

PLANE SYMMETRY COSMOLOGY MODEL OF INTERACTING FIELD IN $f(R, T)$ THEORY

D.D. PAWAR¹ AND R.V. MAPARI²

¹SCHOOL OF MATHEMATICAL SCIENCES, SWAMI RAMANAND TEERTH
MARATHWADA UNIVERSITY, VISHNUPURI, NANDED-431606 (INDIA)

²DEPARTMENT OF MATHEMATICS,
GOVERNMENT VIDARBHA INSTITUTE OF SCIENCE AND HUMANITIES,
AMRAVATI-444604 (INDIA)

E-MAILS: DYPAWAR@YAHOO.COM AND R.V.MAPARI@GMAIL.COM

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ABSTRACT. We have explored a plane symmetric cosmological model in $f(R, T)$ gravity. We have used an interacting field as a source of energy, as well as dark energy. In this case, the interacting field is simply a linear coupling between an electromagnetic field, a massless scalar field, and a charged perfect fluid. Four cases have been described 1) Perfect fluid, 2) Disordered radiation, 3) Dust fluid, and 4) Dark energy. We used the relationship between metric potential to solve field equations. By using equation of state, we observed the pressure and density profiles in various models. We also went over some cosmological and dynamical parameters in depth. Also, we discovered both negative and positive deceleration parameters in different time intervals, indicating the Universe's transition phase.

AMS Classification: 54H20, 83DXX, 83F05, 83D05.

Keywords: $f(R, T)$ theory; Dark energy; Plane symmetry; Interacting field; Massless scalar field; Dust fluid.

1. INTRODUCTION

Man is curious to know the mysterious things of the Universe. For example, what is the age of the Universe?, Origin of the Universe?, Evolution of the Universe?, The reason behind the expansion and acceleration of the Universe?, What is dark matter and dark energy? etc. Scientists and the common man are faced with such and even more perplexing questions about the Universe. For that, there are different philosophies and theories for finding answers to all these astonishing questions. One of the most popular and successful theory is the Big-Bang theory. Because the Big-Bang theory has been able to solve many questions scientifically. Most of the Scientist are working to unravel the mysteries of the Universe by accepting the Big-Bang theory. One of them was Einstein, he discovered the general theory of relativity (GR) in 1915 and got very much attention in the field of cosmology [18]. In 1917, Einstein introduced cosmological constant Λ in the field equation of GR to balance the Universe (to keep static Universe) [17]. After that in 1920 to 1930, Alexander Friedman, Edwin Hubble and Georges Lemaître have observed separately that the Universe is not static rather expanding [20, 25, 31]. Some scientists stated that it is due to the invisible energy which is known as Dark Energy (DE). We have only theoretical observations about DE because it is invisible (DE neither detected nor created in laboratory). Even today, dark energy is the biggest mystery to researchers. Still, it was one of the milestone in the study of the modern cosmology. Because it put the new path in the research of cosmology. Another important observation was made more than half a century later. Scientist observed that the Universe is not only expanding but accelerating as well. Observation of the late-time accelerated expansion of the Universe done by the project type Ia Supernovae and verified by many researchers [24, 41, 42, 43, 48, 57]. Many people from the cosmology field are working on the theoretical model of the dark energy in the Einstein theory and observed that the Universe is dominated by negative pressure known as dark energy. Also, it is noteworthy that researchers have reborn the cosmological constant and established the relation between cosmological constant Λ and dark energy [40, 47]. They have stated that the cosmological constant Λ is most suitable candidate to study the nature of the dark energy. However, there are many options of the theoretical models of the dark energy like, quintessence [46, 60], phantom field [14, 32], K-essence [5, 16], tachyon field [34, 52], quintom [4, 19] and

Chaplygin gas [11, 28]. Nowadays study of late-time acceleration of the Universe and dark energy is the biggest challenge for researchers.

Initially, GR was very much successful to construct the cosmological model and to describe the nature of the Universe. But after observations about accelerated expansion of the Universe, GR has limitations. It means the late-time acceleration of the Universe could not explained by GR. For the purpose, many scientists have proposed the alternative way to study the present epoch of the Universe in the form of modified theory of gravitation. They tried to modify or bypass the Einstein theory and verified for the late-time acceleration of the Universe. We are motivated by one of the theory known as $f(R, T)$ gravity which was proposed by Haarko et al. [22]. Where R is the Ricci scalar and T is a trace of the stress-energy tensor. In which gravitational field equations formed in the metric formalism and in the equation of motion for test particles.

