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MAGNETIZED QUARK AND STRANGE QUARK MATTER IN THE HIGHER DIMENSIONAL SPHERICALLY SYMMETRIC SPACE TIME ADMITTING ONE PARAMETER GROUP OF CONFORMAL MOTIONS

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Abstract. In present paper, we have studied magnetized quark and strange quark matter with conformal motions in higher dimensional spherically symmetric space-time. We found Einstein's eld equations in higher dimensional spherically symmetric space-time using conformal motions. In the framework of higher dimension, various physical quantities have discussed.

AMS Classification: 54H20.

Keywords: Higher dimension; Magnetized quark and Starnge quark matter; Con-formal motion; Bag model.

1. Introduction

Recently, it has been found that magnetic field is present in the universe [Klein et al. (1988), Kronberg et al. (1992, 1994), Wolf et al. (1992), Beck et al. (1996) and Carilli et al. (2004)]. Magnetism is everywhere in the universe: stars, planets, gaseous nebulae and entire galaxies. Neutron stars have the strongest magnetic

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field in the universe. The strenth and structure of cosmic magnetic fields is studied by observation of its polarization, its Faraday rotation and radio continuum emission. Vacca et. al.(2018) have studied magnetic field in galaxy clusters and in the large-scale structure of the universe. Radler et al. (2001) studied the decay of magnetic field penetrating a compact spherical electrically conducting body and continuing in its non-conducting surroundings is systematically. Aktas et.al (2007) studied Space-Time Geometry of magnetized Quark and Strange Quark Matter.

In our model the quark is formed of massive s quarks, massless u, d quark, electrons. Considering non interacting massless quarks in modified bag model, then quark pressure $p_q = \frac{\rho_q}{3}$, where ρ_q is the quark density, the total energy density is $\rho = \rho_q + B$ but total pressure $p = p_q - B$.

Then the equation of state for strange quark matter (Kapusta (1994) and Sotani et. al. (2004))

$$p = \frac{1}{3}(\rho - 4B), \tag{1}$$

where B is the bag constant.

The Chandrasekhar limit has been evaluated for quark stars using the bag model equation of state (1) in (Bannerjee et al.(2000)). The new EOSs for strange matter based on a model of interquark potential is obtained by Dey et al. (1998, 1999). Later on these EOSs have been approximated to the following linear form by Gondek (2000)

$$p = \epsilon(\rho - \rho_0), \tag{2}$$

where ϵ is a constant and ρ_0 is the energy density at zero pressure.

In this work we shall study magnetized quark and strange quark matter with conformal motion in the higher dimensional spherically symmetric space-time by using conformal cillineation. Now Conformal collineation is usually defined by

$$\pounds_\xi g_{ab} = 2\psi g_{ab}, \psi = \psi(x^a) \tag{3}$$

where ξ is vector field and ψ is the arbitrary function of the radial ordinate r. Conformal killing vectors which give a deeper information of the spacetime geometry. Many authors have been studied conformal collineation. Conformal motion in the spherically symmetric space time have been studied by Caley et.al.(1990)

and Maartens et.al.(1995). Mak et.al.(2004) have discussed charged strange quark matter in spherical symmetric space time admitting conformal motion.
The work is organized as follows. In sec. 2 We obtained Einstein's field equations and their solutions via conformal motions in the higher dimensional spherically symmetric space time for magnetized quark matter. In sec. 3 Magnetized strange quark matter solutions of the Einstein's field equations are obtained in higher dimension. In sec. 4 we summerized our result

2. Magnetized quark matter solutions

In ideal MHD, pressure p for (n+2)-velocity u^α, the rest-mass density ρ and magnetic field h^α measured by an observer. The ideal MHD condition, $u_\alpha F^{\alpha\beta} = 0$, where $F^{\alpha\beta}$ is the electromagnetic tensor. The tensor $F^{\alpha\beta}$ and its dual in ideal MHD approximation are given by (Aktas et. al.(2007))

$$F^{\alpha\beta} = \epsilon^{\alpha\beta\gamma\eta} u_\gamma h_\eta, \tag{4}$$

$$F^*_{\alpha\beta} = \frac{1}{2}\epsilon_{\alpha\beta\gamma\eta}F^{\gamma\eta} = u_\beta h_\alpha - u_\alpha h_\beta, \tag{5}$$

where $\epsilon_{\alpha\beta\gamma\eta}$ is the Levi- Civita tensor and magnetic field measured by a normal observer n^α is given by

$$H^\alpha = n_\beta F^{*\beta\alpha} = \alpha(u^t h^\alpha - u^\alpha h^t). \tag{6}$$

