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SPATIALLY HOMOGENEOUS AND ANISOTROPIC TWO FORMS DARK ENERGY COSMOLOGICAL MODEL

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Abstract. We have examined spatially homogeneous and anisotropic cosmological models in the presence of two forms of dark energy (DE). We have calculated the exact solutions of the Einsteins field equations by assuming a special law of variation of Hubbles parameter [3] . Some physical parameters of the model are determined. From the point of kinematical behaviour we have also extended our work by deriving the analytic expressions for the look back time.

AMS Classification: 54H20.

Keywords: Quintessence; Cosmological constant; Bianchi type III space time.

1. Introduction

The recent astronomical observations of type-Ia supernovae experiments, Cosmic Microwave Background (CMB) radiation and large scale structure of the universe [2, 23, 27],[33]-[35] have pointed that the current observable universe is not only expanding but also accelerating and this accelerated expansion of the universe is due to some kind of matter distribution of the universe with large negative pressure known as dark energy (DE). There are the several alternative candidates to the dark energy such as cosmological constant [11], the quintessence [25], phantom field [4], chaplygin gas [14], tachyon field [17] , holographic [28]-[29] etc. are studied for the present accelerated expansion of the universe. The classical models consisting the term of cosmological constant $\wedge$ are of the great importance because of its implications in discussing the present accelerated phase of the universe[26] . According to Sil and Som [30] Cosmological constant is the most useful candidate to explain the origin of the dark energy component in the universe because, for supporting, he has proposed and investigated a model where $\wedge \propto \rho a^3$ and observed that it is very close to present observations. Pawar, et al.[18] have also used the cosmological constant in their cosmological model. According to the model of Kumar and Singh [15], quintessence behaves like cosmological constant by combining positive energy and negative pressure which is suitable for describing the present evolution of the universe. Caldwell et al. [5] have investigated phantom energy model and observed that it can rip apart the milkyway, solar system, the earth and ultimately the molecules, atoms, nuclei and nucleons of which we are composed, before the end of universe in a big rip.

Almost all authors have investigated relativistic cosmological model by using cosmic fluid as a perfect fluid, cosmic fluid as a dark energy, cosmic fluid as a binary mixture of perfect fluid and dark energy [1, 19, 21, 22]. Nowadays, some researchers are working on dark energy cosmological models which contain cosmic fluid either quintessence or cosmological constant or chaplygin gas. No studies are seen so far where both the candidates of the dark energy are taken together but in the present work we have studied spatially homogeneous and anisotropic Bianhi Type III space time by considering the dark energy portion of the universe which consists of cosmological constant form of dark energy and quintessence form of dark energy together [31] . We have obtained the deterministic solution of the field equation by assuming

special law of variation of Hubbles parameter proposed by Berman [3] that yields the constant deceleration parameter. We have extended our work by verifying the consistency of the present model by deriving the kinematical relations such as look back time [20] . Solanke et al.[32] , Dagwal [10] , Dagwal and Pawar [6]-[9] have presented different aspects of dark energy form of cosmological models.
This paper is organized as - Section 2: Metric and the field equations, Section, 3: Solution of the field equations, Section 4: Results and discussion, Section 5: Conclusion.

2. METRIC AND THE FIELD EQUATIONS

Here we have considered the spatially homogeneous and anisotropic Bianchi Type III Metric in the form

$$ds^2 = dt^2 - A^2dx^2 - e^{-2mx}B^2dy^2 - C^2dz^2, \tag{1}$$

where A, B, and C are the function of cosmic time t.
The energy momentum tensor of the source is given by

$$T_{ij} = \rho u_i u_j + (p - \varepsilon\theta) P_{ij} - 2\eta\sigma_{ij}, \tag{2}$$

where ρ is the fluid density, p is the fluid pressure, η is the coefficient of the shear, ε is the coefficient of viscosity and u_i are four velocity vectors satisfying $g_{ij}u^iu^j = 1$. The components of shear tensor, components of the projection tensor and expansion scalar are respectively given by

$$\sigma_{ij} = \frac{1}{2}\left(u_{i;m}P^m_j + u_{i;m}P^m_i\right) - \frac{1}{3}\theta P_{ij}, \tag{3}$$

where $P_{ij} = u_iu_j - g_{ij}$, and $\theta = u^i_{;i}$.

