

STUDY OF FRW TYPE KALUZA-KLEIN DOMAIN WALL COSMOLOGICAL MODELS IN THE PRESENCE OF MASSIVE SCALAR FIELD IN A MODIFIED GRAVITY

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ABSTRACT. This article examines the physical behaviour of the Kaluza-Klein FRW type space-time with $f(R, T)$ gravity proposed by Harko et al. (Phys. Rev. D 84:024020, 2011) in the presence of a massive scalar meson field. Here, "R" stands for the Ricci tensor, while "T" stands for the energy-momentum tensor's trace. We employed the (i) hybrid expansion law (Akarsu et al. JCAP, 01:022, 2014), (ii) the variable deceleration parameter (Mishra et al. Int. J. Theor. Phys. 52:2546, 2013), and (iii) the linearly varying deceleration parameter (Akarsu and Dereli. Int. J. Theor. Phys. 51:612, 2012) to solve the field equations. Additionally, we looked at some of the model's physical and kinematic characteristics. Our models have been found to show a smooth transition from the universe's early slowdown to its current acceleration, and the current value of the deceleration parameter is found to be reasonably consistent with the observational data. The early stages of the universe's evolution will be better understood through the study of domain wall cosmological models.

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1. INTRODUCTION

It is generally known that redshift supernova investigations have supported the notion that the expansion of the universe is accelerating (Riess et al. [1], Perlmutter et al. [2]). The universe's late-time accelerated expansion is thought to be caused by a strong negative pressure known as "dark energy". However, the idea of "dark energy" has remained a mystery up until this point. According to estimates, our universe is made up of about 4% ordinary stuff, 20% dark matter, and 76% dark energy.

Even now, the gravitational phenomenon is explained best by Einstein's theory of gravitation, which is regarded as the most elegant physical theory. However, the latest scenario of the universe's accelerated expansion and dark energy cannot be fully explained by general relativity. In order to explain the universe's late time acceleration and dark energy, numerous changes to Einstein's theory of gravitation have been developed over the past two decades. The $f(R, T)$ theory of gravity proposed by Harko et al. ([3]) and the $f(R)$ gravity developed by Nojiri and Odintsov ([4]) are the two most important theories of gravity among them. These hypotheses were developed to combine early time inflation and late time acceleration. Here, we concentrate on the gravitational constant $f(R, T)$.

The gravitational Lagrangian in the $f(R, T)$ gravity formulation of Harko et al. [3] is provided by an arbitrary function of the Ricci scalar R and the trace of energy momentum tensor T . The Hilbert-Einstein action principle is used to derive the field equations. The source of $f(R, T)$ gravity is the variation of the matter stress energy tensor with respect to the metric. The Hilbert-Einstein type action principle yielded the field equations for this theory, which are provided by

$$(1) \quad S = \frac{1}{16\pi} \left[\int \{ L_m + f(R, T) \} \sqrt{-g} d^4x \right]$$

where L_m is matter Lagrangian density. The stress energy tensor of matter is defined

$$(2) \quad T_{ij} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}L_m)}{\delta g_{ij}}$$

and its trace by $T = g^{ij}T_{ij}$ respectively. There are numerous theoretical models for $f(R, T)$ gravity that differ depending on the source of matter that corresponds to each option of $f(R, T)$.

Now assuming

$$(3) \quad f(R, T) = R + 2f(T)$$

after changing the gravitational field's action S , we arrive at the field equations for this theory as

$$(4) \quad R_{ij} - \frac{1}{2}g_{ij}R = 8\pi T_{ij} + 2f'(T)T_{ij} + [2pf'(T) + f(T)]g_{ij}$$

where $f'(T)$ indicates differentiation with respect to T .

The internal dynamics of the universe remain a mystery to this day. Phase transition and symmetry breaking occur in the early phases of the universe's development, or right after the big bang. Topologically stable defects like cosmic strings, domain walls, and monopoles appear as a result of this process (Kibble [5]). Because they aid in the study of the origin of the universe's large-scale structures, such as galaxies, cosmic strings and domain walls have drawn the attention of numerous researchers Hill et al. [6]. In fact, it is claimed that domain walls created during phase transition following the time of recombination and matter radiation are what cause galaxies to emerge. Several aspects of domain walls have been studied by many authors like Vilenkin [7], Wang [8], Rahaman and Bera [9].

Several researchers have recently looked towards domain wall cosmological models in $f(R, T)$ gravity. Some of them that apply to this discussion will be mentioned. In $f(R, T)$ gravity, Biswal et al. [10] have obtained Kaluza-Klein domain walls. With this theory, non-static plane symmetric domain walls were studied by Shaikh and Wankhede [11], while Bishi et al. [12] looked into domain walls and dark matter cosmological models in $f(R, T) = R + \alpha R^2 + \lambda T$ gravity. In $f(R, T)$ gravity, Katore and Hatkar [13] studied the Bianchi type-III and Kantowski-Sachs domain wall cosmological models whereas Tiwari et al. [14] explored the transit cosmological models with domain walls.

The discussion of scalar field (SF) is important in understanding cosmology. They are crucial to the investigation of general relativity's dark energy hypotheses. Many experts in this field have been drawn to the study of scalar meson fields in general relativity in particular. Particularly remarkable in recent years is the study of large and massless scalar fields combined with gravitational fields. Studying dark energy (DE) models in conjunction with scalar meson fields is also very important. There are two types of scalar meson fields. Massive and zero mass scalar fields are also possible. While huge scalar meson fields describe short-range interactions, massless scalar fields describe long-range interactions. In the context of the anisotropic backdrop in the literature, several researchers have examined DE models with SFs [15]-[31]. Naidu [32] has discussed Bianchi type-*II* modified holographic Ricci DE model in the presence of massive SF. Bianchi type-*V* DE models in the presence of scalar meson fields have been explored by Naidu et al. [33]. Kantowski-Sachs DE model was investigated by Reddy et al. [34] in the presence of the scalar-meson field. Aditya et al. [35] explored the Kaluza-Klein DE model in the Lyra manifold in the presence of large SFs. Aditya and Reddy [36] discussed the Binachi type-*III* model in the presence of massive SF in $f(R, T)$ gravity. Daniel Raju et al. [37]-[39] have looked into a number of aspects of anisotropic DE models with massive SF. While Naidu et al. [41]-[42] studied the Bianchi type *I* and KaluzaKlien cosmological model in the presence of perfect fluid and an attractive massive SF, Aditya et al. [40] investigated the Binachi type $V I_0$ DE model with SF. In their research on dark energy and SF models, Naidu et al. [43], Aditya et al. [44]-[45], and Rao et al. [46]-[47] presented the observational constraints.