From H^α and the antisymmetry of $F^{\alpha\beta}$ we get $n_\alpha H^\alpha = H^t = 0$. The relation between h^α and H^α are

$$h^\beta = \frac{H^\mu u_\mu}{\nu}, \tag{7}$$

$$h^\alpha = \frac{1}{\nu u^t}(H^\mu + H^\alpha u_\mu u^\alpha). \tag{8}$$

The energy momentum tensor by using above variable (Barrow et.al. (2006)) which is written as

$$T^{EM}_{\alpha\beta} = F^\gamma_\beta F_{\alpha\gamma} - \frac{1}{4}g_{\alpha\beta}F_{\alpha\beta}F^{\alpha\beta} = (\frac{1}{2}g_{\alpha\beta} + u_\beta u_\alpha)h^2 - h_\alpha h_\beta, \tag{9}$$

where $h^2 = h^\alpha h_\mu$.
Also, the energy momentum tensor of quark matter due to behavior of perfect fluid

of quark-matter [Back et.al. (2005), Adams at.al. (2005), Adcox at.al. (2005)] is written as

$$T^{Quark}_{\alpha\beta} = (\rho + p)u_\beta u_\alpha + \rho g_{\alpha\beta} \tag{10}$$

where $p = p_q - B$ and $\rho = \rho_q + B$. So, the total energy momentum tensor is $T_{\alpha\beta} = T^{Quark}_{\alpha\beta} + T^{EM}_{\alpha\beta}$, where $T^{Quark}_{\alpha\beta}$ is given by Eq. (10) and $T^{EM}_{\alpha\beta}$ is given by Eq. (9). Hence for magnetized quark matter the total stress-energy tensor becomes

$$T_{\alpha\beta} = (\rho + p + h^2)u_\beta u_\alpha + (p + \frac{h^2}{2})g_{\alpha\beta} - h_\alpha h_\beta. \tag{11}$$

where $u^\alpha = \delta^\alpha_{(n+2)} e^{-\frac{\nu}{2}}$ is the (n+2) velocity and $p_{mag} = \frac{h^2}{2}$ is the magnetic pressure. Also, due to $h_\alpha u^\alpha = 0$ we can choose magnetic flux in the radial direction. Also, Eintsein equation can be expressed as

$$R_{ab} - \frac{1}{2}Rg_{ab} = 8\pi T_{ab}. \tag{12}$$

We consider the higher dimensional spherically symmetric line element in the form

$$ds^2 = e^{\nu(r)}dt^2 - e^{\lambda(r)}dr^2 - r^2 d\Omega^2, \tag{13}$$

with

$$d\Omega^2 = d\theta_1^2 + \sin^2\theta_1 d\theta_2^2 + \sin^2\theta_1 \sin^2\theta_2 d\theta_3^2 + ... + \prod_{i=1}^{n-1} \sin^2\theta_i d\theta_n^2,$$

$$x^{1,2,3,...,(n+1),(n+2)} = r, \theta_1, \theta_2, ..., \theta_{(n+1)}, t.$$

Here we shall use geometrized units so that $8\pi G = c = 1$ and we shall assume the direction of magnetic field in the radial direction. Using the line-element Eq. (13), Eq. (11) and Eq. (12) we get

$$\rho + \frac{h^2}{2} = -e^{-\lambda}\left[\frac{n(n-1)}{2r^2} - \frac{n\lambda'}{2r}\right] + \frac{n(n-1)}{2r^2}, \tag{14}$$

$$-p + \frac{h^2}{2} = -e^{-\lambda}\left[\frac{n\nu'}{2r} + \frac{n(n-1)}{2r^2}\right] + \frac{n(n-1)}{2r^2}, \tag{15}$$

$$p + \frac{h^2}{2} = \frac{e^{-\lambda}}{2}\left[\nu'' - \frac{\nu'\lambda'}{2} + \frac{\nu'^2}{2} - \frac{(n-1)(\lambda' - \nu')}{r} + \frac{(n-1)(n-2)}{r^2}\right]$$

$$- \frac{(n-1)(n-2)}{2r^2}, \tag{16}$$

where a comma $(,)$ denotes partial derivative with respective r.
To get the deterministic solution, we assume that the space-time admits a one-parameter group of conformal motions Eq. (3) i.e.