$f(R, T)$ theory is very popular as an alternative to GR and studied rigorously by many researchers. In which, Pawar et al. [38] have formed the field equations of $f(R, T)$ gravity from Bianchi metric in the presence of Modified Holographic Ricci Dark Energy (MHRDE). They found the Universe is in the phase of accelerated expansion and isotropic throughout the evolution. Some researchers have constructed the cosmological model with Magnetized Strange Quark Matter (MSQM) and found that the $f(R, T)$ theory can describe the late-time acceleration of the Universe [15, 51]. Perfect fluid with heat flow discussed in $f(R, T)$ gravity by [39]. They have noticed the Universe is accelerating and expanding but rate of expansion is decreasing function of cosmic time t . MSQM model discussed in $f(R, T)$ theory [1] and they have observed the model does not approach isotropy. Influence of dark energy in Universe have observed with magnetized domain walls model in $f(R, T)$ theory [2]. Also, domain walls model have studied in $f(R, T)$ gravity [29]. Many researchers found that the $f(R, T)$ theory have the ability to describe the mysterious things of the Universe. Therefore, this theory has been able to attract the attention of cosmologist. Rigorous study have been done of $f(R, T)$ theory with different energy sources and metric by many researchers [8, 21, 35, 37, 49, 54, 61].

Bianchi Universe are used to analyze the cosmological models of the Universe with anisotropic and homogeneous background. Because of pedagogical importance of Bianchi Universe, generally it is used to study the homogeneous and anisotropic

Universe. Because the WMAP data [6, 33] demands to modify the standard cosmological model with $\Lambda > 0$ that support a analogy to the Bianchi morphology [23, 26, 27, 58]. We have evidences that the Universe was not smoothed at the initial time (inflation period of the Universe). It was probably not as smooth as it seems now (may not homogeneous). Hence, in-homogeneous cosmological model has its own importance to study the in-homogeneity of the Universe. In the present paper we have considered the plane symmetry cosmological model of the form given by [7, 59]. Tilted plane symmetric metric studied by Pawar et al. [36]. Katore and Shaikh [30] have discussed plane symmetric cosmological model in Saex-Ballester scalar-tensor theory. Pradhan et al. analyzed plane-symmetric cosmological models for the different contexts of use in general relativity and in Lyra geometry [44, 45]. Bayaskar et al. have obtained a plane-symmetric cosmological model of interacting fields in general relativity [9] and observed that model does not isotropy even for the large cosmic time. Agrawal and Pawar have obtained plane-symmetric cosmological model in $f(R, T)$ theory of gravitation [1] and obtained expanding Universe with Big-Bang singularity. Outline of the paper: Section 2: Metric and the field equations, Section 3: Solution of the field equations, Section 4: Conclusion of the work.

2. THE METRIC AND FIELD EQUATIONS OF $f(R, T)$ THEORY

We considered a plane-symmetric metric of the form,

$$(1) \quad ds^2 = dt^2 - A^2(dx^2 + dy^2) - B^2dz^2.$$

Where A and B are the functions of cosmic time t .

Action principle for the field equations of $f(R, T)$ gravity is,

$$(2) \quad S = \frac{1}{2} \int f(R, T) \sqrt{-g} d^4x + \int L_m \sqrt{-g} d^4x.$$

The gravitational field equations for $f(R, T)$ gravity is given by,

$$(3) \quad \begin{aligned} f_R(R, T)R_{ij} - \frac{1}{2}f(R, T)g_{ij} + (g_{ij}\nabla^i\nabla_j - \nabla_i\nabla_j)f_R(R, T) \\ = \kappa\tilde{T}_{ij} - f_T(R, T)\tilde{T}_{ij} - f_T(R, T)\theta_{ij}. \end{aligned}$$

Where,

$$(4) \quad \theta_{ij} = g^{\alpha\beta} \frac{\partial T_{\alpha\beta}}{\partial g^{ij}}, \quad f_R(R, T) = \frac{\partial f(R, T)}{\partial R} \quad \text{and} \quad f_T(R, T) = \frac{\partial f(R, T)}{\partial T}$$

∇_j is the co-variant derivative. We choose $\kappa = \frac{8\pi G}{c^4}$

Where G is the Newtonian Gravitational constant and c is the speed of light in vacuum. $\tilde{T}_{ij}$ is the standard matter energy-momentum tensor derived from the Lagrangian L_m . We choose matter Lagrangian only for perfect fluid distribution as $L_m = -p$ which has been discussed in [12, 13, 22, 56].