We explore FRW type Kaluza-Klein domain wall cosmological models in conjunction with attractive massive scalar meson fields since they are important and have been the subject of several investigations. Five dimensions are represented by this model. This kind of anisotropic backdrop will be useful in models that attempt to explain the early universe (Witten [48]). The study is divided into the following sections: Section 2 discusses the metric and field equations. The solution of the field equations corresponding to domain wall models and their properties are presented in section 3. We physically discuss the models in section 4 using a graphical depiction. Some few results are discussed in the last section.

2. METRIC AND FIELD EQUATIONS

In this paper, we take a glance at the FRW type Kaluza-Klein five-dimensional anisotropic metric in the form

$$(5) \quad ds^2 = dt^2 - S^2(t) \left\{ \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2\theta d\varphi^2) + (1 - kr^2) d\Psi^2 \right\}$$

where $S(t)$ is the model's five-dimensional average scale factor and $k = -1, 0, +1$ (curvature parameters) for open, flat, and closed models.

The energy-momentum tensor for this domain walls can be written as

$$(6) \quad T_{ij} = \rho (g_{ij} + w_i w_j) + p w_i w_j, \quad w^i w_i = -1$$

where w_i is a unit space-like vector in the same direction, p is the pressure in the direction normal to the plane of the wall, and ρ is the energy density of the wall (Rahaman and Bera [9]). Now in the co moving coordinate system we have form

$$(7) \quad T_1^1 = T_2^2 = T_3^3 = T_4^4 = \rho, \quad T_0^0 = -p, \quad T_j^i = 0, \quad i \neq j$$

where the quantities ρ and p depend on t only. The energy-momentum tensor for the scalar field is

$$(8) \quad T_{ij}^{SF} = \phi_{,i} \phi_{,j} - \frac{1}{2} \left(\phi_{,k} \phi'^k - M^2 \phi^2 \right)$$

where ϕ is the massive SF and M is mass of the SF. The massive SF satisfies the Klein-Gordon equation

$$(9) \quad g^{ij} \phi_{;ij} + M^2 \phi = 0.$$

The field equations for the metric (5) now assume the form with the aid of equations (6) and (7)

$$(10) \quad 3 \left(\frac{\ddot{S}}{S} + \frac{\dot{k}}{S^2} + \frac{\dot{S}^2}{S^2} \right) = -\frac{8\pi}{\lambda_1} \left(\rho - \frac{1}{2} \dot{\phi}^2 + M^2 \phi^2 \right) - \frac{\lambda_2}{\lambda_1} \left(3\rho + \frac{p}{2} + \frac{7}{2} M^2 \phi^2 - \frac{5}{4} \dot{\phi}^2 \right)$$

$$(11) \quad 6 \left(\frac{\dot{k}}{S^2} + \frac{\dot{S}^2}{S^2} \right) = -\frac{8\pi}{\lambda_1} \left(-p + \frac{1}{2} \dot{\phi}^2 + M^2 \phi^2 \right) - \frac{\lambda_2}{\lambda_1} \left(-\frac{p}{2} + 2\rho + \frac{7}{2} M^2 \phi^2 - \frac{1}{4} \dot{\phi}^2 \right)$$

where differentiation with respect to cosmic time t is indicated by an overhead dot.

3. SOLUTION OF FIELD EQUATIONS AND THE MODELS

Now, Eqs. (10)-(11) is a system of two independent equations with four unknowns: $S(t)$, ρ , p , and ϕ . Consequently, we apply the following physically plausible conditions to arrive at a definitive solution

- A power-law relation between scalar field ϕ and average scale factor $S(t)$ of the model is given by (Johri and Sudharsan [49]; Johri and Desikan [50])

$$(12) \quad \phi \propto [S(t)]^n$$

where n is a power index. Here, we explore the relationship between the scalar field and the metric potentials in order to reduce the system's mathematical complexity

$$(13) \quad \phi = \phi_0 [S(t)]^n.$$

This is clearly a consequence of Eq. (12). Numerous authors have thought about the above relationship when studying anisotropic cosmological models with massive scalar fields.

- Many studies have used a constant deceleration parameter to get the model's solutions, which provide a power law for the metric potentials. A positive value of the deceleration parameter represents the early decelerated phase of the universe, whereas a negative value of the deceleration parameter represents the accelerated phase. The universe is not only expanding, but also accelerating right now and slowing down in the past, according to recent measurements of Type Ia supernovae and CMB anisotropies. As a result, the signature of the deceleration parameter must adjust to accommodate for this change. The deceleration parameter thus becomes a time variable rather than a constant. This leads us to make the following average scale factor assumptions, which results in a deceleration parameter that depends on the time.

MODEL-1:

We consider the average scale factor to be (Akarsu et al. [51])

$$(14) \quad S(t) = \exp(\alpha_2 t) t^{\alpha_1}$$

We note that (14) leads to the exponential law cosmology for $\alpha_1 = 0$ and to the power law cosmology for $\alpha_2 = 0$. As a result, particular instances of hybrid expansion-law cosmologies include power-law and exponential-law cosmologies. With this choice of average scale factor, a time-dependent deceleration parameter is produced. With a change from deceleration to acceleration, eras dominated by inflation and radiation are expected. Recently, a number of researchers studied DE cosmological models of the Bianchi type with a hybrid expansion law. From Eqs. (13) and (14), we get the massive SF as

$$(15) \quad \phi = \phi_0 [t^{\alpha_1} \exp(\alpha_2 t)]^n$$

Using Eq. (14) in Eq. (5) we obtain the following model:

$$(16) \quad ds^2 = dt^2 - [t^{\alpha_1} \exp(\alpha_2 t)]^2 \left[\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) + (1 - kr^2) d\Psi^2 \right]$$

with the following dynamical parameters, which are crucial for a physical description of the model, equation (16) represents the FRW type Kaluza-Klein domain wall cosmological model in the presence of massive SFs in $f(R, T)$ gravity.

