

FRW VISCOUS MODIFIED COSMIC CHAPLYGIN GAS COSMOLOGY WITH VARIABLE COSMOLOGICAL CONSTANT Λ

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ABSTRACT. We consider Kaluza–Klein FRW model with modified cosmic Chaplygin gas in the presence of variable cosmological constant Λ by including shear and bulk viscosity. By using numerical analysis we have investigated behavior of energy density ρ and Hubble parameter H . To study effect of bulk and shear viscosity on the cosmological parameters, we have obtained analytical solution for some particular choices of m , n and α . Finally we discussed stability of the model.

AMS Classification: 54H20.

Keywords: Modified cosmic Chaplygin gas; Bulk viscosity; Shear viscosity; Dark energy.

1. INTRODUCTION

A series of astronomical observations such as SNeIa [1, 2], measurements of Cosmic Microwave Background (CMB) [3] from Wilkinson Microwave Anisotropy Probe (WMAP) [5, 6], the data of Large Scale Structure of universe (LSS) from Sloan

Digital Sky Survey (SDSS) [7]-[8] and X-ray experiments [9], over the past decade indicate that our universe has undergone to the accelerating expansion phase. The agent behind acceleration of universe is termed as dark energy. Dark energy is described by number of models and theories [10]-[23]. An interesting model to describe dark energy is Chaplygin gas (CG) [24]-[31], which is extended to generalized Chaplygin gas (GCG) for observational agreements [32]-[36]. Further it is possible to extend GCG to the modified Chaplygin gas (MCG) [37] or modified cosmic Chaplygin gas (MCCG) [38]. We know that the bulk viscosity plays an important role in cosmology [39]-[41]. The idea that Chaplygin gas may has viscosity first proposed by [42] and then developed by [43]-[48].

Kaluza Klein proposed idea that world may have five dimensions and define field equations, which unify both electromagnetism and gravity. Sadeghi et al. (2013) [47] investigated modified cosmic Chaplygin cosmology with non-zero cosmological constant in non-flat universe. By using observational data, they have fixed solution and discussed stability. Saadat (2014) [49] studied interaction between MCCG and pressure less matter in the presence of both bulk and shear viscosities in the framework of arbitrary D-dimensional FRW cosmology and by using first order linear perturbation theory investigated the stability of this model. Samanta and Bishi (2016) [50] formulated Kaluza-Klein FRW model with modified cosmic Chaplygin gas in presence of bulk viscosity by using the EoS of the form

$$(1) \quad p = A\rho - \frac{1}{\rho^\alpha} \left[\frac{B}{1+\omega} - 1 + \left(\rho^{1+\alpha} - \frac{B}{1+\omega} + 1 \right)^{-\omega} \right] - 4\zeta H,$$

where ζ is constant. Samanta and Bishi (2016) [50] obtained time dependent dark energy density, by using exponential function method and discussed the nature of the energy density with and without bulk viscosity. Sharif and Ashraf (2018) [51] examined the thermodynamic features of modified cosmic Chaplygin gas in the context of homogeneous isotropic universe model.

With the motivation of above work here we have studied FRW viscous modified cosmic Chaplygin gas model with variable cosmological constant Λ in flat universe by introducing bulk and shear viscosity.

This paper is organized as follows: In section 2 We have studied the evolution of Chaplygin gas to viscous modified cosmic Chaplygin gas. Section 3 deals with

FRW cosmology. In section 4 we have obtained numerical and analytical solutions of the conservation equation. In section 5 we summarize the results.

2. CHAPLYGIN GAS MODEL

One of the interesting cosmological model to describe universe is based on CG with the following equation of state [24, 25]

$$(2) \quad p = -\frac{B}{\rho},$$

where B is positive constant. The CG overruled the observational data, therefore Eq. (2) has been generalized to,

$$(3) \quad p = -\frac{B}{\rho^\alpha},$$

with $0 < \alpha \leq 1$ which is known as GCG [25]. The GCG can unify dark energy and dark matter. From Eq. (3) it is clear that at early time when energy density is high GCG represents pressureless dust ($p = 0$). In order to have coincidence with observational data, Gonzalez-Diaz [52] gave another extension of GCG called as generalized cosmic Chaplygin gas (GCCG) of the form

$$(4) \quad p = -\frac{1}{\rho^\alpha} \left[\frac{B}{1+\omega} - 1 + \left(\rho^{1+\alpha} - \frac{B}{1+\omega} + 1 \right)^{-\omega} \right].$$

This model is stable and free from unphysical behaviors even when the vacuum fluid satisfies the phantom energy condition [53].

