

TWO FLUIDS COSMOLOGICAL MODEL IN (2+1)-DIMENSIONAL SAEZ-BALLESTER SCALAR-TENSOR THEORY OF GRAVITATION

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ABSTRACT. Two fluids cosmological models with matter and radiating source in $(2 + 1)$ –dimensional Saez-Ballester scalar-tensor theory of gravitation are investigated. In the two-fluid model, one fluid represent the CMB radiation and another fluid represent the matter content of the universe. To get determinate solution of the field equations we have consider the relation between pressure and energy density of the matter field through the gamma law equation of state $p_m = (\gamma - 1)\rho_m$. Some physical and geometric behaviour of the models are also discussed with the uses of mathematical software.

AMS Classification: 54H20.

Keywords: $(2 + 1)$ – dimensional; Two-fluid; Saez-Ballester theory of gravitation.

1. INTRODUCTION

The past two decades have witnessed that there has been a big interest in $(2+1)$ -dimensional gravity, particularly after the illustration of the fact that it's quantum version is solvable by Witten [64] and contains black-hole solutions by Bañados [1]. The study of gravitational theories in dimensions other than four has recently received a lot of attention. The explanations for this are numerous and varied; however, string theory, grand unified theory, and quantum gravity provide the primary inspiration. Factually, that Einstein's field equations exist in two space dimensions and other is one time dimension which considered as the primary explanation for focusing on the $(2+1)$ -dimensions principle of gravitation with no gravitational waves and black holes in the absence of a negative cosmological constant, the Weyl curvature is identically zero, and the theory's weak field limit does not correspond to Newtonian gravity in 2-space aspects, according to general relativity (GR) in $(2+1)$ -dimensions. The innovation of the $(2+1)$ -dimension stationary circularly symmetric black hole solution by [2, 8], which has some characteristics intrinsic in black holes $(3+1)$, sparked interest in the 1990s. While the majority of research in $(2+1)$ -gravity is focused on black hole physics, cosmology has received some attention. Giddings & Barrow [19, 3, 4], investigated some Friedmann-Robertson-Walker (FRW) models in $(2+1)$ -dimensional spacetime. Cornish and Frankel [13] for $k = -1, 0, 1$ constructed solutions for isotropic dust-filled and radiation-dominated universes in $(2+1)$ -dimensional Einstein gravity. Suppose our universe went through a spatially two-dimensional stage, determined by a $(2+1)$ -dimensional dust-filled model, the feasibility to account its current large-scale homogeneity, according to Saslaw [53]. While Garca et al. [18] expand the relationship between $(2+1)$ and $(3+1)$ -FriedmannRobertsonWalker cosmologies; linear, non-linear, and polytropic state equations and on the $(2+1)$ -dimensional FriedmannRobertsonWalker universes: an Ermakov-Pinney equation approach studied by Williams & Kevrekids [65].

Cruz and Martinez [12] derived a flat FRW model in $(2+1)$ -dimensions a field which is homogeneous scalar field in nature that is minimally coupled to gravity. Wang and Abdalla [62] investigated the cosmic holographic theory using $(2+1)$ FRW models. Khadekar [22] discovered holography in a $(2+1)$ -dimensional cosmological model with a generalised equation of state, while Khadekar and Gharad [23]

looked into $(2+1)$ -dimensional cosmological viscous models with G and Λ variables. In the FRW, Khadekar et al. [24] investigated modified Chaplygin gas with bulk viscous cosmology in $(2+1)$ -dimensional spacetime and $(2+1)$ -dimensional cosmological models in $f(R, T)$ gravity with $\Lambda(R, T)$ were investigated by Islam et al. [20].

Many new theories of gravitation have been established over the past few decades which are called alternative theory of gravitation. These gravitational theories include scalar tensor theories suggested by Brans and Dicke [7], Nordvedt [30], Wagoner [63], Rose [52], Dun [17], Saez and Ballester [54], Barber [5], Lau and Prokhovnik [25].

Gravitational theories are categorized under two types namely involving the classic scalar field ϕ which has the inverse dimension of the gravitational constant G with has the scalar field ϕ as the first type. Part of the first stage is the Brans-Decke theory [7] is of paramount importance and the function of the scalar field ϕ is limited to its impact on gravitational field equations. Brans and Decke have developed a scalar-tensor gravitational theory that includes an added scalar field ϕ in addition to the metric tensor g_{ij} and a dimensionless coupling constant ω . The theory is further extended to a general relativity for large values of the constant coupling $\omega > 500$. The second type of theories concerns a dimensionless scalar field. Saez and Ballester [54] have developed a scalar-tensor theory that combines the metric with a dimensionless scalar field in a simple way. This coupling offers a clear description of the weak fields. In spite of the dimensionless existence of the scalar field, an anti-gravity regime exists. This theory suggests a potential way to solve the problem of missing matter in non-flat FRW cosmologies. The Scalar-tensor gravitation theory plays an important role in removing the graceful exit problem in the inflation period [40].

In the earlier literature, cosmological models within the framework of Saez-Ballester scalar-tensor theory of gravitation, have been studied by Singh and Agrawal [56, 57], Ram and Tiwari [43], Singh and Ram [58]. Mohanty and Sahu [28, 29] have studied Bianchi type-VI0 and Bianchi type-I models in Saez-Ballester theory. In recent years, Tripathi et al. [61], Reddy et al. [49, 50, 51], Reddy and Naidu [48], Rao et al. [44, 45, 46], Sahu [55], Singh [59], Pradhan et al. [41, 42], Socorro

and Sabido [60] and Jamil et al. [21] have obtained the solutions in Saez-Ballester scalar-tensor theory of gravitation in different context.

Most of the researchers have studied the Friedmann-Robertson-Walker models with two fluids source [9, 32, 31, 14, 26, 10, 11]. Reddy and Kumar [47], have studied two fluid scenarios for dark energy model in a scalar-tensor theory of gravitation. Two fluids tilted cosmological model in general relativity obtained by Pawar and Dagwal [33]. Axially Bianchi type-I Mesonic cosmological models with two fluid sources in Lyra Geometry formulated by Pawar et al. [34]. According to the scalar-tensor theory of gravitation proposed by Saez and Ballester, Pawar and Dagwal [38, 36, 39] constructed tilted two fluid cosmological models with variable G and Λ in general relativity. They studied tilted Kantowski-Sachs cosmological models with disordered radiation and two fluid cosmological models in scalar-tensor theory of gravitation, while Mete et al. [27] have studied kasner cosmological models with two-fluid source in general relativity. Lately, two fluid axially symmetric cosmological models in $f(R, T)$ theory of gravitation and two-fluid cosmological models in scalar-tensor theory of gravitation presented by Pawar and Dagwal [37, 38]. Dagwal et al. [15] investigated a tilted cosmological model consisting of two types of dark energy and two fluid sources in $F(T)$ theory of gravity.