$$\pounds_{\xi} g_{ab} = \xi_{a;b} + \xi_{b;a} = 2\psi g_{ab}, \tag{17}$$

where ψ is an arbitrary function of r. From Eqs. (13) and (17), we get the following expressions

$$\xi^1 \nu' = \psi, \xi^1 = \frac{\psi r}{2}, \tag{18}$$

$$\lambda' \xi' + 2\xi^1_{,1} = \psi, \xi^{n+2} = C_1 = \text{constant}, \tag{19}$$

where a comma $(,)$ denotes partial derivative with respective r. From Eqs. (18) -(19), we obtain

$$e^{\nu} = C_2^2 r^2, e^{\lambda} = \left(\frac{C_3^2}{\psi}\right)^2, \tag{20}$$

$$\xi^i = \frac{\psi r}{2}\delta^i_1 + C_1 \delta^i_{n+2}, \tag{21}$$

where C_2 and C_3 are constants of integration (Herrera et. al. (1985)).
Now putting Eq. (20) into Eqs. (14) - (16), we get

$$\rho + \frac{h^2}{2} = \frac{n(n-1)}{2r^2}\left[1 - \frac{\psi^2}{C_3^2}\right] - \frac{n\psi\psi'}{C_3^2 r}, \tag{22}$$

$$-p + \frac{h^2}{2} = -\frac{n(n+1)\psi^2}{2C_3^2 r^2} + \frac{n(n-1)}{2r^2}, \tag{23}$$

$$p + \frac{h^2}{2} = \frac{n\psi\psi'}{C_3^2 r} + \frac{n(n-1)\psi^2}{2C_3^2 r^2} - \frac{(n-1)(n-2)}{2r^2}. \tag{24}$$

From Eqs.(22) - (24), we get

$$\rho = -\frac{3n\psi\psi'}{2C_3^2 r} + \frac{n(2-n)\psi^2}{2C_3^2 r^2} + \frac{(n-1)^2}{2r^2}, \tag{25}$$

$$p = \frac{n\psi\psi'}{2C_3^2 r} + \frac{n^2\psi^2}{2C_3^2 r^2} - \frac{(n-1)^2}{2r^2}, \tag{26}$$

$$h^2 = \frac{n\psi\psi'}{C_3^2 r} - \frac{n\psi^2}{C_3^2 r^2} + \frac{(n-1)}{r^2}. \tag{27}$$

Using Eqs. (20), line element given by Eq.(13) becomes

$$ds^2 = C_2^2 r^2 dt^2 - \frac{C_3^2}{\psi^2} dr^2 - r^2 d\Omega^2. \tag{28}$$

According to Eqs. (25)-(27), we can find solution by choice of ψ and from physical consideration ($h^2 = 0, p = 0$.)

2.1. **Case (I):.** If $\psi = \pm(\frac{(n-1)C_3^2}{n} + C_4 r^2)^{\frac{1}{2}}$, then from Eqs. (25) - (27) we get

$$\rho = \frac{(n-1)^2}{2r^2} - \frac{n(n+1)C_4}{2C_3^2}, \tag{29}$$

$$p = \frac{(n-1)}{2r^2} + \frac{n(n+1)C_4}{2C_3^2}, \tag{30}$$

$$h^2 = 0 \tag{31}$$

where C_4 is an integrating constant. In this case, the line element is as follows

$$ds^2 = C_2^2 r^2 dt^2 - \frac{nC_3^2}{(n-1)C_3^2 + nC_4 r^2} dr^2 - r^2 \Omega^2, \tag{32}$$

2.2. **Case (II):.** If $\psi = \pm(\frac{(n-1)^2 C_3^2}{n^2} + \frac{C_5}{r^{2n}})^{\frac{1}{2}}$, then from Eqs. (25) - (27) we get

$$\rho = \frac{(n-1)^2}{nr^2} + \frac{n(n+1)C_5}{C_3^2 r^{2(n+1)}}, \tag{33}$$

$$p = 0, \tag{34}$$

$$h^2 = \frac{(n-1)}{nr^2} - \frac{n(n+1)C_5}{C_3^2 r^{2(n+1)}}, \tag{35}$$

where C_5 is an integrating constant. In this case, the line element is as follows

$$ds^2 = C_2^2 r^2 dt^2 - \frac{n^2 C_3^2 r^{2n}}{(n-1)C_3^2 r^{2n} + n^2 C_5} dr^2 - r^2 d\Omega^2. \tag{36}$$

3. **Magnetized strange quark matter solutions**

By using equations of state (Eq. 1 and 2), we can specify the function $\psi(r)$ a priori for strange quark matter.