Further extension of GCCG is possible with following EoS [54]

$$(5) \quad p = A\rho - \frac{1}{\rho^\alpha} \left[\frac{B}{1+\omega} - 1 + \left(\rho^{1+\alpha} - \frac{B}{1+\omega} + 1 \right)^{-\omega} \right],$$

which is known as modified cosmic Chaplygin gas (MCCG).

Also, one can define viscous modified cosmic Chaplygin gas (VMCCG) by including shear and bulk viscosity in MCCG which gives us [54]

$$(6) \quad p = A\rho - \frac{1}{\rho^\alpha} \left[\frac{B}{1+\omega} - 1 + \left(\rho^{1+\alpha} - \frac{B}{1+\omega} + 1 \right)^{-\omega} \right] - 3\zeta H + 2\eta H,$$

where η and ζ are shear and bulk viscous coefficient respectively.

3. FRW COSMOLOGY

The Kaluza-Klein type Friedmann Robertson Walker (FRW) universe for $k = 1$ is describe by [55]

$$(7) \quad ds^2 = dt^2 - a^2(t) [dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) + d\Psi^2],$$

where $a(t)$ is the scale factor.

The Einstein field equations with time-dependent cosmological is given by [56]

$$(8) \quad R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \Lambda g_{\mu\nu} - 8\pi GT_{\mu\nu},$$

where $R_{\mu\nu}$, R , $g_{\mu\nu}$ are Ricci tensor, Ricci scalar and metric tensor respectively. Assuming that matter filling the universe is in the form of a perfect fluid, then

$$(9) \quad \begin{aligned} T_0^0 &= \rho + \frac{\Lambda}{8\pi G}, \\ T_1^1 &= T_2^2 = T_3^3 = T_4^4 = -p + \frac{\Lambda}{8\pi G}, \end{aligned}$$

where ρ is the density and p is the pressure.

Using line element (7), Einstein's field Eq. (8) and Eq. (9) we obtain following set of differential equations

$$(10) \quad \frac{\dot{a}^2}{a^2} = \frac{8\pi G\rho}{6} + \frac{\Lambda}{6},$$

$$(11) \quad \frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = \frac{-8\pi Gp}{3} + \frac{\Lambda}{3},$$

where dot denotes derivative with respect to cosmic time.

If we assume $8\pi G = 1$ and cosmological constant $\Lambda = \beta_1\rho$, Eqs. (10) and (11) reduces to

$$(12) \quad \frac{\dot{a}^2}{a^2} = \left(\frac{1 + \beta_1}{6} \right) \rho,$$

$$(13) \quad \frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = \frac{p}{3} + \frac{\beta_1\rho}{3}.$$

The conservation equation is given by

$$(14) \quad \dot{\rho} + 4H(\rho + p) = 0.$$

4. VISCOUS MODIFIED COSMIC CHAPLYGIN GAS WITH VARIABLE ζ AND η

In this section we consider the bulk and shear viscous coefficients ζ and η respectively as a linear combination of the form

$$(15) \quad \zeta = \zeta_0 + \zeta_1 \rho^n,$$

$$(16) \quad \eta = \eta_0 + \eta_1 \rho^m,$$

where $m \neq n$.

By using Eqs. (15) and (16) in Eq. (6) we get

$$(17) \quad p = A\rho - \frac{1}{\rho^\alpha} \left[\frac{B}{1+\omega} - 1 + \left(\rho^{1+\alpha} - \frac{B}{1+\omega} + 1 \right)^{-\omega} \right] - 3H(\zeta_0 + \zeta_1 \rho^n) + 2H(\eta_0 + \eta_1 \rho^m).$$

Therefore conservation Eq. (14) reduces to

$$(18) \quad \dot{\rho} + 4(1+A)\sqrt{\frac{1+\beta_1}{6}}\rho^{\frac{3}{2}} - 4\sqrt{\frac{1+\beta_1}{6}}\rho^{\frac{1}{2}-\alpha} \left[\frac{B}{1+\omega} - 1 + \left(\rho^{1+\alpha} - \frac{B}{1+\omega} + 1 \right)^{-\omega} \right] + 4(2\eta_0 - 3\zeta_0) \left(\frac{1+\beta_1}{6} \right) \rho - 12\zeta_1 \left(\frac{1+\beta_1}{6} \right) \rho^{n+1} + 8\eta_1 \left(\frac{1+\beta_1}{6} \right) \rho^{m+1} = 0.$$