The Einsteins field equation as

$$R_{ij} - \frac{1}{2}g_{ij}R + g_{ij}\wedge = -8\pi G\left[\rho u_iu_j + (p - \varepsilon\theta) P_{ij} - 2\eta\sigma_{ij}\right]. \tag{4}$$

From equations (1) -(4) we get

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}-\wedge=-8\pi G\left\{(p-\varepsilon\theta)-2\eta\left[\frac{\dot{A}}{A}-\frac{1}{3}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)\right]\right\}, \tag{5}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}-\wedge=-8\pi G\left\{(p-\varepsilon\theta)-2\eta\left[\frac{\dot{B}}{B}-\frac{1}{3}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)\right]\right\}, \tag{6}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{m^2}{A^2}-\wedge=-8\pi G\left\{(p-\varepsilon\theta)-2\eta\left[\frac{\dot{C}}{C}-\frac{1}{3}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)\right]\right\}, \tag{7}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}-\frac{m^2}{A^2}-\wedge=8\pi G\rho, \tag{8}$$

$$m\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)=0. \tag{9}$$

Integrating equation (9) to get

$$A=c_1B\Rightarrow A=B. \tag{10}$$

For sake of simplicity constant of integration c_1 is chosen to be unity.

Thus from relation (10) system of equations (5) to (8) reduces and takes the form

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}-\wedge=-8\pi G\left\{(p-\varepsilon\theta)-2\eta\left[\frac{\dot{A}}{A}-\frac{1}{3}\left(2\frac{\dot{A}}{A}+\frac{\dot{C}}{C}\right)\right]\right\}, \tag{11}$$

$$2\frac{\ddot{A}}{A}+\frac{\dot{A}^2}{A^2}-\frac{m^2}{A^2}-\wedge=-8\pi G\left\{(p-\varepsilon\theta)-2\eta\left[\frac{\dot{C}}{C}-\frac{1}{3}\left(2\frac{\dot{A}}{A}+\frac{\dot{C}}{C}\right)\right]\right\}, \tag{12}$$

$$\frac{\dot{A}^2}{A^2}+2\frac{\dot{A}\dot{C}}{AC}-\frac{m^2}{A^2}-\wedge=8\pi G\rho. \tag{13}$$

3. SOLUTION OF THE FIELD EQUATIONS

Subtracting equation (12) from (11) and then integrating to get

$$\int\frac{\frac{d}{dt}\left(\frac{\dot{A}}{A}-\frac{\dot{C}}{C}\right)}{\left(\frac{\dot{A}}{A}-\frac{\dot{C}}{C}\right)}dt+\int\left(2\frac{\dot{A}}{A}+\frac{\dot{C}}{C}\right)dt=\int\left[\frac{\frac{m^2}{A^2}}{\left(\frac{\dot{A}}{A}-\frac{\dot{C}}{C}\right)}+2\eta\right]dt. \tag{14}$$

From equation (14) we get

$$\frac{A}{C}=c_2e^{\lambda\int\frac{dt}{A^2C}}, \tag{15}$$

where c_2 being constant of integration.
As per the assumption, a special law of variation of Hubbles parameter proposed by Berman [13] that yields a constant deceleration parameter defined by $q = -\frac{R\ddot{R}}{\dot{R}^2}$ which on integration we get

$$R = (at+b)^{\frac{1}{(q+1)}}, \tag{16}$$

where $a \neq 0, b$ are constants and $(1+q) > 0$ is the condition for an accelerated expansion of the universe.
The average scale factor R as

$$R^3 = ABC = A^2C \qquad (\because A = B) \tag{17}$$

Solving equations (15), (16) and (17) we get the metric potentials as

$$A = B = C_0(at+b)^{\frac{1}{(1+q)}} e^{D_0(at+b)^{\frac{q-2}{q+1}}} \quad \text{and} \quad C = a_0(at+b)^{\frac{1}{(1+q)}} e^{b_0(at+b)^{\frac{q-2}{q+1}}}, \tag{18}$$

where a_0, b_0, C_0 and D_0 are constants.
From equation (1) as

$$\begin{aligned} ds^2 = dt^2 - &\left\{C_0(at+b)^{\frac{1}{1+q}} e^{D_0(at+b)^{\frac{q-2}{q+2}}}\right\}^2 \left[dx^2 + e^{-2mx}dy^2\right] \\ &- \left\{a_0(at+b)^{\frac{1}{1+q}} e^{b_0(at+b)^{\frac{q-2}{q+2}}}\right\}^2 dz^2. \end{aligned} \tag{19}$$