Case (A): Strange quark matter in the BAG model:

From Eqs.(1), (25) and (26) we get

$$\psi(r)=\pm\sqrt{\frac{(n+2)(n-1)^2C_3^2}{n(n^2+2n-2)}-\frac{4C_3^2Br^2}{2n(n+1)}+\frac{C_6}{r^{\frac{2(n^2+2n-2)}{(n+4)}}}}. \tag{37}$$

Substituting Eq. (37) into Eqs. (25), (26) and (27) we have

$$\rho=\frac{(n+1)(n-1)^2}{(n^2+2n-2)r^2}+\frac{n(n^2+2n+1)C_6}{(n+4)C_3^2r^{\frac{2(n^2+3n+3)}{(n+4)}}}+B \tag{38}$$

or

$$\rho_q=\frac{(n+1)(n-1)^2}{(n^2+2n-2)r^2}+\frac{n(n^2+2n+1)C_6}{(n+4)C_3^2r^{\frac{2(n^2+3n+3)}{(n+4)}}}, \tag{39}$$

$$p=\frac{(n-1)^2}{(n^2+2n-2)r^2}+\frac{n(n+1)C_6}{(n+4)C_3^2r^{\frac{2(n^2+3n+3)}{(n+4)}}}-B \tag{40}$$

or

$$p_q=\frac{(n-1)^2}{(n^2+2n-2)r^2}+\frac{n(n+1)C_6}{(n+4)C_3^2r^{\frac{2(n^2+3n+3)}{(n+4)}}} \tag{41}$$

and also

$$h^2=\frac{n(n-1)}{(n^2+2n-2)r^2}-\frac{n(n+1)(n+2)C_6}{(n+4)C_3^2r^{\frac{2(n^2+3n+3)}{(n+4)}}} \tag{42}$$

where C_6 is an integrating constant.

In this case space time geometry of the strange quark matter or star is given by

$$ds^2=C_2^2r^2dt^2-C_3^2\left[\frac{(n+2)(n-1)^2C_3^2}{n(n^2+2n-2)}-\frac{4C_3^2Br^2}{2n(n+1)}+\frac{C_6}{r^{\frac{2(n^2+2n-2)}{(n+4)}}}\right]^{-1}dr^2 \tag{43}$$
$$-r^2d\Omega^2.$$

Case(B): Strange quark matter in the case of linear equation of state:

From Eqs. (2), (25) and (26) we have

$$\psi(r)=\pm\sqrt{\frac{(\epsilon+1)(n-1)^2C_3^2}{n^2(\epsilon+1)-4\epsilon}-\frac{2\epsilon\rho_0C_3^2r^2}{n(n+1)(\epsilon+1)}+\frac{C_7}{r^{\frac{2n(\epsilon+1)-4\epsilon}{(3\epsilon+1)}}}}. \tag{44}$$

Using EQ. (44 into Eqs. (25)-(27)) we get

$$\rho = \frac{(n-1)^2}{2r^2}\left[\frac{2n(\epsilon+1)-4\epsilon}{n^2(\epsilon+1)-4\epsilon}\right] + \frac{\epsilon\rho_0}{(\epsilon+1)} + \frac{n(n+1)C_7}{(3\epsilon+1)C_3^2 r^{\frac{2(n+1)(\epsilon+1)}{(3\epsilon+1)}}}, \tag{45}$$

$$p = \frac{4(n-1)^2\epsilon}{2r^2(n^2(\epsilon+1)-4\epsilon)} - \frac{\epsilon\rho_0}{(\epsilon+1)} + \frac{n(n+1)C_7}{(3\epsilon+1)C_3^2 r^{\frac{2(n+1)(\epsilon+1)}{(3\epsilon+1)}}} \tag{46}$$

and

$$h^2 = \frac{(n-1)}{r^2}\left[\frac{n(\epsilon+1)-4\epsilon}{n^2(\epsilon+1)-4\epsilon}\right] - \frac{n(n+1)(\epsilon+1)C_7}{(3\epsilon+1)C_3^2 r^{\frac{2(n+1)(\epsilon+1)}{(3\epsilon+1)}}} \tag{47}$$

where C_7 is an integrating constant.