The functional $f(R, T)$ have many choices like,

$$(5) \quad f(R, T) = \begin{cases} R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R) f_3(T) \end{cases}$$

We assumed the model $f(R, T) = R + 2f(T)$, which was discussed by Sahoo et. al. 2014 [50], Singh and Bishi 2014 [55]. Where $f(T)$ is an arbitrary function of the trace of the energy-momentum tensor and we choose $f(T) = \lambda T$, Where λ be a constant. The field equations of $f(R, T)$ gravity for linearly coupled charged perfect fluid, source-free electromagnetic field and mass-less scalar fields are,

$$(6) \quad G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij} = (\kappa + 2\lambda)\tilde{T}_{ij} + \lambda(\tilde{T} + 2p)g_{ij}$$

Einstein field equation of GR is given by

$$(7) \quad G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij} - \Lambda g_{ij} = -8\pi T_{ij}$$

Here, Λ is cosmological constant and we used as a dark energy source. Now, Comparing Eqn.(6) with the Einstein field equation, we have

$$(8) \quad \Lambda = \lambda(\tilde{T} + 2p)$$

We considered the source of energy of the gravitational field is an interacting field with dark energy and observed the behavior of the cosmological model in the presence of linearly coupled perfect fluid distribution, mass-less scalar field, and source of a free electromagnetic field. That is,

$$(9) \quad \tilde{T}_{ij} = S_{ij} + T_{ij} + E_{ij}$$

In the Eqn.(9),

S_{ij} is the energy-momentum tensor for perfect fluid distribution and it is given by,

$$(10) \quad S_{ij} = (p + \rho)u_i u_j - g_{ij}p$$

Together with

$$(11) \quad g^{ij}u_iu_j = 1$$

Where, p , ρ , u^i are internal pressure, rest mass density and four-velocity vectors of the distribution respectively.

T_{ij} is the energy-momentum tensor for mass-less scalar field and it is given by,

$$(12) \quad T_{ij} = U_{;i}U_{;j} - \frac{1}{2}g_{ij}U_{;s}U^{;s}$$

Mass-less scalar field U also satisfy,

$$(13) \quad g^{ij}U_{;ij} = \rho_c$$

Where, ρ_c is the charge density, semicolon (;) and comma (,) denotes covariant derivative and partial derivative respectively.

E_{ij} is the electromagnetic energy-momentum tensor given by,

$$(14) \quad E_{ij} = \frac{1}{4\pi}[F_{i\alpha}F_j^\alpha - \frac{1}{4}g_{ij}F_{\alpha\beta}F^{\alpha\beta}]$$

Here F_{ij} is the electromagnetic field tensor obtained from the four potential ϕ_i ,

$$(15) \quad F_{ij} = \phi_{i,j} - \phi_{j,i}$$

$$(16) \quad F^{ij}_{;j} = -4\pi\rho_c u^i$$

In the co-moving transformation system the magnetic field is considered along z -axis only, therefore non-vanishing components of electromagnetic fields F_{ij} are only F_{12} and F_{21} . Also, we have electromagnetic field tensor is anti-symmetric.

The first set of Maxwells equation are,

$$(17) \quad F_{ij,k} + F_{jk,i} + F_{ki,j} = 0$$

Gives,

$$(18) \quad F_{12} = M = constant$$

Now from Eqn.(9), (10), (14) for the metric (1), we have

$$(19) \quad E_1^1 = E_2^2 = -E_3^3 = -E_4^4 = \frac{M^2}{8\pi A^4}$$

$$(20) \quad T_1^1 = T_2^2 = T_3^3 = -T_4^4 = -\frac{\dot{U}^2}{2}$$

$$(21) \quad S_1^1 = S_2^2 = S_3^3 = -p ; \quad S_4^4 = \rho$$

By using Eqn.(19) to (21), the field equations of $f(R, T)$ Eqn.(6), for the metric Eqn.(1) can be reduced as,

$$(22) \quad \frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -(\kappa + 2\lambda) \left[\frac{M^2}{8\pi A^4} - \frac{\dot{U}^2}{2} - p \right] + \lambda \left[-\dot{U}^2 - p + \rho \right]$$

$$(23) \quad \frac{\dot{A}^2}{A^2} + 2\frac{\ddot{A}}{A} = -(\kappa + 2\lambda) \left[-\frac{M^2}{8\pi A^4} - \frac{\dot{U}^2}{2} - p \right] + \lambda \left[-\dot{U}^2 - p + \rho \right]$$

$$(24) \quad \frac{\dot{A}^2}{A^2} + 2\frac{\dot{A}\dot{B}}{AB} = -(\kappa + 2\lambda) \left[-\frac{M^2}{8\pi A^4} + \frac{\dot{U}^2}{2} + \rho \right] - \lambda \left[-\dot{U}^2 - p + \rho \right]$$

Here dot for differentiation w.r.t. time t .