Spatial volume is

$$(17) \quad V = (t^{\alpha_1} \exp(\alpha_2 t))^4$$

The scalar expansion (θ) and average Hubble's parameter (H) is

$$(18) \quad \theta = 3H = \frac{\dot{S}}{S} = 3 \left(\frac{\alpha_1}{t} + \alpha_2 \right)$$

The deceleration parameter is

$$(19) \quad q = -1 + \frac{\alpha_1}{(\alpha_1 + \alpha_2 t)^2}$$

From Eqs. (10), (11), (14) and (15), we get the pressure and energy density of domain wall as

$$\begin{aligned}
p = & \left\{ -2 \frac{\lambda_2}{\lambda_1} \left\{ \frac{3}{t^{\alpha_1} e^{\alpha_2} t} \left(\frac{t^{\alpha_1} \alpha_1^2 e^{\alpha_2} t}{t^2} - \frac{t^{\alpha_1} \alpha_1 e^{\alpha_2} t}{t^2} + 2 \frac{t^{\alpha_1} \alpha_1 e^{\alpha_2} t \alpha_2}{t} + t^{\alpha_1} e^{\alpha_2} t \alpha_2^2 \right) \right. \right. \\
& + \frac{3}{(t^{\alpha_1})^2 (e^{\alpha_2} t)^2} \left(\frac{t^{\alpha_1} \alpha_1 e^{\alpha_2} t}{t} + t^{\alpha_1} e^{\alpha_2} t \alpha_2 \right)^2 + \frac{3k}{(t^{\alpha_1})^2 (e^{\alpha_2} t)^2} \\
& + \frac{8\pi}{\lambda_1} \left\{ \frac{-\phi_0^2 ((t^{\alpha_1} e^{\alpha_2} t)^n)^2 n^2}{2(t^{\alpha_1})^2 (e^{\alpha_2} t)^2} \left(\frac{t^{\alpha_1} \alpha_1 e^{\alpha_2} t}{t} + t^{\alpha_1} e^{\alpha_2} t \alpha_2 \right)^2 \right. \\
& + M^2 \phi_0^2 ((t^{\alpha_1} e^{\alpha_2} t)^n)^2 \left. \right\} + \frac{\lambda_2}{\lambda_1} \left\{ \frac{7}{2} M^2 \phi_0^2 ((t^{\alpha_1} e^{\alpha_2} t)^n)^2 - \frac{5}{4} \frac{\phi_0^2 ((t^{\alpha_1} e^{\alpha_2} t)^n)^2 n^2}{(t^{\alpha_1})^2 (e^{\alpha_2} t)^2} \right. \\
& \times \left(\frac{t^{\alpha_1} \alpha_1 e^{\alpha_2} t}{t} + t^{\alpha_1} e^{\alpha_2} t \alpha_2 \right)^2 \left. \right\} + \left(8 \frac{\pi}{\lambda_1} + 3 \frac{\lambda_2}{\lambda_1} \right) \left\{ \frac{6}{t^{\alpha_1} e^{\alpha_2} t} \left\{ \frac{t^{\alpha_1} \alpha_1^2 e^{\alpha_2} t}{t^2} \right. \right. \\
& - \frac{t^{\alpha_1} \alpha_1 e^{\alpha_2} t}{t^2} + 2 \frac{t^{\alpha_1} \alpha_1 e^{\alpha_2} t \alpha_2}{t} + t^{\alpha_1} e^{\alpha_2} t \alpha_2^2 \left. \right\} + \frac{6k}{(t^{\alpha_1})^2 (e^{\alpha_2} t)^2} \\
& + \frac{8\pi}{\lambda_1} \left(\frac{\phi_0^2 ((t^{\alpha_1} e^{\alpha_2} t)^n)^2 n^2}{2(t^{\alpha_1})^2 (e^{\alpha_2} t)^2} \left(\frac{t^{\alpha_1} \alpha_1 e^{\alpha_2} t}{t} + t^{\alpha_1} e^{\alpha_2} t \alpha_2 \right)^2 + \frac{M^2 \phi_0^2}{(t^{\alpha_1} e^{\alpha_2} t)^{-2n}} \right) \\
& + \frac{\lambda_2}{\lambda_1} \left\{ \frac{7}{2} M^2 \phi_0^2 ((t^{\alpha_1} e^{\alpha_2} t)^n)^2 - \frac{\phi_0^2 ((t^{\alpha_1} e^{\alpha_2} t)^n)^2 n^2}{4(t^{\alpha_1})^2 (e^{\alpha_2} t)^2} \right. \\
& \left. \left(\frac{t^{\alpha_1} \alpha_1 e^{\alpha_2} t}{t} + t^{\alpha_1} e^{\alpha_2} t \alpha_2 \right)^2 \right\} \left. \right\} \times \left(\left(8 \frac{\pi}{\lambda_1} + 3 \frac{\lambda_2}{\lambda_1} \right) \left(8 \frac{\pi}{\lambda_1} + \frac{\lambda_2}{2\lambda_1} \right) + 2 \frac{\lambda_2^2}{\lambda_1^2} \right)^{-1}
\end{aligned}$$

(20)