This paper has led to our investigation on two fluid cosmological models, one fluid represent CMB radiation and another fluid represent the matter. The description in this paper is envisaged as follows: The field equations under the FRW metric are briefed in section II. Section III handles with the solution of the field equations for different case as $\gamma = 1, \frac{3}{2}, \frac{5}{3}, 2$. Section IV deals with discussion & result and conclusion given in section V.

2. FRW MODEL AND THEIR FIELD EQUATIONS

We consider the Friedmann-Robertson-Walker (FRW) universe in $(2+1)$ -dimensional space-time defined as [24],

$$(1) \quad ds^2 = dt^2 - A^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 \right],$$

where, $A(t)$ denotes as the scale factor. The coordinates (t, r, θ) describe the co-moving coordinates and the constant k illustrate the curvature of the space $k = -1, 0, 1$ for open, flat and closed universe respectively.

The Einstein's field equations describe by Saez and Ballester for the combined scalar and tensor field are given below,

$$(2) \quad G_i^j - wV^h \left(V_{,i} V'^j - \frac{1}{2} g_i^j V_{,a} V'^a \right) = T_i^j,$$

and the scalar field satisfies the equation

$$(3) \quad 2V^h V_{;j}^i + hV^{h-1} V_{,a} V'^a = 0,$$

where

$$G_i^j = R_i^j - \frac{1}{2} R g_i^j,$$

is the Einstein tensor, R , R_i^j , h and w are the Ricci scalar, Ricci tensor, an arbitrary exponent and a dimensionless coupling constant respectively.

The energy momentum tensor for the two fluid sources are given by [39],

$$(4) \quad T_i^j = T_i^{(m)j} + T_i^{(r)j},$$

$$(5) \quad T_{ij}^{(m)} = (p_m + \rho_m) u_i^{(m)} u_j^{(m)} - p_m g_{ij}.$$

$$(6) \quad T_{ij}^{(r)} = \frac{3}{2} \rho_r u_i^{(r)} u_j^{(r)} - \frac{1}{2} \rho_r g_{ij}.$$

The three velocity vectors are given by $u_i^m = (1, 0, 0)$ and $u_i^r = (1, 0, 0)$ with $g^{ij} u_i^m u_j^m = 1$ and $g^{ij} u_i^r u_j^r = 1$.

The field equations (2) for metric (1) can be written (Cruz and Martinez [12]) as,

$$(7) \quad \left(\frac{\dot{A}^2 + k}{A^2} \right) + \frac{1}{2} w V^h \dot{V}^2 = -(\rho_m + \rho_r)$$

$$(8) \quad \frac{\ddot{A}}{A} - \frac{1}{2} w V^h \dot{V}^2 = \left(p_m + \frac{1}{3} \rho_r \right)$$

$$(9) \quad \ddot{V} + 2 \frac{\dot{A}}{A} \dot{V} + \frac{h}{2} \frac{\dot{V}^2}{V} = 0$$

where $'(\cdot)'$ describe derivative with respect to cosmic time $'t'$.

With the help of special law of variation of Hubbles parameter proposed by Berman

[6] that relents a constant deceleration parameter model of the universe. We consider only constant deceleration parameter model defined by [16],

$$(10) \quad q = - \left[\frac{a\ddot{a}}{\dot{a}^2} \right],$$

where

$$(11) \quad a = \left[\frac{A^2(t)r}{(1 - kr^2)^{1/2}} \right]^{\frac{1}{2}},$$

is the overall scalar factor.

Here, the constant is considered as negative to analyse if the model does or does not accelerate because of the sign of deceleration parameter. The positive sign $q(> 1)$ reacts to the decelerating model where the negative sign $(-1 < q < 0)$ denotes acceleration and $q = 0$ reacts to expansion with constant velocity.

Put Eqn. (11) into the Eqn. (10), we get

$$(12) \quad A = \left[(1 + q)(ct + d) \right]^{\frac{1}{(1+q)}},$$

where c & d are the integration constant.

This equation involves that the condition of expansion is $1 + q > 0$.

From Eqn. (9), we have the scalar field as,

$$(13) \quad V = \left[\left\{ \frac{-(h+2)c}{2} \left((1+q)(ct+d) \right)^{\frac{-1}{(1+q)}} + \frac{(h+2)c_1}{2} \right\}^{\frac{2}{(h+2)}} \right].$$

where, c_1 is the integration constant.

The standard conservation equation for matter and radiation respectively in the $(2+1)$ -dimension space-time are given as,

$$(14) \quad \dot{\rho}_m + 2 \frac{\dot{A}}{A} (\rho_m + p_m) = 0.$$

$$(15) \quad \dot{\rho}_r + 2 \frac{\dot{A}}{A} (\rho_r + p_r) = 0.$$

3. SOLUTION OF THE FIELD EQUATION

The below is the equation of state denotion of the link between pressure and energy density for matter field through the gamma law is given by [27],

$$(16) \quad p_m = (\gamma - 1)\rho_m, \quad 1 \leq \gamma \leq 2.$$

We have energy density for matter, energy density for radiation, total energy density and pressure for matter given as below.

From Eqn. (14), we have the energy density for matter as,

$$(17) \quad \rho_m = \left[c_2 \left((1+q)(ct+d) \right)^{\frac{-2\gamma}{(1+q)}} \right].$$

Similarly, from Eqn. (15), we obtained the energy density for radiation as

$$(18) \quad \rho_r = \left[c_3 \left((1+q)(ct+d) \right)^{\frac{-3}{(1+q)}} \right].$$

Total energy density from Eqn. (17) & Eqn. (18), we have

$$\rho = (\rho_m + \rho_r)$$

$$(19) \quad \rho = \left[c_4 \left\{ \left((1+q)(ct+d) \right)^{\frac{-2\gamma}{(1+q)}} + \left((1+q)(ct+d) \right)^{\frac{-3}{(1+q)}} \right\} \right].$$

where $c_2 = c_3 = c_4$ are constant of Integration.