For the metric Eqn.(1), the dynamical parameters are defined as follows.

Average scale factor,

$$(25) \quad a(t) = (A^2 B)^{1/3}$$

Spatial Volume,

$$(26) \quad V = A^2 B$$

Directional Hubble parameters,

$$(27) \quad H_x = H_y = \frac{\dot{A}}{A}, \quad H_z = \frac{\dot{B}}{B}$$

Average Hubble Parameter,

$$(28) \quad H = \frac{\dot{a}}{a}$$

Scalar expansion and Shear scalar,

$$(29) \quad \theta = 3H, \quad \sigma^2 = \frac{1}{2} \sigma^{ij} \sigma_{ij}$$

Average anisotropy parameter,

$$(30) \quad \Delta = \frac{1}{3} \sum_{i=1}^3 \left[\frac{H_i - H}{H} \right]^2$$

Where, H_i are the directional Hubble parameters.

The deceleration parameter,

$$(31) \quad q = -1 + \frac{d}{dt} \left(\frac{1}{H} \right)$$

3. SOLUTION OF THE FIELD EQUATIONS

Eqn.(22) to (24) are highly nonlinear differential equations which contained five unknowns A , B , U , p and ρ . We are interested to find them for observation of the Universe (Eqn.(1)) and their physical and dynamical properties. Hence we assumed external constraints for the exact solutions of the obtained field equations (22) to (24). Firstly we take the relation between metric potentials,

$$(32) \quad A = B^n, \text{ where } n \neq 1$$

From Eqn. (32), field equations (22) to (24) reduced as,

$$(33) \quad \ddot{B} + 2n \frac{\dot{B}^2}{B} = \frac{(\kappa + 2\lambda)M^2 B^2}{4\pi(n-1) B^{4n+1}}$$

Above equation can be formed as,

$$(34) \quad \frac{df^2}{dB} + 2\frac{\alpha}{B}f^2 = \frac{(\kappa + 2\lambda)M^2 B^2}{2\pi(n-1) B^{4n+1}}$$

Where, $\alpha = 2n$ and $\dot{B} = f(B)$

From Eqn.(34), we get

$$(35) \quad dt = \left\{ \frac{(\kappa + 2\lambda)M^2}{4\pi(n-1) B^{4n-2}} + \frac{C}{B^{4n}} \right\}^{-1/2} dB$$

Where C is constant of integration.

Now, by using $F_{12} = \text{Const.} = M$ and $u^4 \neq 0$, from Eqn.(16) we have, charge density is zero.

$$(36) \quad \text{i.e.} \quad \rho_c = 0$$

From Eqn.(36) and Eqn.(13), we get

$$(37) \quad \ddot{U} + \left(2\frac{\dot{A}}{A} + \frac{\dot{B}}{B} \right) \dot{U} = 0$$

From Eqn.(32), Eqn.(37), we have

$$(38) \quad \dot{U} = \frac{C_1}{T^{2n+1}}$$

Here, C_1 is the constant of integration.

After some appropriate coordinate transformation Eqn.(1) has the form

$$(39) \quad ds^2 = \left\{ \frac{(\kappa + 2\lambda)M^2}{4\pi(n-1)B^{4n-2}} + \frac{C}{B^{4n}} \right\}^{-1/2} dT^2 - T^{2n} (dx^2 + dy^2) - T^2 dz^2$$

We have obtained some physical and dynamical parameters of the model (39) as follows,

Average scale factor,

$$(40) \quad a(t) = T^{\frac{2n+1}{3}}$$

Spatial Volume,

$$(41) \quad V = T^{2n+1}$$

Directional Hubble parameters, from which we can observed rate of expansion of the Universe in the direction of x, y, z axes are,

$$(42) \quad H_x = H_y = \frac{n}{T^{2n+1}} \left[\frac{(\kappa + 2\lambda)M^2 T^2}{4\pi(n-1)} + C \right]^{1/2}$$

$$(43) \quad H_z = \frac{1}{T^{2n+1}} \left[\frac{(\kappa + 2\lambda)M^2 T^2}{4\pi(n-1)} + C \right]^{1/2}$$

Average Hubble Parameter,

$$(44) \quad H = \frac{2n+1}{3T^{2n+1}} \left[\frac{(\kappa + 2\lambda)M^2 T^2}{4\pi(n-1)} + C \right]^{1/2}$$