By using Eq. (12) in Eq. (18) one can obtain differential equation in terms of Hubble parameter as

$$(19) \quad \dot{H} + \left\{ (1+A) \left(\frac{6H^2}{1+\beta_1} \right) - \left(\frac{6H^2}{1+\beta_1} \right)^{-\alpha} \left[\frac{B}{1+\omega} - 1 + \left[\left(\frac{6H^2}{1+\beta_1} \right)^{1+\alpha} - \frac{B}{1+\omega} + 1 \right]^{-\omega} \right] - 3H \left[\zeta_0 + \zeta_1 \left(\frac{6H^2}{1+\beta_1} \right)^n \right] + 2H \left[\eta_0 + \eta_1 \left(\frac{6H^2}{1+\beta_1} \right)^m \right] \right\} \frac{1+\beta_1}{3} = 0.$$

We have represented nature of time dependent energy density and Hubble parameter in figs. 1 and 2 by solving Eqs. (18) and (19) numerically. Figs. 1 and 2 shows that ρ and H are decreasing functions of time. In fig. 1 (a) we fix $\zeta_0 = \zeta_1 = 0.1$ and observed that increasing value of shear viscosity coefficient increases the value of energy density. Next in fig. 1 (b) we fix $\eta_0 = \eta_1 = 0.1$ and found that energy density decreases with increasing bulk viscosity coefficient. Figs. 2 (a) and (b) represents Hubble parameter decreases with increasing shear viscosity coefficient and increases with increasing bulk viscosity coefficient.

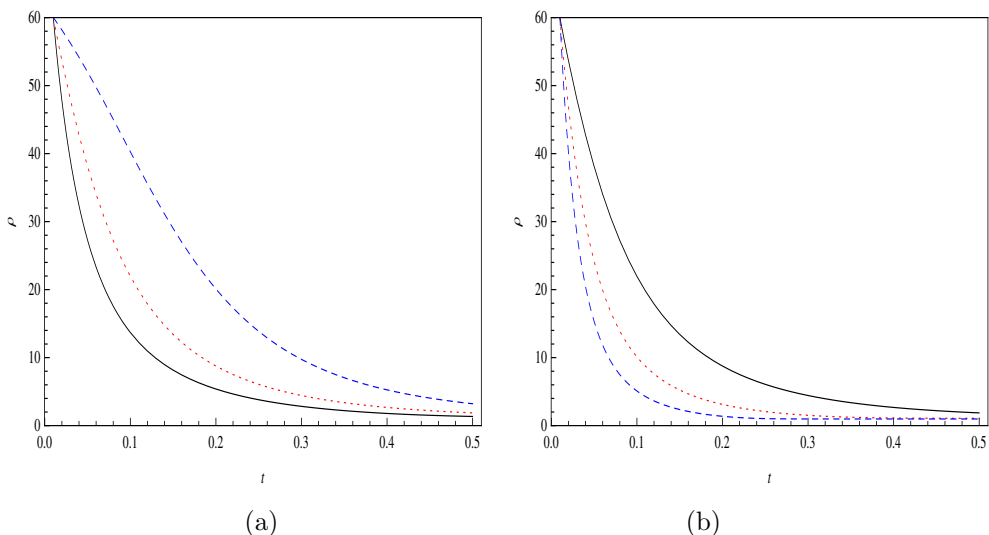


FIGURE 1. Energy density versus time (a) For $B = 3.4$, $\alpha = 0.5$, $\zeta_0 = \zeta_1 = 0.1$, $\omega = 1$, $A = 1/3$, $\beta_1 = 1$, $n = 1/2$, $m = 1$, $\eta_0 = \eta_1 = 0.01$ (solid line), $\eta_0 = \eta_1 = 0.1$ (dotted line), $\eta_0 = \eta_1 = 0.15$ (dashed line) (b) For $B = 3.4$, $\alpha = 0.5$, $\eta_0 = \eta_1 = 0.1$, $\omega = 1$, $A = 1/3$, $\beta_1 = 1$, $n = 1/2$, $m = 1$, $\zeta_0 = \zeta_1 = 0.1$ (solid line), $\zeta_0 = \zeta_1 = 0.5$ (dotted line), $\zeta_0 = \zeta_1 = 1$ (dashed line).

In order to study effect of bulk and shear viscosity on other cosmological parameters we have simplified Eq. (17) by using Binomial expansion as

$$(20) \quad p = A\rho - \frac{1}{\rho^\alpha} \left[\frac{B}{1+\omega} - 1 + \left\{ 1 - \omega \left(\rho^{1+\alpha} - \frac{B}{1+\omega} \right) \right\} \right] - 3H(\zeta_0 + \zeta_1 \rho^n) + 2H(\eta_0 + \eta_1 \rho^m),$$

which can be rewritten as

$$(21) \quad p = (A + \omega)\rho - B\rho^{-\alpha} - 3H(\zeta_0 + \zeta_1 \rho^n) + 2H(\eta_0 + \eta_1 \rho^m).$$