$$\begin{aligned}
\rho = & -\left(8\frac{\pi}{\lambda_1} + \frac{\lambda_2}{2\lambda_1}\right) \left\{ \frac{3}{t^{\alpha_1} e^{\alpha_2} t} \left\{ \frac{t^{\alpha_1} \alpha_1^2 e^{\alpha_2} t}{t^2} - \frac{t^{\alpha_1} \alpha_1 e^{\alpha_2} t}{t^2} \right. \right. \\
& + 2 \frac{t^{\alpha_1} \alpha_1 e^{\alpha_2} t \alpha_2}{t} + t^{\alpha_1} e^{\alpha_2} t \alpha_2^2 \left. \right\} + \frac{3}{(t^{\alpha_1})^2 (e^{\alpha_2} t)^2} \left(\frac{t^{\alpha_1} \alpha_1 e^{\alpha_2} t}{t} + t^{\alpha_1} e^{\alpha_2} t \alpha_2 \right)^2 \\
& + \frac{3k}{(t^{\alpha_1})^2 (e^{\alpha_2} t)^2} + \frac{8\pi}{\lambda_1} \left\{ \frac{-\phi_0^2 ((t^{\alpha_1} e^{\alpha_2} t)^n)^2 n^2}{2(t^{\alpha_1})^2 (e^{\alpha_2} t)^2} \times \left(\frac{t^{\alpha_1} \alpha_1 e^{\alpha_2} t}{t} + t^{\alpha_1} e^{\alpha_2} t \alpha_2 \right)^2 \right. \\
& + M^2 \phi_0^2 ((t^{\alpha_1} e^{\alpha_2} t)^n)^2 \left. \right\} + \frac{\lambda_2}{\lambda_1} \left\{ \frac{7}{2} M^2 \phi_0^2 ((t^{\alpha_1} e^{\alpha_2} t)^n)^2 - \frac{5\phi_0^2 ((t^{\alpha_1} e^{\alpha_2} t)^n)^2 n^2}{4(t^{\alpha_1})^2 (e^{\alpha_2} t)^2} \right. \\
& \times \left(\frac{t^{\alpha_1} \alpha_1 e^{\alpha_2} t}{t} + t^{\alpha_1} e^{\alpha_2} t \alpha_2 \right)^2 \left. \right\} - \frac{\lambda_2}{\lambda_1} \left\{ \frac{6}{t^{\alpha_1} e^{\alpha_2} t} \left\{ \frac{t^{\alpha_1} \alpha_1^2 e^{\alpha_2} t}{t^2} - \frac{t^{\alpha_1} \alpha_1 e^{\alpha_2} t}{t^2} \right. \right. \\
& + 2 \frac{t^{\alpha_1} \alpha_1 e^{\alpha_2} t \alpha_2}{t} + t^{\alpha_1} e^{\alpha_2} t \alpha_2^2 \left. \right\} + \frac{6k}{(t^{\alpha_1})^2 (e^{\alpha_2} t)^2} + \frac{8\pi}{\lambda_1} \left\{ \frac{\phi_0^2 ((t^{\alpha_1} e^{\alpha_2} t)^n)^2 n^2}{2(t^{\alpha_1})^2 (e^{\alpha_2} t)^2} \right. \\
& \times \left(\frac{t^{\alpha_1} \alpha_1 e^{\alpha_2} t}{t} + t^{\alpha_1} e^{\alpha_2} t \alpha_2 \right)^2 + M^2 \phi_0^2 ((t^{\alpha_1} e^{\alpha_2} t)^n)^2 \left. \right\} \\
& + \frac{\lambda_2}{\lambda_1} \left\{ \frac{7}{2} M^2 \phi_0^2 ((t^{\alpha_1} e^{\alpha_2} t)^n)^2 - \frac{1}{4} \frac{\phi_0^2 ((t^{\alpha_1} e^{\alpha_2} t)^n)^2 n^2}{(t^{\alpha_1})^2 (e^{\alpha_2} t)^2} \right. \\
& \times \left(\frac{t^{\alpha_1} \alpha_1 e^{\alpha_2} t}{t} + t^{\alpha_1} e^{\alpha_2} t \alpha_2 \right)^2 \left. \right\} \times \left(\left(8\frac{\pi}{\lambda_1} + 3\frac{\lambda_2}{\lambda_1} \right) \left(8\frac{\pi}{\lambda_1} + \frac{\lambda_2}{2\lambda_1} \right) + 2\frac{\lambda_2^2}{\lambda_1^2} \right)^{-1}
\end{aligned}$$

(21)

MODEL-2:

Here, we assume that the average scale factor proposed by Mishra et al. [52], which results in the following time-dependent deceleration parameter

$$(22) \quad S(t) = [\sinh(\beta_1 t)]^{\frac{1}{\beta_2}}.$$

Recently, several authors have discussed various models using the above mentioned average scale factor (Santhi et al. [19]; Rao and Prasanthi [53]).

From Eqs. (15) and (22), we get the scalar field as

$$(23) \quad \phi = \phi_0 [\sinh(\beta_1 t)]^{\frac{n}{\beta_2}}$$

Using Eq. (22) in Eq. (5) we obtain the following model:

$$(24) \quad ds^2 = dt^2 - [\sinh(\beta_1 t)]^{\frac{2}{\beta_2}} \left\{ \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) + (1 - kr^2) d\Psi^2 \right\}$$

In the presence of massive SFs in $f(R, T)$ gravity, the FRW type Kaluza-Klein domain wall cosmological model is described by equation (24) with the following dynamical parameters that aid in a physical description of the model.

Spatial volume is

$$(25) \quad V = [\sinh(\beta_1 t)]^{\frac{4}{\beta_2}}$$

The scalar expansion (θ) and average Hubble's parameter (H) is

$$(26) \quad \theta = 3H = \frac{3\beta_1}{\beta_2} \coth \beta_1 t$$

The deceleration parameter is

$$(27) \quad q = -1 + \beta_2 (\operatorname{sech}(\beta_1 t))^2$$

From Eqs. (10), (11), (22) and (23), we get the pressure and energy density of domain wall as