From Eqn.(14), we obtained the pressure for matter,

$$(20) \quad p_m = \left[(\gamma - 1)c_2 \left((1+q)(ct+d) \right)^{\frac{-2\gamma}{(1+q)}} \right].$$

The Hubbles parameter are given as,

$$(21) \quad H = \left[4c \left((1+q)(ct+d) \right)^{-1} \right].$$

Scalar of expansion are define by,

$$(22) \quad \theta = u_{;i}^i = 2H = \left[8c \left((1+q)(ct+d) \right)^{-1} \right].$$

The shear scalar as,

$$(23) \quad \sigma^2 = \left[3c^2 \left((1+q)(ct+d) \right)^{-2} \right].$$

The density parameter for matter and radiation respectively are given by,

$$(24) \quad \Omega_m = \frac{\rho_m}{2H^2} = \left[\frac{1}{32c} \left((1+q)(ct+d) \right)^{\frac{2(1+q)-2\gamma}{(1+q)}} \right].$$

$$(25) \quad \Omega_r = \frac{\rho_r}{2H^2} = \left[\frac{1}{32c} \left((1+q)(ct+d) \right)^{\frac{(2q-1)}{(1+q)}} \right].$$

Total density parameter for matter and radiation are derived from Eqn. (24) & Eqn. (25) as,

$$(26) \quad \Omega = \Omega_m + \Omega_r$$

$$(26) \quad \Omega = \left[\frac{1}{32c} \left\{ \left((1+q)(ct+d) \right)^{\frac{2(1+q)-2\gamma}{(1+q)}} + \left((1+q)(ct+d) \right)^{\frac{(2q-1)}{(1+q)}} \right\} \right].$$

Case-(i): Dust model: when $\gamma = 1$ i.e $p_m = 0$.

Energy density of matter from Eqn. (17) becomes

$$(27) \quad \rho_m = \left[c_2 \left((1+q)(ct+d) \right)^{\frac{-2}{(1+q)}} \right].$$

Energy density of radiation from Eqn. (18) gives,

$$(28) \quad \rho_r = \left[c_3 \left((1+q)(ct+d) \right)^{\frac{-3}{(1+q)}} \right].$$

Total energy density obtained from Eqn. (27) & Eqn. (28) given as,

$$\rho = (\rho_m + \rho_r)$$

$$(29) \quad \rho = \left[c_4 \left\{ \left((1+q)(ct+d) \right)^{\frac{-2}{(1+q)}} + \left((1+q)(ct+d) \right)^{\frac{-3}{(1+q)}} \right\} \right].$$

We have drawn the behaviour of Hubble parameter H , the scalar of expansion θ for matter and radiation and the shear scalar σ^2 for matter and radiation with respect to cosmic time ' t ' in the Fig.(1). It is observed that the Hubble parameter H declines as the cosmic time raises and monotonically becomes constant on later time ' t ' whereas we have seen that the scalar of expansion θ for matter and radiation is declines as the cosmic time raises and approaches to an infinitesimal constant at

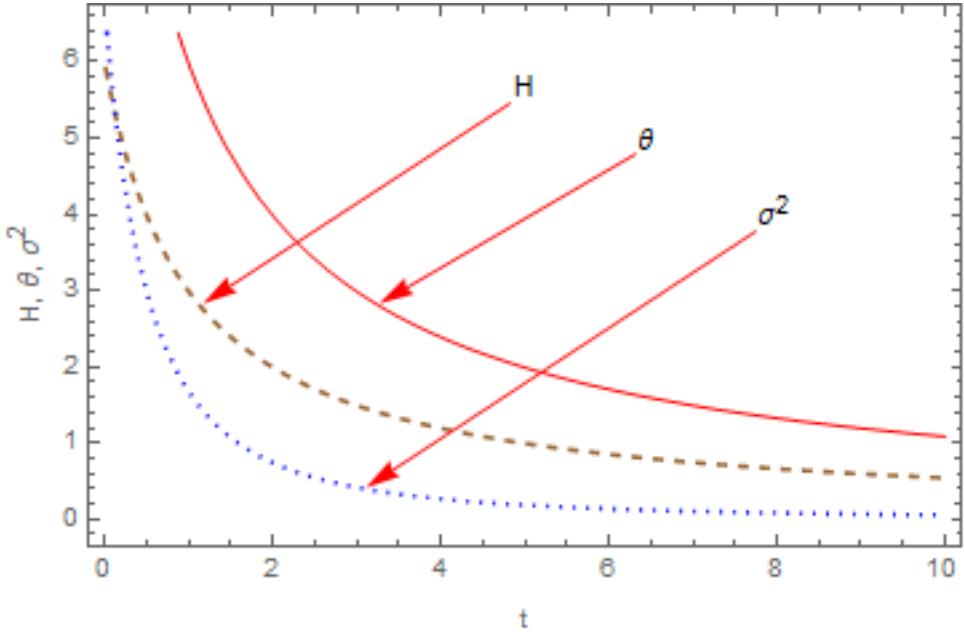


FIGURE 1. Hubble parameter H , scalar of expansion θ & the shear scalar σ^2 is described against time for the different values of the constants $c = 0.1, d = 0.1, q = -0.33, \gamma = 1$.

late time ' t' ' and the shear scalar σ^2 for matter and radiation declines as the cosmic time raises and slightly becomes constant on later time ' t' '.

