

ANALYSIS OF VARIOUS PARAMETERS OF LIMITED SOURCE QUEUEING MODEL HAVING SINGLE SERVER BY USING BINOMIAL AND UNIFORM DISTRIBUTION

¹DAMODHAR F. SHASTRAKAR, ²SHARAD S. POKLEY, ³SAJID ANWAR

¹RESEARCH SCHOLAR,

N. S. SCIENCE & ARTS COLLEGE BHADRAWATI, DIST. CHANDRAPUR, MAHARASHTRA, INDIA

²ASSOCIATE PROFESSOR, MATHEMATICS DEPARTMENT

KAVIKULGURU INSTITUTE OF TECHNOLOGY AND SCIENCE, RAMTEK, MAHARASHTRA, INDIA

³PROFESSOR, MATHEMATICS DEPARTMENT,

ANJUMAN COLLEGE OF ENGINEERING & TECHNOLOGY, NAGPUR, MAHARASHTRA, INDIA

E-MAILS: DFSHASTRAKAR@YAHOO.COM, SHARADPOKLEY@GMAIL.COM AND

SAJIDANWAR0616@GMAIL.COM

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ABSTRACT. A limited source Queueing model having single server is considering here to discuss its various parameters. The number of arriving customers in this model is fixed. Since the arriving customers are already fixed and system is busy in serving the customers one by one, the arrival rate of next coming customer gets reduced. This reduces the further congestion. The service discipline in the model is general discipline. As the arriving customers in the system are limited, binomial distribution is used to deal with the arrival rate of customers and service rate of server. Uniform distribution is applied to the inter-arrival time and service time. By using the binomial distribution, formula is derived to find the expected waiting time of a customer in the queue. The derived formula and little's formula are then used together to study the various parameters of the Queueing model. Finally comparing the results calculated by the newly derived formulae and existing formulae.

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Keywords: Limited source; General discipline; Utilization factor; Arrival rate; Service rate; Inter-arrival time; Service time.

1. INTRODUCTION

As per the detailed discussion of various performance measures of Limited Source Multiple- Server Queuing Model [13], we apply the same approach here to study the limited Source Single Server Queuing Model. The main aim to study the Queuing analysis is to increase the satisfaction level of customers by reducing their waiting time [9]. For the same we discussed here the various parameters of Limited Source Queuing Model having Single Server. The arriving customers in this model are limited and General Service Discipline is applied here to serve the customers. Since the arriving customers are already fixed and system is busy in serving the customers one by one, gets reduce the arrival rate of next coming customer. This reduces the further congestion. Machine repairing is one of the common example of this model. The servicemen who are servicing the machine are considered as servers and machines are considered as customers. This type of related work is carried out in [1, 5, 6, 7]. As in the system the arriving Population is fixed, the arrival rate of customers and service rate of server act in accordance with Binomial Distribution, whereas the inter-arrival time and service time accompany with Uniform Distribution [10, 11, 12]. A new formula is derived to find the expected waiting time of a customer in the queue. Using this formula and Littles Formula together the others parameters of the model are calculated, studied and analyzed. Finally interpret the conclusion of comparative study of the results by derived formulae and existing formulae.

2. METHODOLOGY

Let R be the fixed number of customers. The mean arrival rate of the customer is α and the mean service rate of the server is β . Service Discipline is General.

$$(1) \quad \text{Utilization factor of server } \rho = \alpha/\beta$$

3. PROBABILITY OF x CUSTOMERS ARRIVED/SERVED IN TIME t [10] [12] [13]

Since the arriving customers are fixed in both Single Server and Multiple-Server of Finite Source Queuing Model, this part is similar as in [13]. By Binomial Distribution, probability of x customers arriving in time t is given by [13]

$$(2) \quad P(x \text{ customers arrived in time } t) = \binom{R}{x} \left(\frac{\alpha(R-x)t}{R} \right)^x \left(1 - \frac{\alpha(R-x)t}{R} \right)^{R-x}, \quad t < \frac{R}{\alpha(R-x)}$$

The above equation assures the time interval for the arrival of R number of customers within $0 \leq t \leq [R/\alpha(R-x)]$. Probability of x customers served in time t is given by

$$(3) \quad P(x \text{ customers served in time } t) = \binom{R}{x} \left(\frac{\beta t}{R} \right)^x \left(1 - \frac{\beta t}{R} \right)^{R-x}, \quad t < \frac{R}{\beta}$$

This above equation assures the time interval to serve R customers is $0 \leq t \leq [R/\beta]$.