$$\begin{aligned}
p = & \left\{ -2 \frac{\lambda_2}{\lambda_1} \left\{ \frac{3}{(\sinh(\beta_1 t))^{\beta_2-1}} \left\{ \frac{(\sinh(\beta_1 t))^{\beta_2-1} \beta_1^2 (\cosh(\beta_1 t))^2}{\beta_2^2 (\sinh(\beta_1 t))^2} \right. \right. \right. \\
& + \frac{(\sinh(\beta_1 t))^{\beta_2-1} \beta_1^2}{\beta_2} - \frac{(\sinh(\beta_1 t))^{\beta_2-1} \beta_1^2 (\cosh(\beta_1 t))^2}{\beta_2 (\sinh(\beta_1 t))^2} \left. \left. \right\} + 3 \frac{\beta_1^2 (\cosh(\beta_1 t))^2}{\beta_2^2 (\sinh(\beta_1 t))^2} \right. \\
& + \frac{3k}{\left\{ (\sinh(\beta_1 t))^{\beta_2-1} \right\}^2} + 8 \frac{\pi}{\lambda_1} \left\{ -\frac{1}{2} \frac{\phi_0^2 \left\{ (\sinh(\beta_1 t))^{\beta_2-1} \right\}^{2n} n^2 \beta_1^2 (\cosh(\beta_1 t))^2}{\beta_2^2 (\sinh(\beta_1 t))^2} \right. \\
& + M^2 \phi_0^2 \left\{ (\sinh(\beta_1 t))^{\beta_2-1} \right\}^{2n} \left. \right\} + \frac{\lambda_2}{\lambda_1} \left\{ \frac{7}{2} M^2 \phi_0^2 \left\{ (\sinh(\beta_1 t))^{\beta_2-1} \right\}^{2n} \right. \\
& - \frac{5}{4} \frac{\phi_0^2 \left\{ (\sinh(\beta_1 t))^{\beta_2-1} \right\}^{2n} n^2 \beta_1^2 (\cosh(\beta_1 t))^2}{\beta_2^2 (\sinh(\beta_1 t))^2} \left. \right\} \left. \right\} + \left\{ 8 \frac{\pi}{\lambda_1} + 3 \frac{\lambda_2}{\lambda_1} \right\} \\
& \times \left\{ 6 \frac{1}{(\sinh(\beta_1 t))^{\beta_2-1}} \left\{ \frac{(\sinh(\beta_1 t))^{\beta_2-1} \beta_1^2 (\cosh(\beta_1 t))^2}{\beta_2^2 (\sinh(\beta_1 t))^2} + \frac{(\sinh(\beta_1 t))^{\beta_2-1} \beta_1^2}{\beta_2} \right. \right. \\
& - \frac{(\sinh(\beta_1 t))^{\beta_2-1} \beta_1^2 (\cosh(\beta_1 t))^2}{\beta_2 (\sinh(\beta_1 t))^2} \left. \left. \right\} + 6 \frac{k}{\left\{ (\sinh(\beta_1 t))^{\beta_2-1} \right\}^2} \right. \\
& + 8 \frac{\pi}{\lambda_1} \left\{ \frac{1}{2} \frac{\phi_0^2 \left\{ (\sinh(\beta_1 t))^{\beta_2-1} \right\}^{2n} n^2 \beta_1^2 (\cosh(\beta_1 t))^2}{\beta_2^2 (\sinh(\beta_1 t))^2} \right. \\
& + M^2 \phi_0^2 \left\{ (\sinh(\beta_1 t))^{\beta_2-1} \right\}^{2n} \left. \right\} + \frac{\lambda_2}{\lambda_1} \left\{ \frac{7}{2} M^2 \phi_0^2 \left\{ (\sinh(\beta_1 t))^{\beta_2-1} \right\}^{2n} \right. \\
& - \frac{\phi_0^2 \left\{ (\sinh(\beta_1 t))^{\beta_2-1} \right\}^{2n} n^2 \beta_1^2 (\cosh(\beta_1 t))^2}{4 \beta_2^2 (\sinh(\beta_1 t))^2} \left. \right\} \left. \right\} \\
& \times \left\{ \left\{ 8 \frac{\pi}{\lambda_1} + 3 \frac{\lambda_2}{\lambda_1} \right\} \left\{ 8 \frac{\pi}{\lambda_1} + \frac{1}{2} \frac{\lambda_2}{\lambda_1} \right\} + 2 \frac{\lambda_2^2}{\lambda_1^2} \right\}^{-1}
\end{aligned}$$

(28)

$$\begin{aligned}
\rho = & \left\{ - \left\{ 8 \frac{\pi}{\lambda_1} + \frac{1}{2} \frac{\lambda_2}{\lambda_1} \right\} \left\{ 3 \frac{1}{(\sinh(\beta_1 t))^{\beta_2 - 1}} \left\{ \frac{(\sinh(\beta_1 t))^{\beta_2 - 1} \beta_1^2 (\cosh(\beta_1 t))^2}{\beta_2^2 (\sinh(\beta_1 t))^2} \right. \right. \right. \\
& + \frac{(\sinh(\beta_1 t))^{\beta_2 - 1} \beta_1^2}{\beta_2} - \left. \left. \frac{(\sinh(\beta_1 t))^{\beta_2 - 1} \beta_1^2 (\cosh(\beta_1 t))^2}{\beta_2 (\sinh(\beta_1 t))^2} \right\} + 3 \frac{\beta_1^2 (\cosh(\beta_1 t))^2}{\beta_2^2 (\sinh(\beta_1 t))^2} \right. \\
& + 3 \frac{k}{\left\{ (\sinh(\beta_1 t))^{\beta_2 - 1} \right\}^2} + 8 \frac{\pi}{\lambda_1} \left\{ - \frac{1}{2} \frac{\phi_0^2 \left\{ (\sinh(\beta_1 t))^{\beta_2 - 1} \right\}^{2n} n^2 \beta_1^2 (\cosh(\beta_1 t))^2}{\beta_2^2 (\sinh(\beta_1 t))^2} \right. \\
& + M^2 \phi_0^2 \left\{ (\sinh(\beta_1 t))^{\beta_2 - 1} \right\}^{2n} \left. \right\} + \frac{\lambda_2}{\lambda_1} \left\{ \frac{7}{2} M^2 \phi_0^2 \left\{ (\sinh(\beta_1 t))^{\beta_2 - 1} \right\}^{2n} \right. \\
& - \left. \left. \frac{5}{4} \frac{\phi_0^2 \left\{ (\sinh(\beta_1 t))^{\beta_2 - 1} \right\}^{2n} n^2 \beta_1^2 (\cosh(\beta_1 t))^2}{\beta_2^2 (\sinh(\beta_1 t))^2} \right\} \right\} \\
& - \frac{\lambda_2}{\lambda_1} \left\{ 6 \frac{1}{(\sinh(\beta_1 t))^{\beta_2 - 1}} \left\{ \frac{(\sinh(\beta_1 t))^{\beta_2 - 1} \beta_1^2 (\cosh(\beta_1 t))^2}{\beta_2^2 (\sinh(\beta_1 t))^2} + \frac{(\sinh(\beta_1 t))^{\beta_2 - 1} \beta_1^2}{\beta_2} \right. \right. \\
& - \left. \left. \frac{(\sinh(\beta_1 t))^{\beta_2 - 1} \beta_1^2 (\cosh(\beta_1 t))^2}{\beta_2 (\sinh(\beta_1 t))^2} \right\} + 6 \frac{k}{\left\{ (\sinh(\beta_1 t))^{\beta_2 - 1} \right\}^2} \right. \\
& + \frac{8\pi}{\lambda_1} \left\{ \frac{\phi_0^2 \left\{ (\sinh(\beta_1 t))^{\beta_2 - 1} \right\}^{2n} n^2 \beta_1^2 (\cosh(\beta_1 t))^2}{2\beta_2^2 (\sinh(\beta_1 t))^2} + M^2 \phi_0^2 \left\{ (\sinh(\beta_1 t))^{\frac{2n}{\beta_2}} \right\} \right\} \\
& + \frac{\lambda_2}{\lambda_1} \left\{ \frac{7}{2} M^2 \phi_0^2 \left\{ (\sinh(\beta_1 t))^{2n\beta_2 - 1} \right\} - \frac{\phi_0^2 \left\{ (\sinh(\beta_1 t))^{2n\beta_2 - 1} \right\} n^2 \beta_1^2}{4\beta_2^2 (\tanh(\beta_1 t))^2} \right\} \left. \right\} \\
& \times \left\{ \left\{ 8 \frac{\pi}{\lambda_1} + 3 \frac{\lambda_2}{\lambda_1} \right\} \left\{ 8 \frac{\pi}{\lambda_1} + \frac{1}{2} \frac{\lambda_2}{\lambda_1} \right\} + 2 \frac{\lambda_2^2}{\lambda_1^2} \right\}^{-1}
\end{aligned}$$