The density parameter for matter and radiation are obtained from Eqn. (24) & Eqn. (25) respectively as,

$$(30) \quad \Omega_m = \left[\frac{1}{32c} \left((1+q)(ct+d) \right)^{\frac{2q}{(1+q)}} \right].$$

and

$$(31) \quad \Omega_r = \left[\frac{1}{32c} \left((1+q)(ct+d) \right)^{\frac{(2q-1)}{(1+q)}} \right].$$

Total density parameter for matter and radiation are derived from Eqn. (30) & Eqn. (31) as,

$$\Omega = \Omega_m + \Omega_r$$

$$(32) \quad \Omega = \left[\frac{1}{32c} \left\{ \left((1+q)(ct+d) \right)^{\frac{2q}{(1+q)}} + \left((1+q)(ct+d) \right)^{\frac{(2q-1)}{(1+q)}} \right\} \right].$$

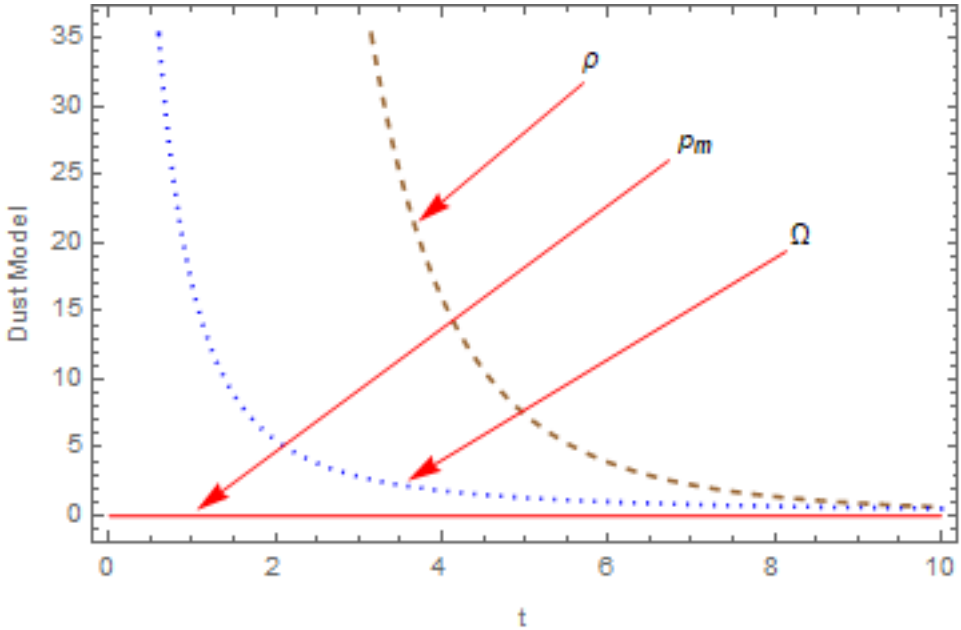


FIGURE 2. Dust model for total energy density, pressure for matter and density parameter is described against time for the values of the constants $c = 0.1, d = 0.1, q = -0.33, \gamma = 1$.

We have drawn the behaviour of pressure for matter p_m , total energy density ρ and density parameter Ω for matter and radiation with respect to cosmic time ' t ' for dust model represented in Fig.(2). It is observed that the pressure for matter is constant throughout the universe, total energy density declines as the cosmic time raises and monotonically becomes constant on later time ' t ' whereas we have seen that the density parameter for matter and radiation is declines as the cosmic time raises and approaches to an infinitesimal constant at late time ' t '.

Case-(ii): Radiation Universe: when $\gamma = \frac{3}{2}$

Energy density of matter from Eqn. (17) becomes,

$$(33) \quad \rho_m = \left[c_2 \left((1+q)(ct+d) \right)^{\frac{-3}{(1+q)}} \right].$$

Energy density of radiation from Eqn. (18) gives,

$$(34) \quad \rho_r = \left[c_3 \left((1+q)(ct+d) \right)^{\frac{-3}{(1+q)}} \right].$$

Total energy density from Eqn. (33) & Eqn. (34) are given as,

$$\rho = (\rho_m + \rho_r)$$

$$(35) \quad \rho = \left[2c_4 \left\{ \left((1+q)(ct+d) \right)^{\frac{-3}{(1+q)}} \right\} \right].$$

Pressure for matter obtained from Eqn. (20) given as,

$$(36) \quad p_m = \left[\frac{1}{2} c_2 \left\{ \left((1+q)(ct+d) \right)^{\frac{-3}{(1+q)}} \right\} \right].$$

The density parameter for matter and radiation are derived from Eqn. (24) & Eqn. (25) respectively as,

$$(37) \quad \Omega_m = \left[\frac{1}{32c} \left\{ \left((1+q)(ct+d) \right)^{\frac{(2q-1)}{(1+q)}} \right\} \right].$$

and

$$(38) \quad \Omega_r = \left[\frac{1}{32c} \left\{ \left((1+q)(ct+d) \right)^{\frac{(2q-1)}{(1+q)}} \right\} \right].$$

Total density parameter for matter and radiation are derived from Eqn. (37) & Eqn. (38) as,

$$\Omega = \Omega_m + \Omega_r$$

$$(39) \quad \Omega = \left[\frac{1}{16c} \left\{ \left((1+q)(ct+d) \right)^{\frac{(2q-1)}{(1+q)}} \right\} \right].$$

We have drawn the behaviour of pressure for matter p_m , total energy density ρ and density parameter Ω for matter and radiation with respect to cosmic time

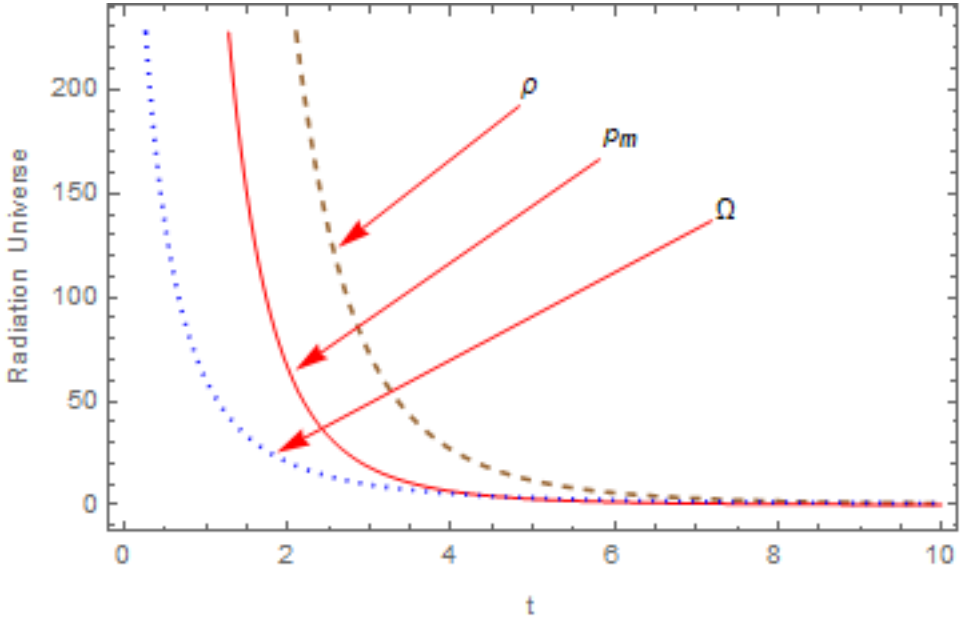


FIGURE 3. Radiation Universe for total energy density, pressure for matter and density parameter is described against time for the different values of the constants $c = 0.1, d = 0.1, q = -0.33, \gamma = \frac{3}{2}$.