By using Eqs. (12) and (21) in conservation Eq. (14) we get

$$(22) \quad \dot{\rho} + 4(1 + A + \omega) \sqrt{\frac{1 + \beta_1}{6}} \rho^{\frac{3}{2}} - 4B \sqrt{\frac{1 + \beta_1}{6}} \rho^{-\alpha + \frac{1}{2}} + (8\eta_0 - 12\zeta_0) \left(\frac{1 + \beta_1}{6} \right) \rho - 12\zeta_1 \left(\frac{1 + \beta_1}{6} \right) \rho^{n+1} + 8\eta_1 \left(\frac{1 + \beta_1}{6} \right) \rho^{m+1} = 0.$$

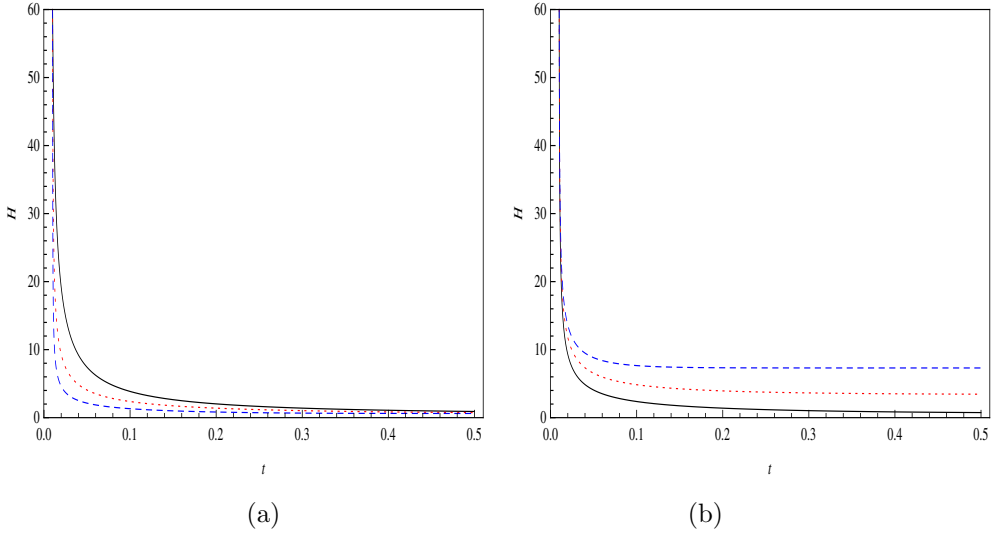


FIGURE 2. Hubble parameter versus time (a) For $B = 3.4$, $\alpha = 0.5$, $\zeta_0 = \zeta_1 = 0.1$, $\omega = 1$, $A = 1/3$, $\beta_1=1$, $n = 1/2$, $m = 1$, $\eta_0 = \eta_1 = 0.01$ (solid line), $\eta_0 = \eta_1 = 0.1$ (dotted line), $\eta_0 = \eta_1 = 0.15$ (dashed line)(b) $B = 3.4$, $\alpha = 0.5$, $\eta_0 = \eta_1 = 0.1$, $\omega = 1$, $A = 1/3$, $\beta_1=1$, $n = 1/2$, $m = 1$, $\zeta_0 = \zeta_1 = 0.1$ (solid line), $\zeta_0 = \zeta_1 = 0.5$ (dotted line), $\zeta_0 = \zeta_1 = 1$ (dashed line).

It is difficult to find analytical solution of Eq. (22), hence we fix $\alpha = \frac{1}{2}$, $n = -\frac{1}{2}$ and $m = 1$.

Then above nonlinear Eq. (22) reduces to

$$\begin{aligned} \dot{\rho} + 4(1 + A + \omega)\sqrt{\frac{1 + \beta_1}{6}}\rho^{\frac{3}{2}} - 4B\sqrt{\frac{1 + \beta_1}{6}} + (8\eta_0 - 12\zeta_0)\left(\frac{1 + \beta_1}{6}\right)\rho - \\ (23) \quad 12\zeta_1\left(\frac{1 + \beta_1}{6}\right)\rho^{\frac{1}{2}} + 8\eta_1\left(\frac{1 + \beta_1}{6}\right)\rho^2 = 0. \end{aligned}$$

This can be rewritten as

$$(24) \quad \dot{\rho} + c_1\rho^{\frac{3}{2}} - c_2 + c_3\rho - c_4\rho^{\frac{1}{2}} + c_5\rho^2 = 0,$$

where,

$$c_1 = 4(1 + A + \omega)\sqrt{\frac{1 + \beta_1}{6}},$$

$$\begin{aligned}
c_2 &= 4B\sqrt{\frac{1+\beta_1}{6}}, \\
c_3 &= (8\eta_0 - 12\zeta_0) \left(\frac{1+\beta_1}{6} \right), \\
c_4 &= 12\zeta_1 \left(\frac{1+\beta_1}{6} \right), \\
c_5 &= 8\eta_1 \left(\frac{1+\beta_1}{6} \right).
\end{aligned}$$