Equation of the conservation for the given metric (1) is found to be

$$\dot{\rho} + \left(2\frac{\dot{A}}{A} + \frac{\dot{C}}{C}\right)(\rho + P) = 0. \tag{20}$$

We have taken the equation of state as

$$p_\phi = \omega_\phi \rho_\phi. \tag{21}$$

Thus from the equation (20) with the help of the equations (18) and (21) we respectively have

$$\rho_\phi = c_1(at+b)^{\frac{-3\left(1+\omega_\phi\right)}{q+1}}, \quad p_\phi = c_1\omega_\phi(at+b)^{\frac{-3\left(1+\omega_\phi\right)}{q+1}}, \tag{22}$$

where $c_1$is constant of integration.
The energy density and pressure are presented in Fig (1) and Fig (2) respectively.

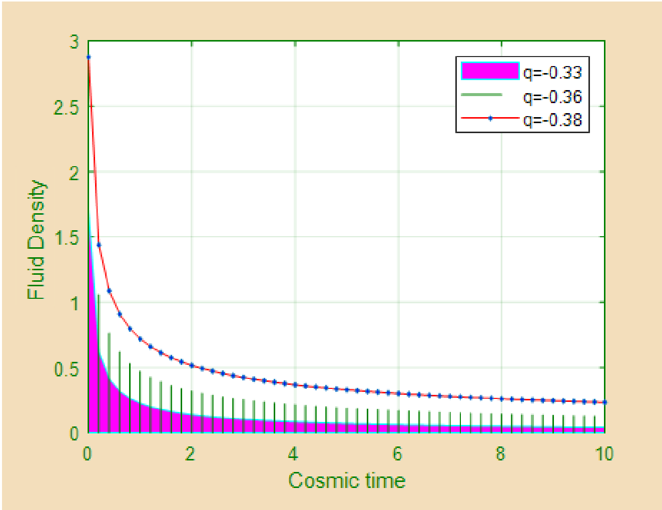

FIGURE 1. Variation of the density against cosmic time by setting the values$c_1 = 9.1$, $\omega_\phi = 9.10$, $q = (-0.33, -0.36, -0.38)$, $a = 150$, $b = 10$.

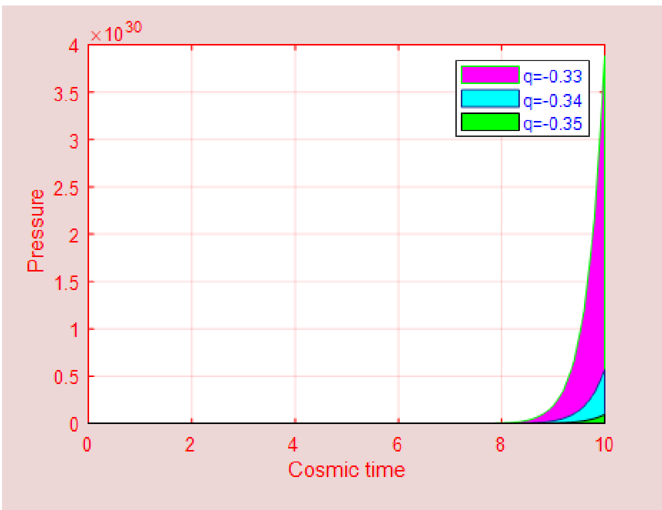

FIGURE 2. Variation of the pressure against cosmic time by setting the values $c_1 = 9.1$, $\omega_\phi = 2.10$, $q = (-0.33, -0.34, -0.35)$, $a = 1$, $b = 0.1$.

The energy density decreases monotonically where as pressure increases in the context of the cosmic time.

Equation (13), (18) and (22) gives the cosmological term $\wedge$ as

$$\wedge = \left(\frac{a}{1+q}\right)^2 \left\{ \frac{3}{(at+b)^2} - \frac{3{D_0}^2(q-2)^2}{(at+b)^{\frac{6}{1+q}}} \right\} - \frac{m^2}{C_0^2(at+b)^{\frac{2}{1+q}} e^{2D_0(at+b)^{\frac{q-2}{q+1}}}} - \frac{8\pi G c_1}{(at+b)^{\frac{3\left(1+\omega_\phi\right)}{q+1}}}. \tag{23}$$