' t' for radiation universe represented in Fig.(3). It is observed that the pressure for matter declines as the cosmic time raises and monotonically becomes constant on later time ' t' ' whereas we have seen that total energy density declines as the cosmic time raises and approaches to an infinitesimal constant at late time ' t' ' and the density parameter for matter and radiation is declines as the cosmic time raises and slightly becomes constant on later time ' t' '.

Case-(iii): Hard Universe: when $\gamma \in \left(\frac{4}{3}, 2\right)$, let $\gamma = \frac{5}{3}$
Energy density of matter from Eqn. (17) becomes,

$$(40) \quad \rho_m = \left[c_2 \left\{ \left((1+q)(ct+d) \right)^{\frac{-10}{3(1+q)}} \right\} \right]$$

Energy density of radiation from Eqn. (18) gives,

$$(41) \quad \rho_r = \left[c_3 \left\{ \left((1+q)(ct+d) \right)^{\frac{-5}{(1+q)}} \right\} \right].$$

Total energy density are given from Eqn. (40) & Eqn. (41) as,

$$\rho = (\rho_m + \rho_r)$$

$$(42) \quad \rho = \left[c_4 \left\{ \left((1+q)(ct+d) \right)^{\frac{-10}{3(1+q)}} + \left((1+q)(ct+d) \right)^{\frac{-5}{(1+q)}} \right\} \right].$$

Pressure for matter obtained from Eqn. (20) given as,

$$(43) \quad p_m = \left[\frac{2}{3} c_2 \left\{ \left(2(1+q)(ct+d) \right)^{\frac{-10}{3(1+q)}} \right\} \right].$$

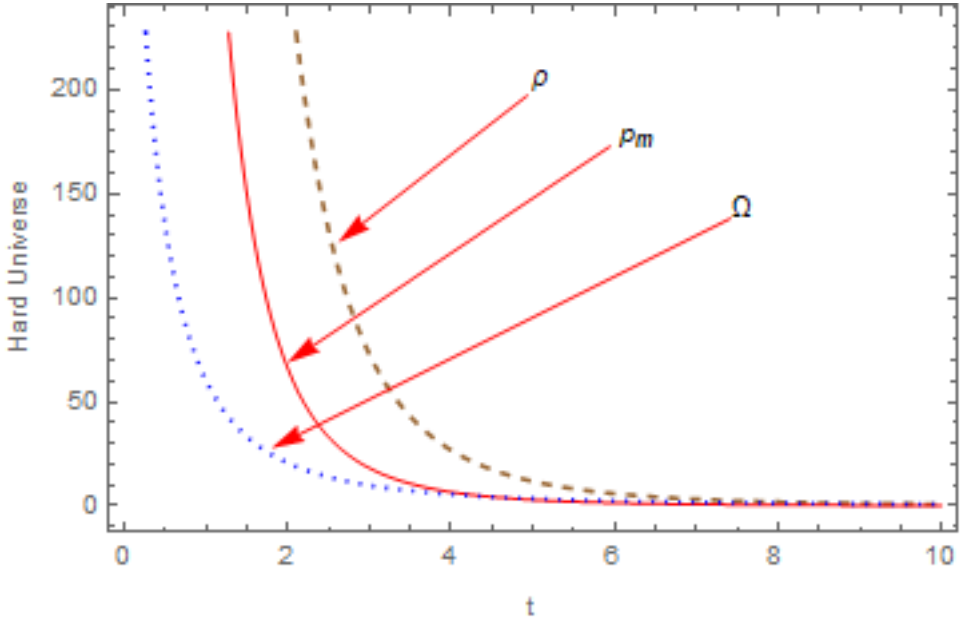


FIGURE 4. Hard Universe for total energy density, pressure for matter and density parameter is described against time for the values of the constants $c = 0.1, d = 0.1, q = -0.33, \gamma = \frac{5}{3}$.

We have drawn the behaviour of pressure for matter p_m , total energy density ρ and density parameter Ω for matter and radiation with respect to cosmic time t for hard universe in represented Fig.(4). It is observed that the pressure for matter declines as the cosmic time raises and monotonically becomes constant on later time ' t ' whereas we have seen that total energy density declines as the cosmic time

raises and approaches to an infinitesimal constant at late time t' and the density parameter for matter and radiation is declines as the cosmic time raises and slightly becomes constant on later time t' .

The density parameter for matter and radiation are given from Eqn. (24) & Eqn. (25) respectively as,

$$(44) \quad \Omega_m = \left[\frac{1}{32c} \left\{ \left((1+q)(ct+d) \right)^{\frac{6q-4}{3(1+q)}} \right\} \right].$$

and

$$(45) \quad \Omega_r = \left[\frac{1}{32c} \left\{ \left((1+q)(ct+d) \right)^{\frac{(2q-1)}{(1+q)}} \right\} \right].$$

Total density parameter for matter and radiation are derived from Eqn. (45) & Eqn. (46) as,

$$(46) \quad \Omega = \Omega_m + \Omega_r$$

$$(46) \quad \Omega = \left[\frac{1}{32c} \left\{ \left((1+q)(ct+d) \right)^{\frac{6q-4}{3(1+q)}} + \left((1+q)(ct+d) \right)^{\frac{(2q-1)}{(1+q)}} \right\} \right].$$

Case-(iv): Zeldovich Universe: when $\gamma = 2$

Energy density of matter from Eqn. (17) becomes,

$$(47) \quad \rho_m = \left[c_2 \left\{ \left((1+q)(ct+d) \right)^{\frac{-4}{(1+q)}} \right\} \right].$$