On integrating Eq. (24) we get

$$(25) \quad t = \frac{-2X \ln(\sqrt{\rho} - X)}{4c_5 X^3 + 3c_1 X^2 + 2c_3 X - c_4},$$

where X is root of the equation

$$(26) \quad c_5 X^4 + c_1 X^3 + c_3 X^2 - c_4 X - c_2 = 0.$$

Let $P_1 = 4c_5 X^3 + 3c_1 X^2 + 2c_3 X - c_4$, then Eq. (25) reduces to

$$(27) \quad t = \frac{-2X \ln(\sqrt{\rho} - X)}{P_1},$$

by simplifying Eq. (27) we can obtain energy density as

$$(28) \quad \rho = \left(X + e^{\frac{-P_1}{2X}t} \right)^2.$$

By using Eqs. (28) in Eq. (12) and (21) we get

$$(29) \quad H = \sqrt{\frac{1+\beta_1}{6}} \left(X + e^{\frac{-P_1}{2X}t} \right),$$

$$\begin{aligned}
(30) \quad p &= (A + \omega) \left(X + e^{\frac{-P_1}{2X}t} \right)^2 - B \left(X + e^{\frac{-P_1}{2X}t} \right)^{-2\alpha} + \left\{ 2 \left[\eta_0 + \eta_1 \left(X + e^{\frac{-P_1}{2X}t} \right)^2 \right] \right. \\
&\quad \left. - 3 \left[\zeta_0 + \zeta_1 \left(X + e^{\frac{-P_1}{2X}t} \right)^{-1} \right] \right\} \sqrt{\frac{1+\beta_1}{6}} \left(X + e^{\frac{-P_1}{2X}t} \right).
\end{aligned}$$

Fig. 3 (a) and (b) shows behavior of dark energy density with respect to time for increasing values of ζ_0, ζ_1, η_0 and η_1 respectively and observed that increasing shear viscosity decreases the value of energy density while increasing bulk viscosity increase the value of energy density. In fig. 4 (a) and (b), we have plotted Hubble parameter and observed that H is decreasing function of time, but the value of viscosity plays vital role in increasing or decreasing value of H . Fig. 5 (a) and (b)

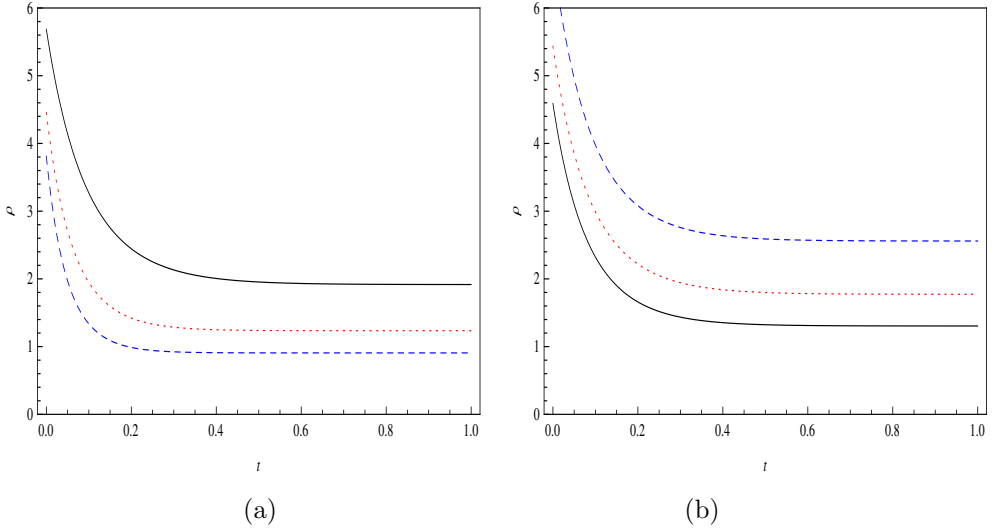


FIGURE 3. Energy Density versus time (a) For $\alpha = 0.5$, $\zeta_0 = \zeta_1 = 0.5$, $\omega = 1$, $A = 1/3$, $\beta_1=1$, $n = -1/2$, $m = 1$, $\eta_0 = \eta_1 = 0.01$ (solid line), $\eta_0 = \eta_1 = 0.2$ (dotted line), $\eta_0 = \eta_1 = 0.5$ (dashed line)(b) $\alpha = 0.5$, $\eta_0 = \eta_1 = 0.1$, $\omega = 1$, $A = 1/3$, $\beta_1=1$, $n = -0.5$, $m = 1$, $\zeta_0 = \zeta_1 = 0.1$ (solid line), $\zeta_0 = \zeta_1 = 0.5$ (dotted line), $\zeta_0 = \zeta_1 = 0.8$ (dashed line).

shows the effect of shear and bulk viscosity on pressure.