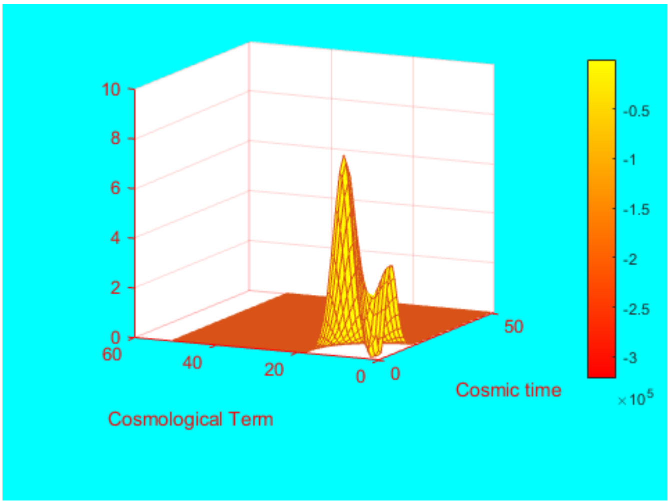

FIGURE 3. Variation of the cosmological term against cosmic time by setting the values $c_1 = 9.1,\ \ \omega_\phi = 10.10,\ \ q = -0.33,\ \ a = 1,\ \ b = 1, m = -1, c_0 = 1, D_0 = 6.34.$

Thus equation (11) with the equations (18), (22) and (23) gives

(24)

$$\varepsilon = \frac{-a}{24\pi\,(q+1)} \left[\frac{2{D_0}^2(q-2)^2}{(at+b)^{\frac{5-q}{1+q}}} + \frac{4}{(at+b)} \right] - \frac{(1+q)\,m^2}{24a\pi G{C_0}^2(at+b)^{\frac{1-q}{1+q}} e^{2D_0(at+b)^{\frac{q-2}{q+1}}}} - \frac{c_1\,(1+q)\,(1-\omega_\phi)}{3a(at+b)^{\frac{2-q+3\omega_\phi}{q+1}}} - \frac{2\eta D_0\,(q-2)}{3(at+b)^{\frac{2-q}{q+1}}}.$$

The scalar expansion θ and the shear scalar σ^2 for the model (19) with the help of the equation (18) are respectively given by

$$\theta = \frac{2\dot{A}}{A} + \frac{\dot{C}}{C} = \frac{3a}{(1+q)(at+b)}, \tag{25}$$

$$\sigma^2 = \frac{2}{3}\left(\frac{\dot{A}}{A} - \frac{\dot{C}}{C}\right)^2 = 6\left[\frac{aD_0(q-2)}{(1+q)(at+b)^{\frac{3}{q+1}}}\right]^2.$$

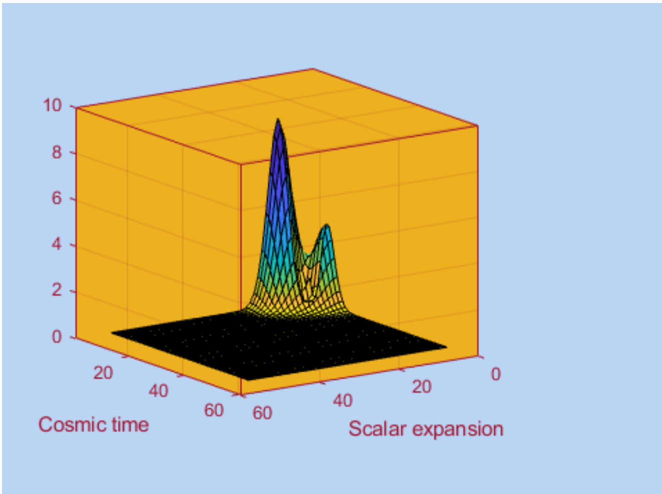

FIGURE 4. Variation of the scalar expansion against cosmic time by setting the values $a = 1,\ b = 10,\ q = -0.33$.