Energy density of radiation are derived from Eqn. (18) gives,

$$(48) \quad \rho_r = \left[c_3 \left\{ \left((1+q)(ct+d) \right)^{\frac{-3}{(1+q)}} \right\} \right].$$

Total energy density are derived from Eqn. (47) and Eqn. (48) as,

$$(49) \quad \rho = (\rho_m + \rho_r)$$

$$(49) \quad \rho = \left[c_4 \left\{ \left((1+q)(ct+d) \right)^{\frac{-4}{(1+q)}} + \left((1+q)(ct+d) \right)^{\frac{-3}{(1+q)}} \right\} \right].$$

Pressure for matter obtained from Eqn. (20) given as,

$$(50) \quad p_m = \left[c_2 \left\{ \left((1+q)(ct+d) \right)^{\frac{-4}{(1+q)}} \right\} \right].$$

The density parameter for matter and radiation are derived from Eqn. (24) & Eqn. (25) respectively as,

$$(51) \quad \Omega_m = \left[\frac{1}{32c} \left\{ \left((1+q)(ct+d) \right)^{\frac{2(q-1)}{(1+q)}} \right\} \right].$$

and

$$(52) \quad \Omega_r = \left[\frac{1}{32c} \left\{ \left((1+q)(ct+d) \right)^{\frac{(2q-1)}{(1+q)}} \right\} \right].$$

Total density parameter for matter and radiation are given as,

$$\Omega = \Omega_m + \Omega_r$$

$$(53) \quad \Omega = \left[\frac{1}{32c} \left\{ \left((1+q)(ct+d) \right)^{\frac{2(q-1)}{(1+q)}} + \left((1+q)(ct+d) \right)^{\frac{(2q-1)}{(1+q)}} \right\} \right].$$

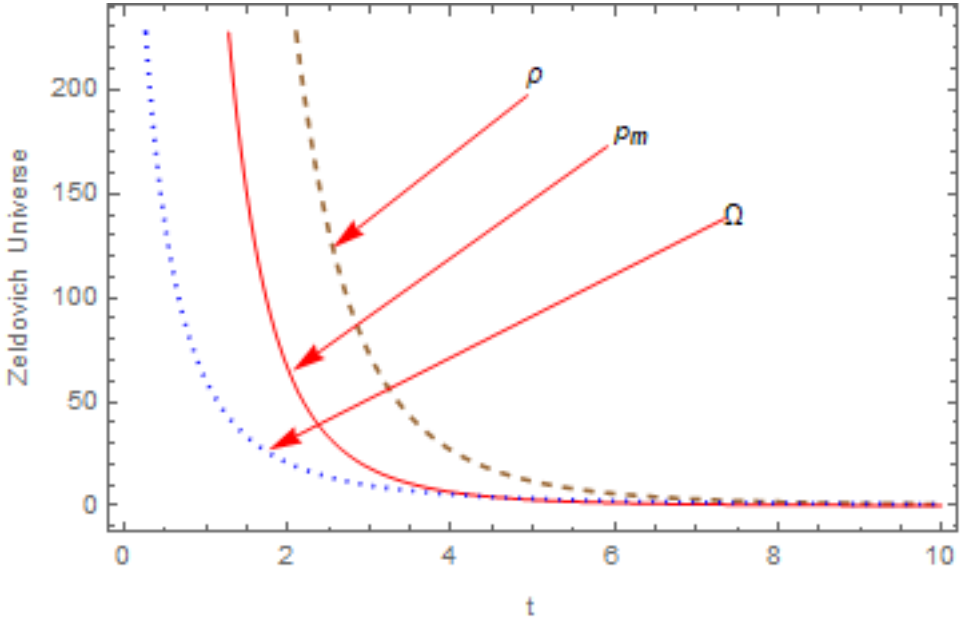


FIGURE 5. Zeldovich model for total energy density, pressure for matter and density parameter is stated against time for the different values of the constants $c = 0.1, d = 0.1, q = -0.33, \gamma = 2$.

We have concluded the behaviour of pressure for matter p_m , total energy density ρ and density parameter Ω for matter and radiation with respect to cosmic time

t for Zeldovich model in represented Fig.(5). It is identify that the pressure for matter declines as the cosmic time raises and monotonically becomes constant on later time ' t ' whereas we have seen that total energy density declines as the cosmic time raises and approaches to an infinitesimal constant at late time ' t ' and the density parameter for matter and radiation is declines as the cosmic time raises and slightly becomes constant on later time ' t '.

4. DISCUSSION AND CONCLUSION

This paper involves our investigation on two-fluid cosmological models whereas one fluid represent CMB radiation and another fluid represent the matter content of the universe and obtained the result of the field equations in $(2+1)$ -dimensional Saez-Ballester theory of gravitation. The behaviour of matter density, radiation density and pressure has been discussed.

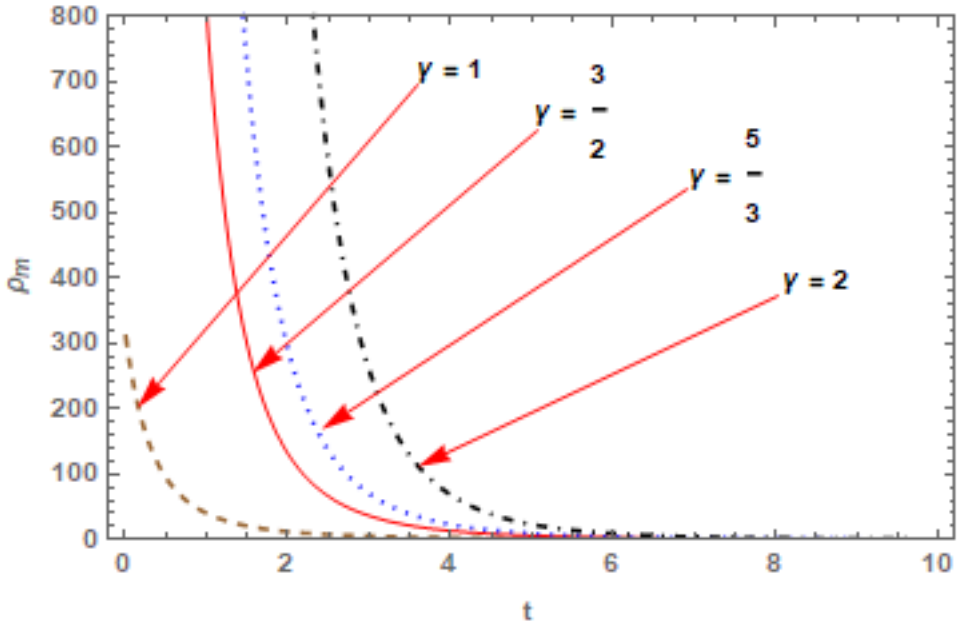


FIGURE 6. Total Energy density ρ for matter & radiation is stated against time for the different values of the constants $c = 0.1, d = 0.1, q = -0.33, \gamma = 1, \frac{3}{2}, \frac{5}{3}, 2$.