The deceleration parameter is given by,

$$(31) \quad q = -1 - \frac{\dot{H}}{H^2}.$$

On differentiating Eq. (29) we get

$$(32) \quad \dot{H} = \sqrt{\frac{1+\beta_1}{6}} \left(\frac{-P_1}{2X} \right) e^{\frac{-P_1}{2X}t}.$$

Using Eqs. (29) and (32) in (31) we get

$$(33) \quad q = -1 + \frac{P_1 e^{-\frac{P_1}{2X}t}}{2X \sqrt{\frac{1+\beta_1}{6}} \left(X + e^{\frac{-P_1}{2X}t} \right)^2}.$$

The deceleration parameter versus cosmic time is plotted in fig. 6. One can observe from fig. 6, for particular choices of unknowns we get $q = -1$ that represent Λ CDM model.

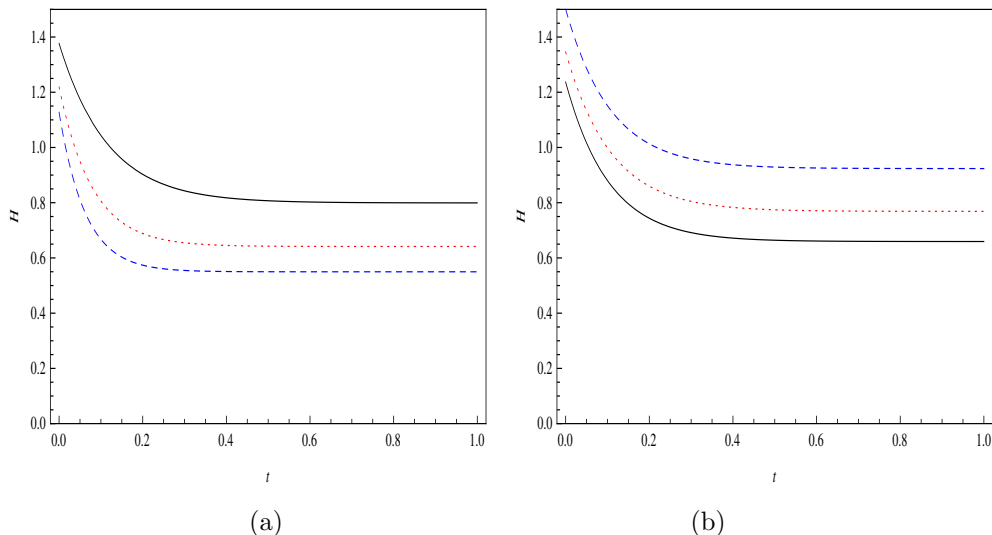


FIGURE 4. Hubble parameter versus time (a) For $\alpha = 0.5$, $\zeta_0 = \zeta_1 = 0.5$, $\omega = 1$, $A = 1/3$, $\beta_1=1$, $n = -1/2$, $m = 1$, $(\eta_0 = 0.01, \eta_1 = 0.01)$ (Black), $(\eta_0 = 0.2, \eta_1 = 0.2)$ (Red), $(\eta_0 = 0.5, \eta_1 = 0.5)$ (Blue) (b) $\alpha = 0.5$, $\eta_0 = \eta_1 = 0.1$, $\omega = 1$, $A = 1/3$, $\beta_1=1$, $n = -0.5$, $m = 1$, $\zeta_0 = \zeta_1 = 0.1$ (Black), $\zeta_0 = \zeta_1 = 0.5$ (Red), $\zeta_0 = \zeta_1 = 0.8$ (Blue).

Next we study the stability of model by calculating square of sound speed.

$$(34) \quad C_s^2 = \frac{dp}{d\rho} \geq 0.$$

On differentiating Eq. (21) for $\alpha = 1/2$, $n = -1/2$ and $m = 1$ we get

$$(35) \quad C_s^2 = A + \omega + \frac{B}{2}\rho^{-\frac{3}{2}} + \sqrt{\frac{1+\beta_1}{6}} \left[\frac{1}{2}(2\eta_0 - 3\zeta_0)\rho^{-\frac{1}{2}} + 3\eta_1\rho^{\frac{1}{2}} \right],$$

where value of ρ can be obtained from Eq. (28).