The directional Hubbles parameters for the model (19) are

$$H_x = H_y \Rightarrow \frac{\dot{A}}{A} = \frac{\dot{B}}{B} \Rightarrow \frac{a}{(1+q)}\left\{\frac{1}{(at+b)} + \frac{D_0(q-2)}{(at+b)^{\frac{3}{1+q}}}\right\}, \tag{26}$$

$$H_z = \frac{\dot{C}}{C} = \frac{a}{(1+q)}\left\{\frac{1}{(at+b)} + \frac{b_0(q-2)}{(at+b)^{\frac{3}{1+q}}}\right\}. \tag{27}$$

Similarly mean generalized Hubble parameter is given by

$$H = \frac{1}{3}(H_x + H_y + H_z) = \frac{a}{(1+q)(at+b)}, \tag{28}$$

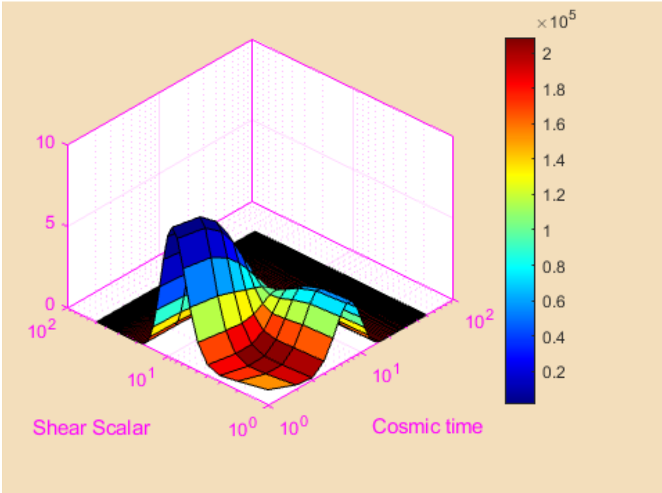

FIGURE 5. Evolution of the shear scalar versus cosmic time by setting the values $a = 1,\ b = 10,\ q = -0.33, D_0 = 10$.

where $b_0 = -2D_0$.
Also

$$\theta = \frac{3a}{(1+q)(at+b)}. \tag{29}$$

Thus the relation $\theta = 3H$ is verified.
The mean anisotropy parameter Δ for the model (19) takes the form

$$\Delta = \frac{1}{3}\sum\left(\frac{H_i - H}{H}\right)^2 = 2\left[\frac{D_0\,(q-2)}{(at+b)^{\frac{2-q}{1+q}}}\right]^2. \tag{30}$$

In the present paper we have extended our work to investigate the consistency of our model (19) with the observational parameters through the kinematics tests by studying the astronomical phenomena such as look back time red shift of our model.

Look-Back Time Redshift:
The look back time is considered by

$$\Delta t = t_0 - t(z) = \int_R^{R_0} \frac{dR}{\dot{R}}, \tag{31}$$

where R_0 is the current scale factor of the universe.
From equations (16) and (28) we have

$$1+z=\left(\frac{at_0+b}{at+b}\right)^{\frac{1}{1+q}}. \tag{32}$$

But when $t=0, H=H_0$ equation (28) gives

$$H_0\left(1+q\right)=\frac{a}{b}. \tag{33}$$

Thus after a little manipulation with the equations (32) and (33) and neglecting the higher power as well as product terms tt_0in the above expansion we get

$$\left(1+q\right)H_0\left(t_0-t\right)=1-\left(1+z\right)^{-(1+q)}. \tag{34}$$

Taking $z\to\infty$ in above equation we get

$$H_0\left(t_0-t\right)=\frac{1}{(1+q)}\Rightarrow\left(t_0-t\right)=\frac{H_0^{-1}}{(1+q)}. \tag{35}$$

Thus the present age of the universe is

$$t_0=\frac{H_0^{-1}}{(1+q)}\qquad\left(\because t=0\right), \tag{36}$$

where H_0 is the Hubble constant at current.

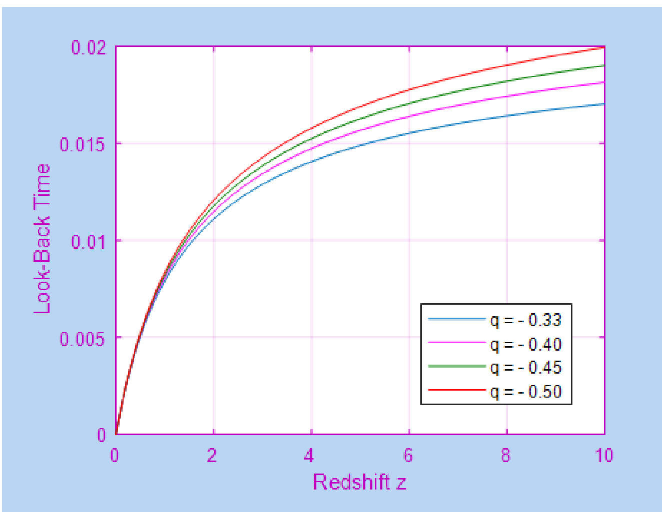

Figure 6. Evolution of look-back time versus z .