It is shown that the total energy density for matter and radiation ρ with respect to cosmic time ' t ' in Fig.(6). It shows that the total energy density declines as time raises and approaches to an infinitesimal constant at late times in all four universes.

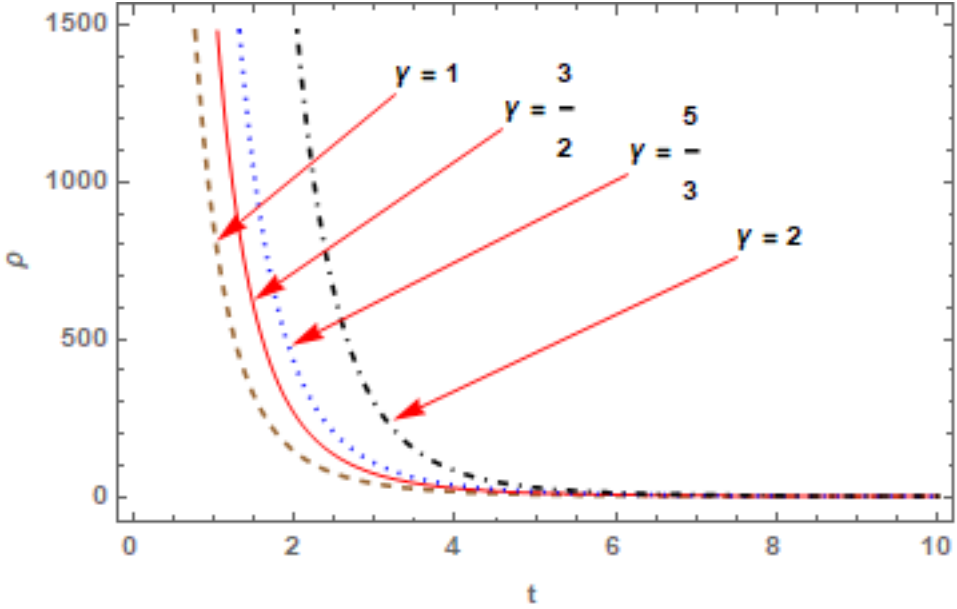


FIGURE 7. Energy density ρ_m for matter is stated against time for the different values of the constants $c = 0.1, d = 0.1, q = -0.33, \gamma = 1, \frac{4}{3}, \frac{5}{3}, 2$.

We have concluded the behaviour of energy density for matter ρ_m with respect to cosmic time ' t ' in Fig.(7) for all four universes i.e. $\gamma = 1$ (Dust universe), $\gamma = \frac{3}{2}$ (Radiation universe), $\gamma = \frac{5}{3}$ (Hard universe), $\gamma = 2$ (Zeldovich universe). It is identity that the energy density declines as time raises and approaches to an infinitesimal constant at late times in all the universes.

We solved field equations with the uses of negative constant deceleration parameter and EoS. The model are expanding and shearing universe. The two-fluid big-bang models which is under the cosmic evolutions results better than two sequences of a single fluid model. The models have been taken based on the assumption of

both the fluids being present throughout the cosmic evolution where one fluid dominates the other one. Energy density for radiation has the similar behaviour in all the universe. The model has near singularity at $(ct + d = 0)$ and it starts with big bang and stops, when cosmic time is infinite. The model does not approach isotropy.

We discussed different type of universes as follows:

Dust model: During the initial stage energy density for matter are infinite whereas energy density for matter evades when cosmic time is infinite. The pressure for matter is zero. The density parameter is constant for $q = \frac{1}{2}$. The energy density for matter and radiation decrease monotonically with respect to cosmic time and tend to a constant value in the large-time limit. The density parameter is constant at $q = 1$.

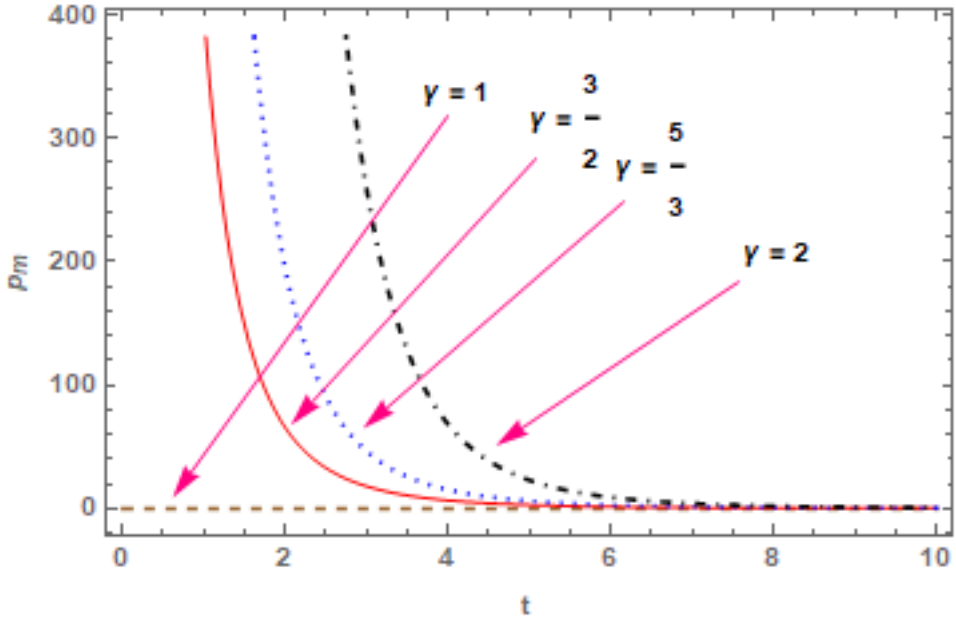


FIGURE 8. Pressure for matter p_m is described against time for the different values of the constants $c = 0.1, d = 0.1, q = -0.33, \gamma = 1, \frac{3}{2}, \frac{5}{3}, 2$.