The behavior of C_s^2 is plotted in fig. 7 for increasing bulk and shear viscosity, which shows that, C_s^2 increases initially and reaches to constant value at late time. It means the VMCCG model is stable everywhere. Fig. 7 (a) and (b), shows that increase or decrease in the value of C_s^2 depends on shear and bulk viscosity.

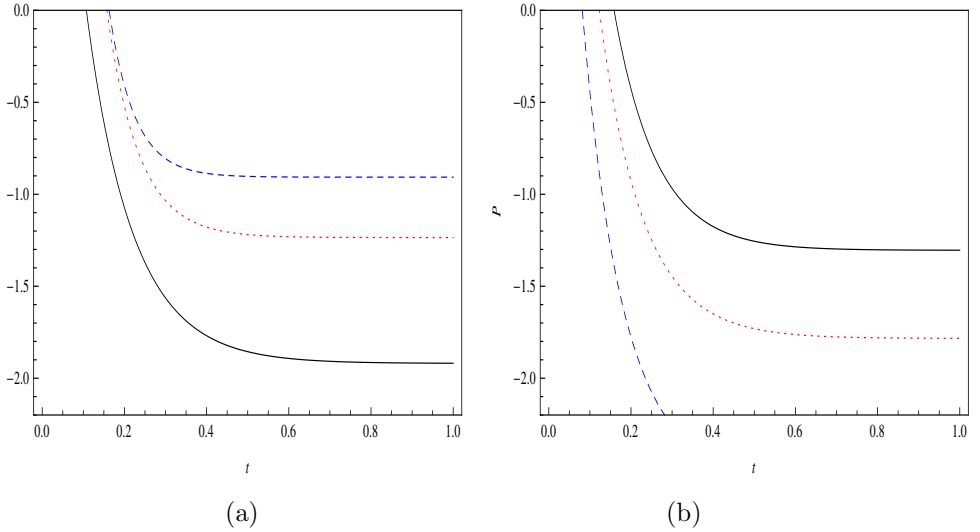


FIGURE 5. Pressure versus time (a) For $\alpha = 0.5$, $\zeta_0 = \zeta_1 = 0.5$, $\omega = 1$, $A = 1/3$, $\beta_1 = 1$, $n = -1/2$, $m = 1$, $\eta_0 = \eta_1 = 0.01$ (solid line), $\eta_0 = \eta_1 = 0.2$ (dotted line), $\eta_0 = \eta_1 = 0.5$ (dashed line) (b) $\alpha = 0.5$, $\eta_0 = \eta_1 = 0.1$, $\omega = 1$, $A = 1/3$, $\beta_1 = 1$, $n = -1/2$, $m = 1$, $\zeta_0 = \zeta_1 = 0.1$ (solid line), $\zeta_0 = \zeta_1 = 0.5$ (dotted line), $\zeta_0 = \zeta_1 = 0.8$ (dashed line).

5. DISCUSSION

In the present work we consider VMCCG with $\Lambda = \beta_1 \rho$ in the context of KK theory of gravitation. Here, we have considered bulk and shear viscosity coefficients as a linear combination which is defined in Eq. (15) and (16) and obtained conservation Eq. (18) by using field Eq. (12). By using numerical analysis we have solved Eqs. (18) and (19) and investigated the behavior of energy density and Hubble parameter, which is represented graphically for different values of ζ , η , β and ω in figs. 1 to 4. It is observed that energy density decreases with increasing ζ , β and ω and increases with increasing shear coefficient η . To obtain the solutions of non-linear differential equation we choose $\alpha = \frac{1}{2}$, $n = -\frac{1}{2}$ and $m = 1$. Therefore the Eq. (22) gives us equation as defined in Eq. (23), on integration which gives us time dependent energy density ρ . By using this ρ we have obtained other time dependent