Scalar expansion and Shear scalar,

$$(45) \quad \theta = \frac{2n+1}{T^{2n+1}} \left[\frac{(\kappa + 2\lambda)M^2 T^2}{4\pi(n-1)} + C \right]^{1/2}$$

$$(46) \quad \sigma^2 = \frac{(n-1)^2}{2T^{2n+2}} \left[\frac{(\kappa + 2\lambda)M^2 T^2}{4\pi(n-1)} + C \right]$$

Average anisotropy parameter,

$$(47) \quad \Delta = \frac{-2(n-1)^2}{(2n+1)^2}$$

The deceleration parameter,

$$(48) \quad q = -1 + T^{2n} \left[\frac{(\kappa + 2\lambda)M^2T^2}{4\pi(n-1)} + C \right]^{-1/2} - T^{2n+2} \frac{(\kappa + 2\lambda)M^2}{4\pi(n-1)} \left[\frac{(\kappa + 2\lambda)M^2T^2}{4\pi(n-1)} + C \right]^{-3/2}$$

We have deceleration parameter given by Eqn.(48), in which we observed, it has positive as well as negative value in the particular time interval and setting the values of constant. Therefore, We have discussed the exact solution of the pressure and density in four models. We used the second constraint in the form of equation of state for Zeldovich fluid or stiff fluid, disordered radiation, dust fluid and dark energy along with the relation between metric potentials which we have already assumed in Eqn.(32).

3.1. Zeldovich Fluid or Stiff Fluid

For the exact solution of the field equations (22) to (24) we used the second constraint in the form of EoS for stiff fluid (which is the relation between pressure and density), we have

$$(49) \quad p = \rho$$

Using Eqn.(49) in the Eqn.(23), the pressure and density of the model (Eqn.(39)) is,

$$(50) \quad p = \rho = \left[\frac{3n^2 - 2n}{\kappa + 2\lambda} \right] \frac{1}{T^2} + \left[\frac{\lambda}{\kappa + 2\lambda} - \frac{1}{2} \right] \frac{C_1^2}{T^{4n+2}} - \frac{M^2}{8\pi T^{4n}}$$

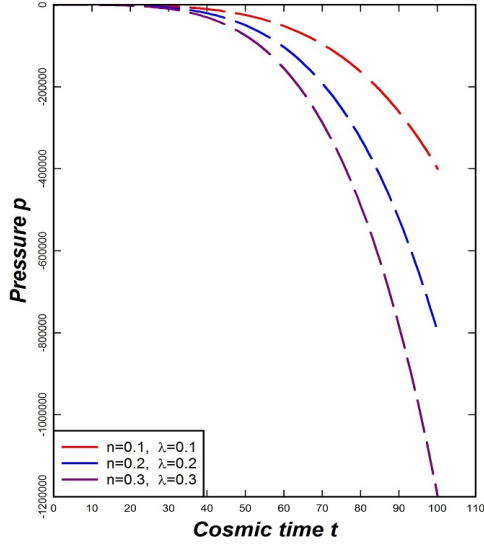


FIGURE 1. Variation of pressure for stiff fluid against cosmic time t by setting the values $C_1 = M = \kappa = 1$ and $n = \lambda = 0.1, 0.2, 0.3$

3.2. Disordered Radiation

For disordered radiation EoS is,

$$(51) \quad \rho = 3p$$

Using Eqn. (51) in the Eqn. (23), the pressure and density of the model (39) is,

$$(52) \quad p = \left[\frac{3n^2 - 2n}{\kappa + 4\lambda} \right] \frac{1}{T^2} + \left[\lambda - \frac{\kappa + 2\lambda}{2} \right] \frac{C_1^2}{(\kappa + 4\lambda)T^{4n+2}} - \frac{(\kappa + 2\lambda)M^2}{8\pi(\kappa + 4\lambda)T^{4n}}$$

$$(53) \quad \rho = \frac{1}{3} \left\{ \left[\frac{3n^2 - 2n}{\kappa + 4\lambda} \right] \frac{1}{T^2} + \left[\lambda - \frac{\kappa + 2\lambda}{2} \right] \frac{C_1^2}{(\kappa + 4\lambda)T^{4n+2}} - \frac{(\kappa + 2\lambda)M^2}{8\pi(\kappa + 4\lambda)T^{4n}} \right\}$$