(29)

MODEL-3:

Taking into account an appropriate selection of integrating constants, the linearly variable deceleration parameter (Akarsu and Dereli [54]) results in

$$(30) \quad S(t) = \exp\left(\frac{2}{\gamma_2} \operatorname{arctanh}\left(\frac{\gamma_1 t}{\gamma_2} - 1\right)\right)$$

where, for simplicity, $\gamma_1 > 0$, $\gamma_2 \geq 0$. Consideration should be given to the solutions for $\gamma_1 > 0$ and $\gamma_2 > 0$ not only because they are easy, but also because they are consistent with the known universe. In this case, $\gamma_1 > 0$ indicates that we are dealing with accelerating motion. Recently, some authors have looked into alternate theories of gravitation that include cosmological models with a linearly variable deceleration parameter.

From Eqs. (13) and (30), we get the scalar field as

$$(31) \quad \phi = \phi_0 \exp\left(\frac{2n}{\gamma_2} \operatorname{arctanh}\left(\frac{\gamma_1 t}{\gamma_2} - 1\right)\right)$$

Using Eq. (30) in Eq. (5) we obtain the following model:

$$(32) \quad \begin{aligned} ds^2 = & dt^2 - \exp\left(\frac{4}{\gamma_2} \operatorname{arctanh}\left(\frac{\gamma_1 t}{\gamma_2} - 1\right)\right) \left\{ \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2\theta d\phi^2) \right. \\ & \left. + (1 - kr^2) d\Psi^2 \right\} \end{aligned}$$

Equation (32) explains the FRW type Kaluza-Klein domain wall cosmological model with the following dynamical parameters, which are crucial for the physical explanation of the model.

Spatial volume, scalar expansion (θ) and average Hubble's parameter (H) are

$$(33) \quad V = \exp\left(\frac{8}{\gamma_2} \operatorname{arctanh}\left(\frac{\gamma_1 t}{\gamma_2} - 1\right)\right)$$

$$(34) \quad \theta = 3H = \frac{6\gamma_2}{\gamma_1 t (2\gamma_2 - \gamma_1 t)}$$

The deceleration parameter is

$$(35) \quad q = -\gamma_1 t + \gamma_2 - 1$$

From Eqs. (10), (11), (30) and (31), we get the pressure and energy density of domain wall as

$$\begin{aligned}
 p = & \left\{ -2 \frac{\lambda_2}{\lambda_1} \left\{ 31 \left\{ 4 \frac{\gamma_1^2}{\gamma_2^3} e^{2 \frac{1}{\gamma_2} \operatorname{arctanh}\left(\frac{\gamma_1 t}{\gamma_2} - 1\right)} \left\{ \frac{\gamma_1 t}{\gamma_2} - 1 \right\} \left\{ - \left(\frac{\gamma_1 t}{\gamma_2} - 1 \right)^2 + 1 \right\} \right\}^{-2} \right. \right. \\
 & + 4 \frac{\gamma_1^2}{\gamma_2^4} e^{2 \frac{1}{\gamma_2} \operatorname{arctanh}\left(\frac{\gamma_1 t}{\gamma_2} - 1\right)} \left\{ - \left(\frac{\gamma_1 t}{\gamma_2} - 1 \right)^2 + 1 \right\}^{-2} \left. \left\{ e^{2 \frac{1}{\gamma_2} \operatorname{arctanh}\left(\frac{\gamma_1 t}{\gamma_2} - 1\right)} \right\}^{-1} \right. \\
 & + 12 \frac{\gamma_1^2}{\gamma_2^4} \left\{ - \left(\frac{\gamma_1 t}{\gamma_2} - 1 \right)^2 + 1 \right\}^{-2} + 3k \left\{ e^{2 \frac{1}{\gamma_2} \operatorname{arctanh}\left(\frac{\gamma_1 t}{\gamma_2} - 1\right)} \right\}^{-2} \\
 & + 8 \frac{\pi}{\lambda_1} \left\{ -2 \frac{\phi_0^2 n^2 \gamma_1^2}{\gamma_2^4} \left\{ \left\{ e^{2 \frac{1}{\gamma_2} \operatorname{arctanh}\left(\frac{\gamma_1 t}{\gamma_2} - 1\right)} \right\}^n \right\}^2 \right. \\
 & X \left\{ - \left(\frac{\gamma_1 t}{\gamma_2} - 1 \right)^2 + 1 \right\}^{-2} + M^2 \phi_0^2 \left\{ \left\{ e^{2 \frac{1}{\gamma_2} \operatorname{arctanh}\left(\frac{\gamma_1 t}{\gamma_2} - 1\right)} \right\}^n \right\}^2 \left. \right\} \\
 & + \frac{\lambda_2}{\lambda_1} \left\{ \frac{7}{2} M^2 \phi_0^2 \left\{ \left\{ e^{2 \frac{1}{\gamma_2} \operatorname{arctanh}\left(\frac{\gamma_1 t}{\gamma_2} - 1\right)} \right\}^n \right\}^2 \right. \\
 & \left. \left. - 5 \frac{\phi_0^2 n^2 \gamma_1^2}{\gamma_2^4} \left\{ \left\{ e^{2 \frac{1}{\gamma_2} \operatorname{arctanh}\left(\frac{\gamma_1 t}{\gamma_2} - 1\right)} \right\}^n \right\}^2 \left\{ - \left(\frac{\gamma_1 t}{\gamma_2} - 1 \right)^2 + 1 \right\}^{-2} \right\} \right\}
 \end{aligned}
 \tag{36}$$