4. ARRIVAL AND SERVICE TIME DISTRIBUTION [10] [13]

Arrival Time Distribution and Service Time Distribution follow Uniform Distribution [13]

5. PROBABILITY OF x CUSTOMERS IN THE SYSTEM BY USING THE SYSTEM OF DIFFERENTIAL DIFFERENCE EQUATIONS FOR SINGLE SERVER LIMITED SOURCE QUEUING MODEL

System will have a Probability of containing x number of customers at a time $(t + \omega t)$,

$$\begin{aligned} P_x(t + \omega t) = & P_x(t) \{ \text{Prob}(\text{zero arrival in } \omega t \text{ \& zero departure in } \omega t) \\ & + P_{x+1}(t) \{ \text{Prob}(\text{zero arrival in } \omega t \text{ \& one departure in } \omega t) \\ & + P_{x-1}(t) \{ \text{Prob}(\text{one arrival in } \omega t \text{ \& zero departure in } \omega t) \} \} \end{aligned}$$

in which ωt is very small.

Using equations (2) and (3) for a small time ωt , yields

$$P_x(t + \omega t) = P_x(t) \left\{ \left(1 - \frac{\alpha(R-x)\omega t}{R} \right)^R \left(1 - \frac{\beta\omega t}{R} \right)^R \right\}$$

$$\begin{aligned}
& +P_{x+1}(t) \left\{ \left(1 - \frac{\alpha(R-x-1)\omega t}{R} \right)^R R \left(\frac{\beta\omega t}{R} \right) \left(1 - \frac{\beta\omega t}{R} \right)^{R-1} \right\} \\
& +P_{x-1}(t) \left\{ R \left(\frac{\alpha(R-x+1)\omega t}{R} \right) \left(1 - \frac{\alpha(R-x+1)\omega t}{R} \right)^{R-1} \left(1 - \frac{\beta\omega t}{R} \right)^R \right\}
\end{aligned}$$

On simplifying we get [10],

$$(4) \quad P'_x(t) = \alpha(R-x+1)P_{x-1}(t) + \beta P_{x+1}(t) - [\alpha(R-x) + \beta]P_x(t)$$

In the steady-state condition we get, the Probability of x customers in the system [10],

$$(5) \quad P_x = \frac{R!}{(R-x)!} \left(\frac{\alpha}{\beta} \right)^x P_0, \quad x = 0, 1, 2, 3, \dots, R$$

Where server stands idle [10],

$$(6) \quad P_0 = 1 \div \left[\sum_{x=0}^R \frac{R!}{(R-x)!} \left(\frac{\alpha}{\beta} \right)^x \right]$$

6. WAITING TIME DISTRIBUTION OF A CUSTOMER IN THE QUEUE FOR SINGLE SERVER LIMITED SOURCE QUEUING MODEL [4]

In the steady-state condition, the waiting time distribution of each customer is identical and a continuous random variable. Let T be the time required by the server to serve all the customers who are already in the system. Let $F_T(t)$ be the Probability distribution function of T then, using equation (6)

$$(7) \quad F_T(0) = P_0 = 1 \div \left[\sum_{x=0}^R \frac{R!}{(R-x)!} \left(\frac{\alpha}{\beta} \right)^x \right]$$

If a customer is arriving for service and there are already $(x \geq 1)$ customers present in the system, then the arriving customer will get service after the service completion of all the customers in the system. As the Calling Population is limited to R , arriving customer is getting service after the service completion of $1 \leq x \leq (R-1)$ customers.

Let $f_x(t)$ be the Probability function for $1 \leq x \leq (R-1)$ customers

$$\begin{aligned}
f_x(t) = & \int_{x=1}^{R-1} P_x [\text{Prob } \{(x-1) \text{ customers got service at time } t\} \\
& \times \text{Prob } \{\text{one customer is under service during time } \omega t\}]
\end{aligned}$$

where ωt is very small [4, 8].

Since β is the mean service rate of a server then Probability of a customer under service during time $\omega t = \beta \omega t$

Using equations (3) and (5), yields

$$f_x(t) = \sum_{x=1}^{R-1} \frac{R!}{(R-x)!} \left(\frac{\alpha}{\beta}\right)^x P_0 \binom{R-1}{x-1} \left(\frac{\beta t}{R-1}\right)^{x-1} \left(1 - \frac{\beta t}{R-1}\right)^{(R-1)-(x-1)} \beta \omega t$$

Therefore

$$(8) \quad f_x(t) = \sum_{x=1}^{R-1} \frac{R!}{(R-x)!} \left(\frac{\alpha}{\beta}\right)^x P_0 \binom{R-1}{x-1} \left(\frac{\beta t}{R-1}\right)^{x-1} \left(1 - \frac{\beta t}{R-1}\right)^{(R-x)} \beta \omega t$$

and from equations (7) and (8), we get

$$\begin{aligned} F_T(t) &= P(T \ll t) \\ &= F_T(0) + \int_0^t \sum_{x=1}^{R-1} \frac{R!}{(R-x)!} \left(\frac{\alpha}{\beta}\right)^x P_0 \left[\binom{R-1}{x-1} \left(\frac{\beta t}{R-1}\right)^{x-1} \left(1 - \frac{\beta t}{R-1}\right)^{(R-x)} \right] \beta dt \end{aligned}$$

By differentiating with respect to t , we get Probability density function of waiting time distribution is

$$F'_T(t) = 0 + \int_0^{R-1} \frac{R!}{(R-x)!} \left(\frac{\alpha}{\beta}\right)^x P_0 \left[\binom{R-1}{x-1} \left(\frac{\beta t}{R-1}\right)^{x-1} \left(1 - \frac{\beta t}{R-1}\right)^{(R-x)} \right] \beta$$

($\because$ from equation (7), $F_T(0)$ is independent of t).