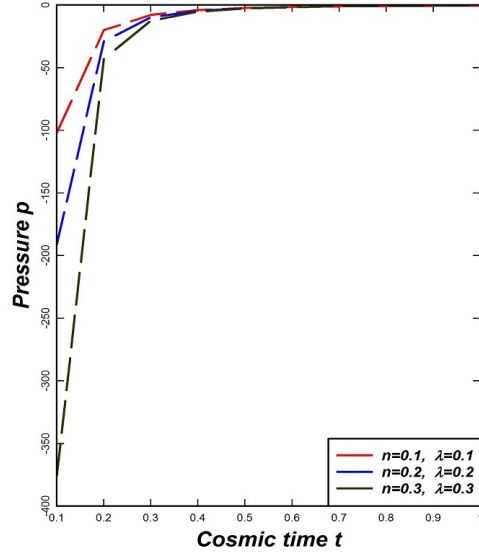


FIGURE 2. Variation of pressure for disordered radiation against cosmic time t by setting the values $C_1 = M = \kappa = 1$ and $n = \lambda = 0.1, 0.2, 0.3$

3.3. Dust fluid

For dust fluid, we have

$$(54) \quad p = 0$$

Using Eqn.(54) in the Eqn.(24), the density of the model (Eqn.(39)) is,

$$(55) \quad \rho = \left[\frac{\lambda}{\kappa + 3\lambda} - \frac{\kappa + 2\lambda}{2} \right] \frac{C_1^2}{T^{4n+2}} + \frac{(\kappa + 2\lambda)M^2}{8\pi(\kappa + 3\lambda)T^{4n}} - \frac{n^2}{(\kappa + 3\lambda)T^2} - \frac{2n}{(\kappa + 3\lambda)T}$$

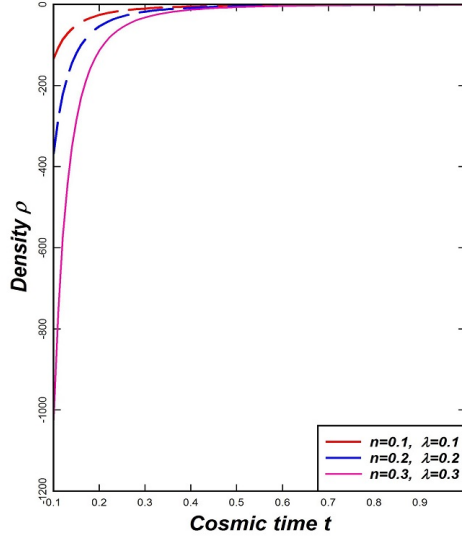


FIGURE 3. Variation of density for dust fluid against cosmic time t by setting the values $C_1 = M = \kappa = 1$ and $n = \lambda = 0.1, 0.2, 0.3$

3.4. Dark Energy

For dark energy EoS parameter $\omega = -1$, we have

$$(56) \quad p = -\rho$$

Using Eqn.(56) in Eqn.(23), the pressure and density of the model is

$$(57) \quad p = -\frac{1}{N} \left[\frac{M^2}{8\pi T^{4n}} + \frac{C_1^2 N}{2T^{4n+2}} - \frac{n(3n-2)}{(\kappa+2\lambda)T^2} \right]$$

Where, $N = 1 - \frac{2\lambda}{\kappa+2\lambda}$

$$(58) \quad \rho = \frac{1}{N} \left[\frac{M^2}{8\pi T^{4n}} + \frac{C_1^2 N}{2T^{4n+2}} - \frac{n(3n-2)}{(\kappa+2\lambda)T^2} \right]$$

Using Eqn.(57), (19), (20), (21) in Eqn.(8), we obtained Cosmological constant Λ as,

$$(59) \quad \Lambda = \lambda \left[\frac{M^2}{4\pi N T^{4n}} - \frac{2n(3n-2)}{(\kappa+2\lambda)N T^2} \right]$$

Graphical observations:

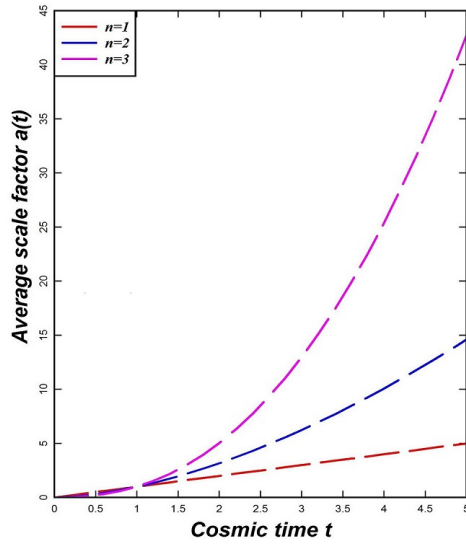


FIGURE 4. Variation of Average scale factor ($a(t)$) against cosmic time t by setting the values $n = 1, 2, 3$