In this case space time geometry of the strange quark matter or star as follows

(48)

$$ds^2 = C_2^2 r^2 dt^2 - C_3^2\left[\frac{(\epsilon+1)(n-1)^2C_3^2}{n^2(\epsilon+1)-4\epsilon} - \frac{2\epsilon\rho_0 C_3^2 r^2}{n(n+1)(\epsilon+1)} + \frac{C_7}{r^{\frac{2n(\epsilon+1)-4\epsilon}{(3\epsilon+1)}}}\right]^{-1} dr^2$$

$$-r^2 d\Omega^2.$$

4. Concluding Remarks

In our present paper, we have examined magnetized quark and strange quark matter in the spherically symmetric space-time via conformal motion in the context of higher dimensional space-time. The obtained solutions generalize to higher dimensions the well known results in four dimensional space-time (Aktas et. al.(2007)). It is observed that the difference is significant at least in the principle to the analogous situation in four dimensional space-time. The solutions reduce to the four dimensional form when $n = 2$ those of Aktas et. al.(2007). For $r < r_0$ we have observed that e^{ν} and e^{λ} are non negative, continuous and non-singular .

In case (I) of magnetized quark matter, we get only quark matter solution without magnetic field. And in case (II) we obtained the solution at $p = 0$. We also studied spherically symmetric strange quark matter in the bag model in the presence of magnetic field via one parameter group of conformal motion in the frame work of higher dimensional space time. In case (A), at the surface of the star, we have

$q \to (n+2)B$ as $p \to 0$ and the gradients $\frac{d\rho}{dr}$ and $\frac{dp}{dr}$ should be negative, we get

$$\frac{d\rho}{dr} = \frac{-2(n+1)(n-1)^2}{(n^2+2n-2)r^3} - \frac{2n(n+1)(n+2)(n^2+2n+1)C_6}{(n+4)^2 C_3^2 r^{\frac{(2n^2+7n+8)}{(n+4)}}} \tag{49}$$

$$\frac{dp}{dr} = \frac{-2(n-1)^2}{(n^2+2n-2)r^3} - \frac{2n(n+1)^2(n+2)C_6}{(n+4)^2 C_3^2 r^{\frac{(2n^2+7n+8)}{(n+4)}}} \tag{50}$$

In the case of magnetized strange quark matter in the Bag model if we take $n = 2$ and $C_6 = 0$ and electric field instead of magnetic field , the equations in our solution reduce to those of Mark and Harko (2004). In case (B), for the strange quark matter in BAG model and in the case of linear equation of state we obtained the same geometry. Also in this case, the gradients $\frac{d\rho}{dr}$ and $\frac{dp}{dr}$ should be negative, we get

$$\frac{d\rho}{dr} = \frac{(n-1)^2}{r^3}\left[\frac{2n(\epsilon+1)-4\epsilon}{n^2(\epsilon+1)-4\epsilon}\right] - \frac{2n(n+1)^2(\epsilon+1)C_7}{(3\epsilon+1)^2 C_3^2 r^{\frac{2(n+1)(\epsilon+1)+(3\epsilon+1)}{(3\epsilon+1)}}} \tag{51}$$

$$\frac{dp}{dr} = \frac{-4(n-1)^2\epsilon}{r^3(n^2(\epsilon+1)-4\epsilon)} - \frac{2n(n+1)^2\epsilon(\epsilon+1)C_7}{(3\epsilon+1)^2 C_3^2 r^{\frac{2(n+1)(\epsilon+1)+(3\epsilon+1)}{(3\epsilon+1)}}} \tag{52}$$

For the case (A) and (B) we have given graphical representation in fig. (1) to fig. (6) for the values of ρ, $\frac{dp}{dr}$, $\frac{d\rho}{dr}$, h Vs r.

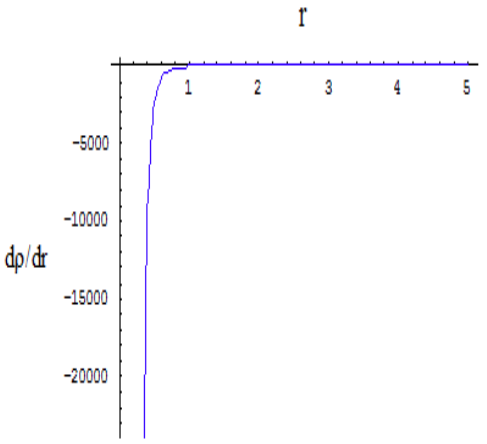

FIGURE 1. Relation between the scale factor $\frac{d\rho}{dr}$ and cosmic time r of different cases for $\epsilon = 0.463, C_3 = 1, C_7 = 1$

