phase and eventually moves into an accelerating era (Fig. 4). We observed that the transition of the universe from decelerating phase to the accelerating phase occurred at $0.3 < \mathrm{z} < 0.8$. This is in good agreement with cosmological observations discussed by Capozziello et al. [73] and Muthukrishna and Parkinson [74]. The present value of the deceleration

parameter in our model is $q \approx -0.82$ which is following the observational data (Capozziello et al. [77]; Amirhashchi and Amirhashchi [78]) given as $q = -0.930 \pm 0.218$ $(BAO + Masers + TDSL + Pantheon + H_z)$ and $q = -1.2037 \pm 0.175$ $(BAO + Masers + TDSL + Pantheon + H_0 + H_z)$. It can be seen that the SF shows some significant effect on the deceleration parameter and hence on the accelerating expansion of our cosmological model.

- Our model's behavior was investigated using statefinder phase analysis. As can be seen, our model behaves similarly to how the Chaplygin gas and phantom model behaves, and finally, the model approaches the ΛCDM model (Fig. 5). We can observe that the track of our model gradually approaches the steady state model (Fig. 6). According to Singh and Kumar [76] and Aditya et al. [65], this behavior is very comparable to SF models.

Finally, we may say that all cosmic parameters have been found to agree with recent cosmological experimental observations. The significance of massive SFs in the development of domain walls in contemporary cosmology will thus be better understood with the support of our anisotropic model.

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