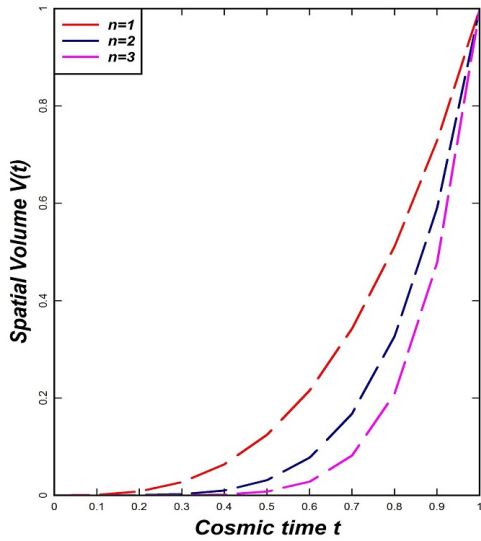


FIGURE 5. Variation of Spatial Volume (V) against cosmic time t by setting the values $n = 1, 2, 3$

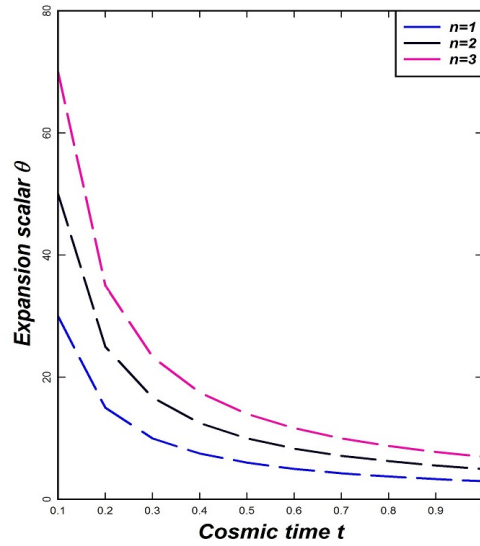


FIGURE 6. Variation of Expansion Scalar (θ) against cosmic time t by setting the values $n = 1, 2, 3$

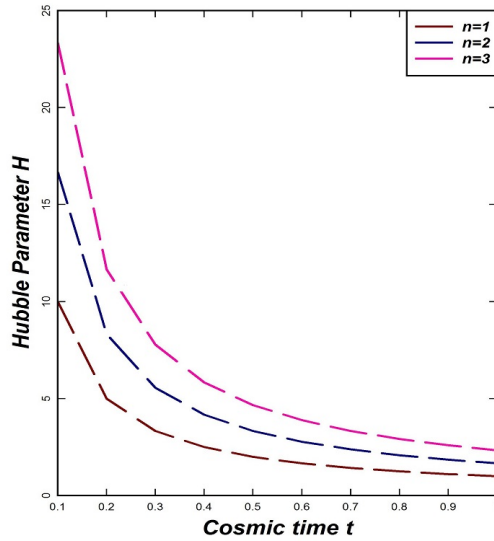


FIGURE 7. Variation of Hubble Parameter (H) against cosmic time t by setting the values $n = 1, 2, 3$

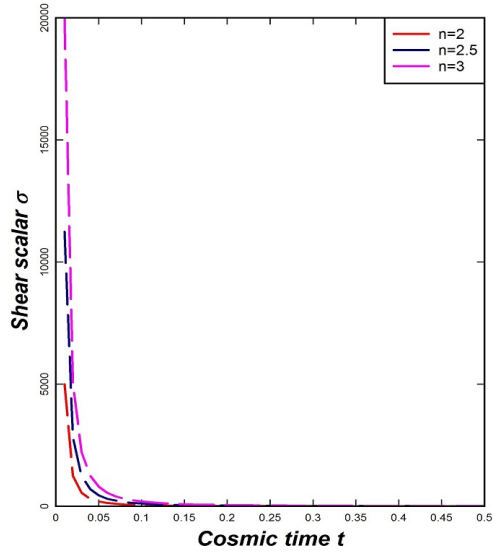


FIGURE 8. Variation of Shear scalar (σ) against cosmic time t by setting the values $n = 1, 2, 3$

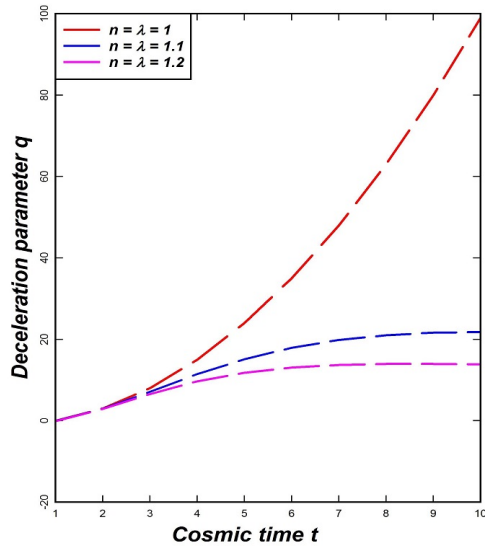


FIGURE 9. Variation of Deceleration Parameter (q) against cosmic time t ($1 < t < 10$) by setting the values $C = M = \kappa = 1$