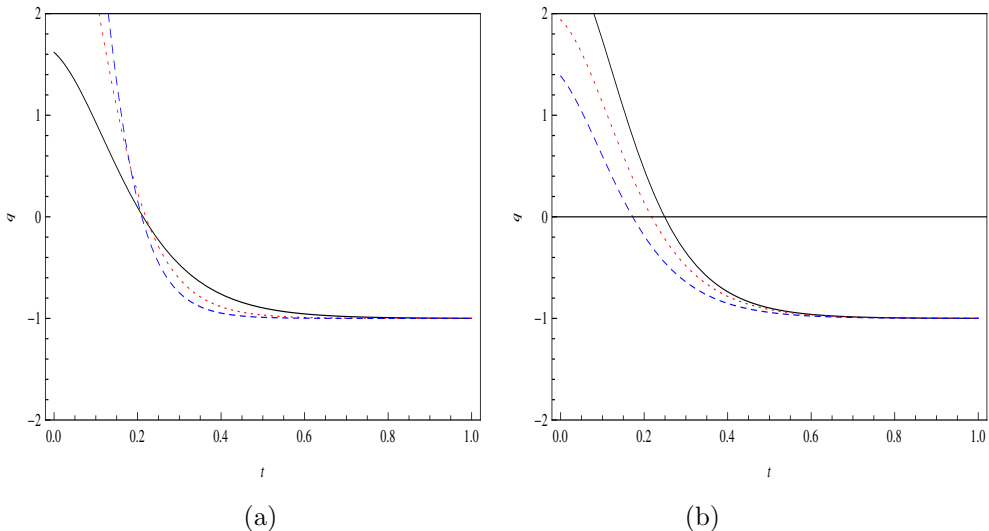


FIGURE 6. Deceleration parameter with respect to time (a) For $\alpha = 0.5$, $\zeta_0 = \zeta_1 = 0.5$, $\omega = 1$, $A = 1/3$, $\beta_1 = 1$, $n = -1/2$, $m = 1$, $(\eta_0 = 0.01, \eta_1 = 0.01)$ (Black), $(\eta_0 = 0.2, \eta_1 = 0.2)$ (Red), $(\eta_0 = 0.5, \eta_1 = 0.5)$ (Blue) (b) $\alpha = 0.5$, $\eta_0 = \eta_1 = 0.1$, $\omega = 1$, $A = 1/3$, $\beta_1 = 1$, $n = -0.5$, $m = 1$, $\zeta_0 = \zeta_1 = 0.1$ (Black), $\zeta_0 = \zeta_1 = 0.5$ (Red), $\zeta_0 = \zeta_1 = 0.8$ (Blue).

cosmological parameters and studied the effect of bulk and shear viscosity on these parameters in higher dimension.

Fig. 6 shows graphical representation of deceleration parameter (q), as expected we have obtained $q \rightarrow -1$ at the late times. It means our model represent Λ CDM model. Also we have studied stability of model and found that, it is completely stable.

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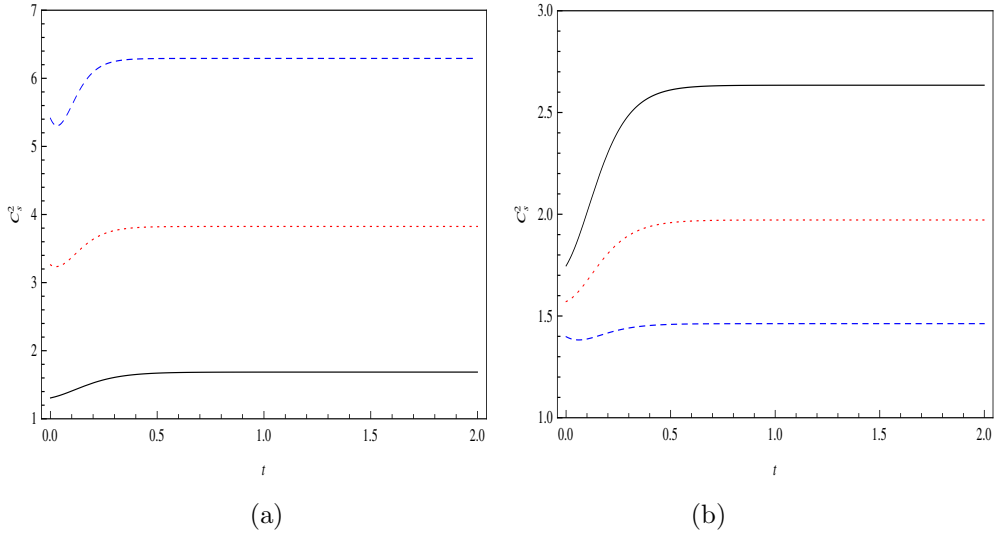


FIGURE 7. Squared speed of sound versus time (a) For $\alpha = 0.5$, $\zeta_0 = \zeta_1 = 0.5$, $\omega = 1$, $A = 1/3$, $\beta_1 = 1$, $n = -1/2$, $m = 1$, $\eta_0 = \eta_1 = 0.01$ (solid line), $\eta_0 = \eta_1 = 0.2$ (dotted line), $\eta_0 = \eta_1 = 0.5$ (dashed line) (b) $\alpha = 0.5$, $\eta_0 = \eta_1 = 0.1$, $\omega = 1$, $A = 1/3$, $\beta_1 = 1$, $n = -1/2$, $m = 1$, $\zeta_0 = \zeta_1 = 0.1$ (solid line), $\zeta_0 = \zeta_1 = 0.5$ (dotted line), $\zeta_0 = \zeta_1 = 0.8$ (dashed line).

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