4. RESULTS AND DISCUSSION

- The pressure and energy density are presented in Fig (2) and Fig (1) respectively. The energy density decreases monotonically whereas the pressure increases against t.
- The cosmological constant shown in Fig (3), raise with the context of t. When $t \to -\frac{b}{a}$, the cosmological constant has a big rip singularity.
- Fig. (4) and Fig. (5) present the evolution of scalar expansion and Shear scalar. The scalar expansion and Shear scalar grow quadratically with respective t. Equation (25) shows big rip at $t \to -\frac{b}{a}$ and big bang at $t \to \infty$. The θ and σ^2 are in big rip and big bang singularity represented in Fig. (4) and Fig. (5) respectively.
- Fig.6 represents the evolution of look-back time with respective z by using the range $q = (-0.33, -0.40, -0.45, -0.50)$ and $H_0 = 70$. The range of H_0 is amid $50 - 100\,\text{kms}^{-1}\text{Mpc}^{-1}$.
- Observational Constraints-:
  Various authors have constrained on the H_0 and q with the assist of present epidemiological data. Freedman et al.[12] have calculated a value of $H_0 =$ 728 km/s/Mpc . Gumjudpai [13] has obtained H_0 and $\frac{1}{1+q}$ for WMAP7 and WMAP7 + BAO + H(z) data sets respectively. Kumar [16] has developed $H(z)$ by usesing 15 data points and 557SNIa data points. Rani et al. [24] have constrained the value of Hubble parameter and deceleration parameter q for flat power law cosmological model. In the present work , we have used value of Hubble parameter and deceleration parameter q, for different datasets (Rani et al. [24]) as given in following table

Table1.The data sets for the range of deceleration parameter q and Hubble parameter H_0.

Data	q	H_0(km/s/Mpc)	$\frac{1}{1+q}$
H(z)	$-0.18^{+0.12}_{-0.12}$	$68.43^{+2.84}_{-2.80}$	-
SNeIa	$-0.38^{+0.05}_{-0.05}$	$69.18^{+0.55}_{-0.54}$	-
WMAP7	-	$70.3^{+2.5}_{-2.5}$	$0.99^{+0.04}_{-0.04}$
WMAP7+BAO+H(z)	-	$70.4^{+1.4}_{-1.4}$	$0.99^{+0.02}_{-0.02}$

According to table 1, we have presented Fig.7.

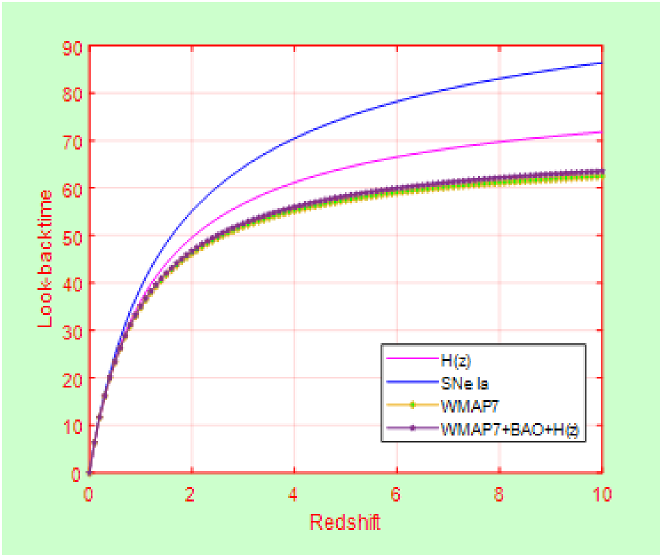

FIGURE 7. Evolution of look-back time with respective redshift z.

5. CONCLUSION

In the present paper we have investigated Bianchi Type III space time by assuming the special law of Hubbles parameter in the presence of two forms of DE. All the parameters which we have determined of the model (19) are found to be the function of cosmic time. For the derived model, the bulk viscosity is discovered to be the diminishing function of t and vanishes at an infinitely large time, similarly energy density for the scalar field is discovered to be the diminishing function of t which indicates that the quintessence model of DE declines. The cosmological term is also the function of cosmic time. Our study also reveals the astrophysical phenomena such as look back time red shift for the deterministic model and observed that such model is compatible with recent observations.

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