$$\begin{aligned}
\rho = & \left\{ - \left(8 \frac{\pi}{\lambda_1} + \frac{1}{2} \frac{\lambda_2}{\lambda_1} \right) \left\{ 31 \left\{ 4 \frac{\gamma_1^2}{\gamma_2^3} e^{2 \frac{1}{\gamma_2} \operatorname{arctanh} \left(\frac{\gamma_1 t}{\gamma_2} - 1 \right)} \left(\frac{\gamma_1 t}{\gamma_2} - 1 \right) \left(- \left(\frac{\gamma_1 t}{\gamma_2} - 1 \right)^2 + 1 \right)^{-2} \right. \right. \right. \\
& + 4 \frac{\gamma_1^2}{\gamma_2^4} e^{2 \frac{1}{\gamma_2} \operatorname{arctanh} \left(\frac{\gamma_1 t}{\gamma_2} - 1 \right)} \left(- \left(\frac{\gamma_1 t}{\gamma_2} - 1 \right)^2 + 1 \right)^{-2} \left. \left. \left. \right\} \left(e^{2 \frac{1}{\gamma_2} \operatorname{arctanh} \left(\frac{\gamma_1 t}{\gamma_2} - 1 \right)} \right)^{-1} \right. \right. \\
& + 12 \frac{\gamma_1^2}{\gamma_2^4} \left(- \left(\frac{\gamma_1 t}{\gamma_2} - 1 \right)^2 + 1 \right)^{-2} + 3k \left(e^{2 \frac{1}{\gamma_2} \operatorname{arctanh} \left(\frac{\gamma_1 t}{\gamma_2} - 1 \right)} \right)^{-2} \\
& + 8 \frac{\pi}{\lambda_1} \left\{ - 2 \frac{\phi_0^2 n^2 \gamma_1^2}{\gamma_2^4} \left(\left(e^{2 \frac{1}{\gamma_2} \operatorname{arctanh} \left(\frac{\gamma_1 t}{\gamma_2} - 1 \right)} \right)^n \right)^2 \left(- \left(\frac{\gamma_1 t}{\gamma_2} - 1 \right)^2 + 1 \right)^{-2} \right. \\
& + M^2 \phi_0^2 \left(\left(e^{2 \frac{1}{\gamma_2} \operatorname{arctanh} \left(\frac{\gamma_1 t}{\gamma_2} - 1 \right)} \right)^n \right)^2 \left. \right\} + \frac{\lambda_2}{\lambda_1} \left\{ \frac{7}{2} M^2 \phi_0^2 \left(\left(e^{2 \frac{1}{\gamma_2} \operatorname{arctanh} \left(\frac{\gamma_1 t}{\gamma_2} - 1 \right)} \right)^n \right)^2 \right. \\
& - 5 \frac{\phi_0^2 n^2 \gamma_1^2}{\gamma_2^4} \left(\left(e^{2 \frac{1}{\gamma_2} \operatorname{arctanh} \left(\frac{\gamma_1 t}{\gamma_2} - 1 \right)} \right)^n \right)^2 \left(- \left(\frac{\gamma_1 t}{\gamma_2} - 1 \right)^2 + 1 \right)^{-2} \left. \right\} \\
& - \frac{\lambda_2}{\lambda_1} \left\{ 61 \left\{ 4 \frac{\gamma_1^2}{\gamma_2^3} e^{2 \frac{1}{\gamma_2} \operatorname{arctanh} \left(\frac{\gamma_1 t}{\gamma_2} - 1 \right)} \left(\frac{\gamma_1 t}{\gamma_2} - 1 \right) \left(- \left(\frac{\gamma_1 t}{\gamma_2} - 1 \right)^2 + 1 \right)^{-2} \right. \right. \\
& + 4 \frac{\gamma_1^2}{\gamma_2^4} e^{2 \frac{1}{\gamma_2} \operatorname{arctanh} \left(\frac{\gamma_1 t}{\gamma_2} - 1 \right)} \left(- \left(\frac{\gamma_1 t}{\gamma_2} - 1 \right)^2 + 1 \right)^{-2} \left. \left. \right\} \left(e^{2 \frac{1}{\gamma_2} \operatorname{arctanh} \left(\frac{\gamma_1 t}{\gamma_2} - 1 \right)} \right)^{-1} \right. \\
& + 6k \left(e^{2 \frac{1}{\gamma_2} \operatorname{arctanh} \left(\frac{\gamma_1 t}{\gamma_2} - 1 \right)} \right)^{-2} + 8 \frac{\pi}{\lambda_1} \left\{ 2 \frac{\phi_0^2 n^2 \gamma_1^2}{\gamma_2^4} \left(\left(e^{2 \frac{1}{\gamma_2} \operatorname{arctanh} \left(\frac{\gamma_1 t}{\gamma_2} - 1 \right)} \right)^n \right)^2 \right. \\
& \times \left(- \left(\frac{\gamma_1 t}{\gamma_2} - 1 \right)^2 + 1 \right)^{-2} + M^2 \phi_0^2 \left(\left(e^{2 \frac{1}{\gamma_2} \operatorname{arctanh} \left(\frac{\gamma_1 t}{\gamma_2} - 1 \right)} \right)^n \right)^2 \left. \right\} \\
& + \frac{\lambda_2}{\lambda_1} \left\{ \frac{7}{2} M^2 \phi_0^2 \left(\left(e^{2 \frac{1}{\gamma_2} \operatorname{arctanh} \left(\frac{\gamma_1 t}{\gamma_2} - 1 \right)} \right)^n \right)^2 - \frac{\phi_0^2 n^2 \gamma_1^2}{\gamma_2^4} \left(\left(e^{2 \frac{1}{\gamma_2} \operatorname{arctanh} \left(\frac{\gamma_1 t}{\gamma_2} - 1 \right)} \right)^n \right)^2 \right. \\
& \left. \left. \left(- \left(\frac{\gamma_1 t}{\gamma_2} - 1 \right)^2 + 1 \right)^{-2} \right\} \right\} \times \left(\left(8 \frac{\pi}{\lambda_1} + 3 \frac{\lambda_2}{\lambda_1} \right) \left(8 \frac{\pi}{\lambda_1} + \frac{1}{2} \frac{\lambda_2}{\lambda_1} \right) + 2 \frac{\lambda_2^2}{\lambda_1^2} \right)^{-1}
\end{aligned}$$

(37)

4. PHYSICAL DISCUSSION

We have presented the trajectories of the dynamical parameters in our models along with the necessary arbitrary constant values in order to illustrate how they behave.