The behaviour of pressure for matter p_m related to cosmic time ' t ' represented in Fig.(8), which observed that the pressure throughout the dust model remains

constant while in the remaining three universes it declines as time raises and approaches to an infinitesimal constant at late times t' .

Radiation Universe: The energy density for matter and radiation tend to zero as cosmic time tends to infinite. The pressure for matter vanishes for large value of cosmic time but initial stage the pressure for matter is infinite. Total density parameter is constant for $q = 1$. Total energy density and pressure for matter declines monotonically with respect to time and tend to a constant value in the large time.

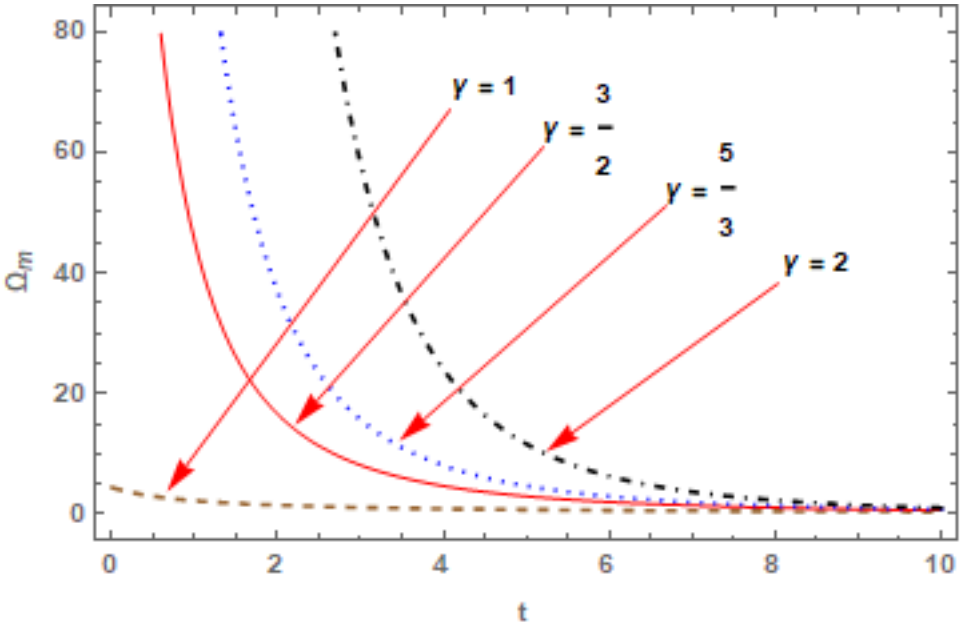


FIGURE 9. The density parameters for matter Ω_m is shown against time for the different values of the constants $c = 0.1, d = 0.1, q = -0.33, \gamma = 1, \frac{3}{2}, \frac{5}{3}, 2$.

Fig.(9) represented that the behaviour of density parameters for matter Ω_m with respect to cosmic time t which is declines as time raises for all four universes i.e. $\gamma = 1$ (Dust universe), $\gamma = \frac{3}{2}$ (Radiation universe), $\gamma = \frac{5}{3}$ (Hard universe), $\gamma = 2$ (Zeldovich universe) and tends to an infinitesimal constant at late times.

Hard Universe: When $t \rightarrow \infty$, the energy density for matter and radiation, the pressure for matter are zero but initial epoch total energy density & pressure for matter are infinite. The density parameter for matter and radiation are constant for $q = \frac{3}{2}$ & $q = 1$.

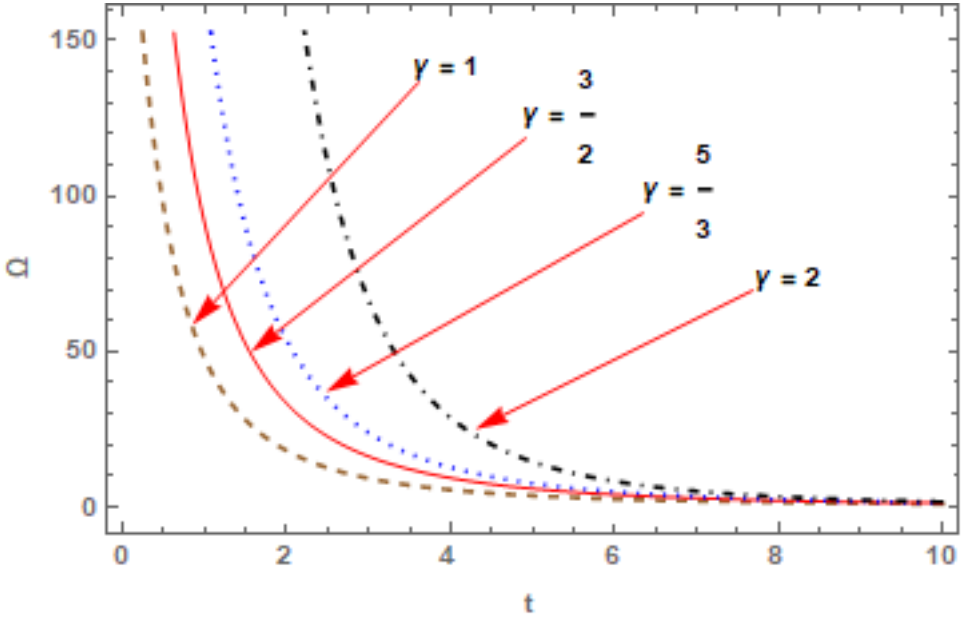


FIGURE 10. Total density parameters Ω for matter & radiation is stated against time for the different values of the constants $c = 0.1, d = 0.1, q = -0.33, \gamma = 1, \frac{3}{2}, \frac{5}{3}, 2$.

The behaviour of total density parameters for matter and radiation Ω with respect to cosmic time t shown in Fig.(10), which describe that the total density parameters declines as time raises and tends to an infinitesimal constant at late times for all four universes.

Zeldovich Universe: At $q = 2$ & $q = 1$, the density parameter for matter & radiation are constant.

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CONFLICT OF INTEREST

Authors declare there is no conflict of interest.

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