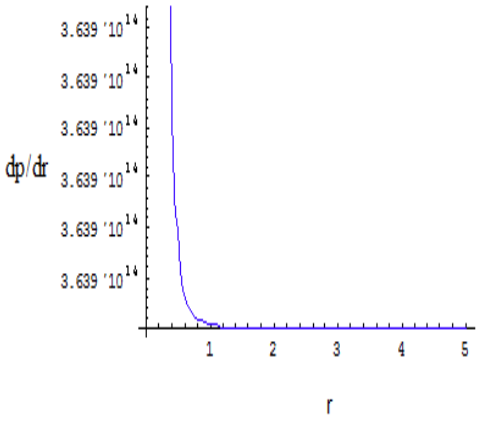

FIGURE 2. Relation between the scale factor $\frac{dp}{dr}$ and cosmic time r of different cases for $\epsilon_0 = 0.463, C_3 = 1, C_7 = 1$

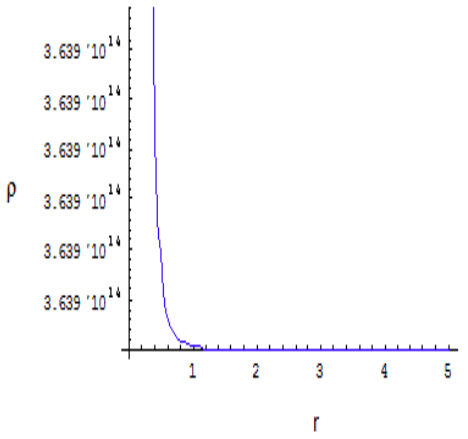

FIGURE 3. Relation between the scale factor ρ and cosmic time r of different cases for $\epsilon_0 = 0.463, C_3 = 1, C_7 = 1$

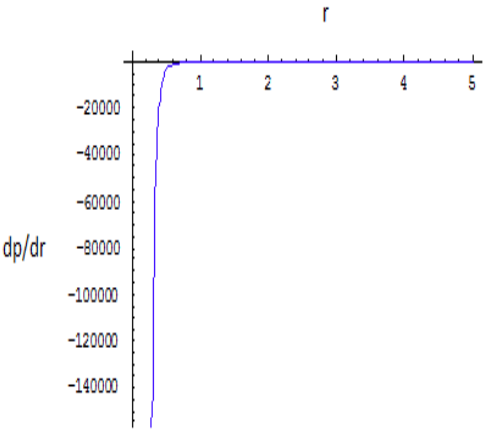

FIGURE 4. Relation between the scale factor $\frac{dp}{dr}$ and cosmic time r of different cases for $\epsilon_0 = 0.463, C_3 = 1, C_6 = 1$

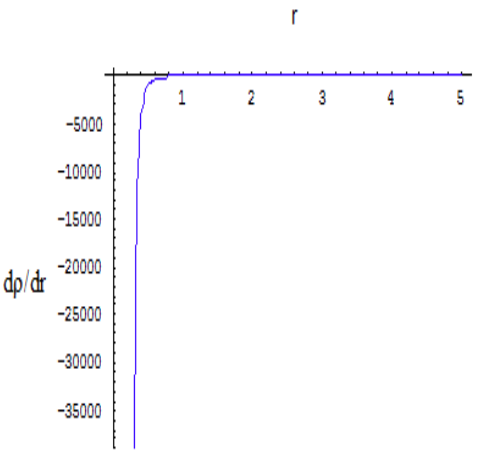

FIGURE 5. Relation between the scale factor $\frac{d\rho}{dr}$ and cosmic time r of different cases for $\epsilon_0 = 0.463, C_3 = 1, C_6 = 1$

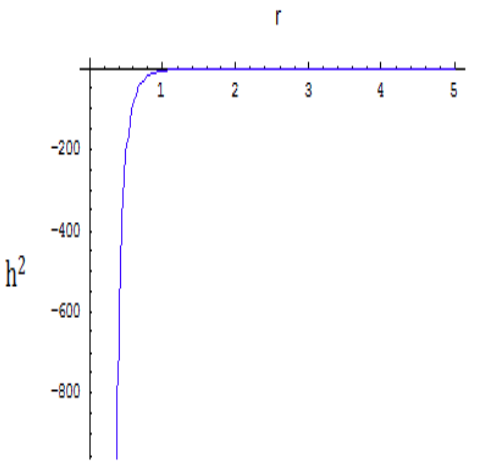

FIGURE 6. Relation between the scale factor h^2 and cosmic time r of different cases for $\epsilon_0 = 0.463, C_3 = 1, C_7 = 1$

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