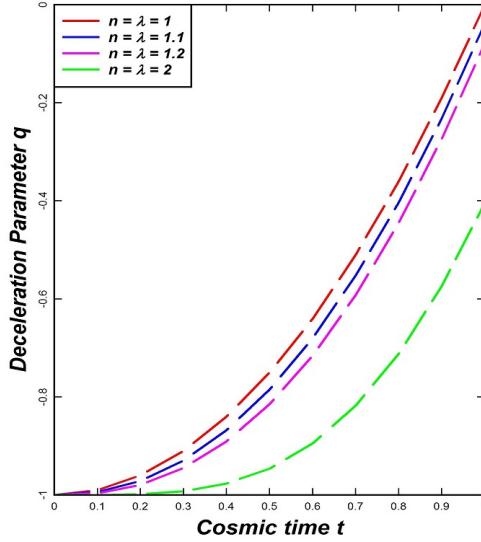


FIGURE 10. Variation of Deceleration Parameter (q) against cosmic time t ($0 < t < 1$) by setting the values $C = M = \kappa = 1$

4. CONCLUSION

We have explored a plane symmetric cosmological model in $f(R, T)$ gravity. We have used an interacting field as a source of energy, as well as dark energy. In the paper we have described four cases 1) Perfect fluid, 2) Disordered radiation, 3) Dust fluid, and 4) Dark energy. We have used the relation between metric potential to solve field equations.

We have noticed that the nature of the pressure and density behaves differently in different models. The pressure of a stiff fluid decreases as a function of time t , but it is infinite at the start (Fig.1). We also noticed that the pressure is negative in nature. For disordered radiation, the pressure is infinity at the initial epoch. It is increasing for time $0 < t < 1$ and decreasing for $t > 1$ (Fig.2). For dust fluid we have seen pressure is zero therefore we observed density of the model. From Fig.3 we have observed density for dust fluid is increasing for $0 < t < 1$ and it is decreasing as time increases ($t > 1$). For dark energy model we found the pressure is negative in nature. It is noteworthy that the pressure starts from large negative value as cosmic time attain its early stage (initial time) [$p \rightarrow -\infty$ as $t \rightarrow 0$] which resembles

with the observation of Ahmed and Pradhan [3]. The cosmological constant Λ is the decreasing function of cosmic time t and it approaches to small positive value at late-time. The present model compatible for the profile of vacuum energy density Λ [53].

We have observed that the average scale factor or cosmic scale factor increases with time increases (Fig.4). Spatial volume is zero at an initial time $t = 0$ and it increases gradually as time increases, which is consistent with the idea of expanding Universe. It means Universe expansion starts with finite volume and it is expanding with time increases (Fig.5). At an initial time rate of expansion is infinite but after that, the rate of expansion is decreasing as time increases (Fig.6) and it will be zero for large time which is agreed with the results of [62].

The directional Hubble parameters as well as average Hubble parameter and shear scalar are the function of the cosmic time t and have a singularity at initial time $t = 0$, decreases as time increases and it tends to zero for large t . In this regard we have good agreement with the results of [39]. Also, the average Hubble parameter H and shear scalar σ decreases as time increases (Fig.7 and Fig.8), but the positive value of expansion scalar and Hubble parameter throughout the evolution shows that the Universe is expanding gradually. Here Anisotropy parameter is zero only for $n = 1$ that means it is shear free otherwise, nonzero throughout the evolution for all $n \neq 0$ and not free from shear this shows that our model does not approach isotropy. The value of average anisotropic parameter is representing anisotropic model, which is compatible with the current observations.

As positive value of deceleration parameter shows decelerating Universe whereas negative value shows accelerating Universe. In the present study, from the Fig.9 we have found positive value of deceleration parameter for $t > 1$ and setting particular values of constants. To visualize the scenario, we plot the graph for $1 < t < 10$. It means Universe is decelerating. Also, from Fig.10 we have found negative value of deceleration parameter for $0 < t < 1$ and setting particular values of constants. But it increases as time traveled from zero to 1 and approach towards positive value. It means Universe is accelerating in this time interval. Thus, the model we have chosen agreed with the recent results by [41, 42] and [48] in the time interval $0 < t < 1$.

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