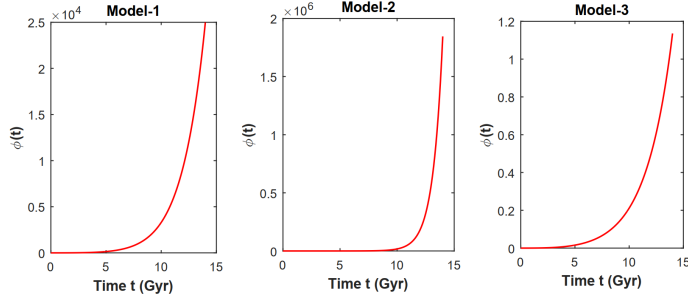


FIGURE 1. Plot of scalar field versus cosmic time for $\alpha_1 = 0.65$, $\alpha_2 = 0.09$, $\phi_0 = 1.75$, $n = 3.5$, $\beta_1 = 0.5$, $\beta_2 = 1.5$, $\gamma_1 = 0.1$ and $\gamma_2 = 1.3$.

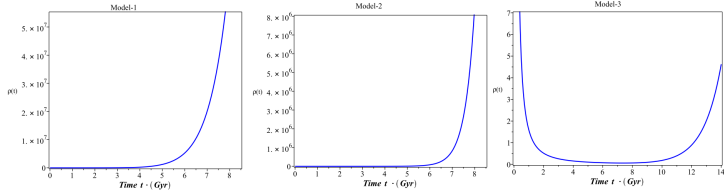


FIGURE 2. Plot of energy density versus cosmic time for $\alpha_1 = 0.65$, $\alpha_2 = 0.09$, $\phi_0 = 1.75$, $n = 3.5$, $\lambda_1 = -0.1$, $\lambda_2 = 1.3$, $M = 1.85$, $k = 1$, $\beta_1 = 0.5$, $\beta_2 = 1.5$, $\gamma_1 = 0.1$ and $\gamma_2 = 1.3$.

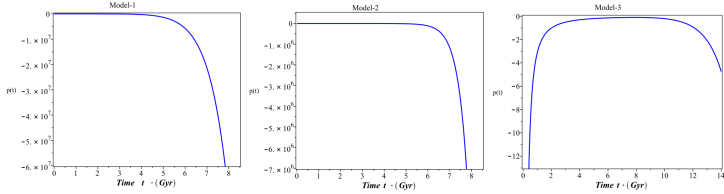


FIGURE 3. Plot of pressure versus cosmic time for $\alpha_1 = 0.65$, $\alpha_2 = 0.09$, $\phi_0 = 1.75$, $n = 3.5$, $\lambda_1 = -0.1$, $\lambda_2 = 1.3$, $M = 1.85$, $k = 1$, $\beta_1 = 0.5$, $\beta_2 = 1.5$, $\gamma_1 = 0.1$ and $\gamma_2 = 1.3$.

Scalar field variation is seen in Figure 1 as a function of cosmic time "t". It is evident that the scalar field is an increasing function of cosmic time and is always positive. The behaviour of the energy density and pressure of the domain wall with respect to cosmic time is shown in Figures 2 and 3. We notice that while pressure is a decreasing function of time, energy density is an increasing and positive function

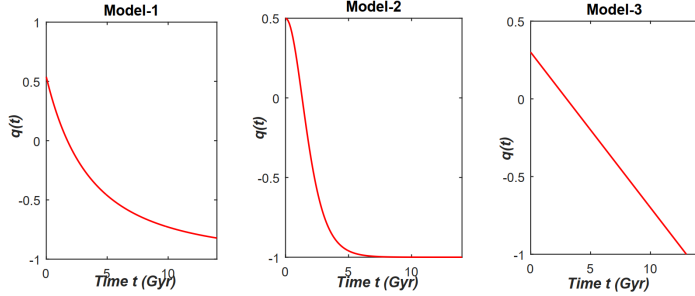


FIGURE 4. Plots of deceleration parameter versus cosmic time for $\alpha_1 = 0.65$, $\alpha_2 = 0.09$, $\beta_1 = 0.5$, $\beta_2 = 1.5$, $\gamma_1 = 0.1$ and $\gamma_2 = 1.3$.

of cosmic time. Pressure builds up over time from small negative values to large negative values. Since the discovery of the universe's accelerated expansion, it has been recognized that the dark energy type of substance with extremely strong negative pressure is to blame for the current cosmic acceleration. The behaviour of the deceleration parameter in terms of cosmic time for each of the three models is shown in Figure 4. From the early decelerated ($q > 0$) phase to the current accelerating ($q < 0$) phase, it can be seen that all three models are changing. It is important to note that all open ($k = -1$), closed ($k = +1$), and flat ($k = 0$) models of the universe exhibit very comparable parameter behaviour.

5. CONCLUSION

The FRW type Kaluza-Klein domain wall cosmological models in $f(R, T)$ theory of gravity are examined in this study in the context of massive scalar fields. Using the hybrid expansion law of the average scale factor and modifying deceleration parameters, three different models have been built. It is evident that none of the three models' spatial volumes vanish at the beginning epoch, proving that there is no point-type singularity at $t=0$. All of the physical and kinematical parameters that belong to each model have been identified, and it has been noted that they all diverge at the first epoch. As can be seen, model-1's spatial volume is constant whereas all other models' spatial volumes grow infinite as t approaches infinity. Therefore, in this particular scenario (i.e., model-1), the universe grows at a constant rate indefinitely. All three models' scalar fields are increasing functions

of cosmic time that continue to be positive. All three of the models that are discussed here show a smooth change from the universe's decelerated to accelerated phases. Additionally, the three models' current values of deceleration parameter-match the most recent observational data, which is $q = -0.930 \pm 0.218$ (BAO + Masers +TDSL+Pantheon+ H_z) [55]. Three models' domain wall pressure displays negative behaviour that is quite similar to domain wall model behaviour discovered by a number of researchers (Bishi et al. [12]; Tiwari et al. [14]). Understanding domain wall models with massive scalar fields in the $f(R, T)$ theory of gravity will be made easier by the study of the models described here.

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