Therefore

$$(9) \quad F'_T(t) = \beta P_0 \sum_{x=1}^{R-1} \frac{R!}{(R-x)!} \left(\frac{\alpha}{\beta}\right)^x \binom{R-1}{x-1} \left(\frac{\beta t}{R-1}\right)^{x-1} \left(1 - \frac{\beta t}{R-1}\right)^{(R-x)}$$

Now, time interval to serve $(R-1)$ number of customers is $0 \ll t < \frac{R-1}{\beta}$.

Therefore, Expected waiting time of a customer in the queue can be denoted by

$$T_q = \int_0^{R-1/\beta} t \times F'_T(t) dt$$

and using equation (9), we arrive at

$$\begin{aligned} T_q &= \int_0^{R-1/\beta} t \times \beta P_0 \sum_{x=1}^{R-1} \frac{R!}{(R-x)!} \left(\frac{\alpha}{\beta}\right)^x \left[\binom{R-1}{x-1} \left(\frac{\beta t}{R-1}\right)^{x-1} \left(1 - \frac{\beta t}{R-1}\right)^{(R-x)} \right] dt \\ &= \beta P_0 \sum_{x=1}^{R-1} \frac{R!}{(R-x)!} \left(\frac{\alpha}{\beta}\right)^x \binom{R-1}{x-1} \left(\frac{\beta}{R-1}\right)^{x-1} \int_0^{R-1/\beta} t^x \left(1 - \frac{\beta t}{R-1}\right)^{(R-x)} dt \end{aligned}$$

To solve the integration using substitution

$$\frac{\beta t}{R-1} = y \implies t = \left(\frac{R-1}{\beta}\right) y, \quad dt = \left(\frac{R-1}{\beta}\right) dy, \quad 0 \leq y \leq 1$$

Therefore

$$\begin{aligned} T_q &= \beta P_0 \\ &\times \sum_{x=1}^{R-1} \frac{R!}{(R-x)!} \left(\frac{\alpha}{\beta}\right)^x \binom{R-1}{x-1} \left(\frac{\beta}{R-1}\right)^{x-1} \int_0^1 \left(\frac{R-1}{\beta} y\right)^x (1-y)^{(R-x)} \left(\frac{R-1}{\beta}\right) dy \\ &= \beta P_0 \sum_{x=1}^{R-1} \frac{R!}{(R-x)!} \left(\frac{\alpha}{\beta}\right)^x \binom{R-1}{x-1} \left(\frac{R-1}{\beta}\right)^2 \int_0^1 y^x (1-y)^{(R-x)} dy \\ &= \beta P_0 \sum_{x=1}^{R-1} \frac{R!}{(R-x)!} \left(\frac{\alpha}{\beta}\right)^x \frac{(R-1)!}{(x-1)!(R-x)!} \left(\frac{R-1}{\beta}\right)^2 \beta'(x+1, R-x+1) \\ &= P_0 \frac{(R-1)^2}{\beta} \sum_{x=1}^{R-1} \frac{R!}{(R-x)!} \left(\frac{\alpha}{\beta}\right)^x \frac{(R-1)!}{(x-1)!(R-x)!} \frac{x!(R-x)!}{(R+1)!} \\ &= \frac{(R-1)!(R-1)^2 P_0}{\beta(R+1)} \sum_{x=1}^{R-1} \frac{x}{(R-x)!} \left(\frac{\alpha}{\beta}\right)^x \end{aligned}$$

Hence, the expected waiting time of a customer in the queue,

$$(10) \quad T_q = \frac{(R-1)!(R-1)^2 P_0}{\beta(R+1)} \sum_{x=1}^{R-1} \frac{x}{(R-x)!} \left(\frac{\alpha}{\beta}\right)^x$$

7. EXAMPLE FOR ANALYSIS OF VARIOUS PARAMETERS OF QUEUEING MODEL

We take the same readings from [13] having data of five customers given in following Table 1 and use it to find the values of various parameters by using new derived formulae and existing formulae to compare the results. Arriving customers be limited to $R=5$. Here we consider 'customers per hour' as 'cph'.

8. CALCULATIONS [2, 3, 4, 8]

(1) Utilization factor of server, (from equation (1))

$$\rho = \frac{\alpha}{\beta} = \frac{0.1834}{0.3505} = 0.5233$$

TABLE 1. Data of five customers used for single server limited source Queuing Model. [13]

A	B	C	D	E	F	G
1	(Between 1 & 2) = 6	0.1667	3	0.3333	0.1834	0.3505
2	(Between 2 & 3) = 5	0.2	2.5	0.4		
3	(Between 3 & 4) = 5	0.2	3.5	0.2857		
4	(Between 4 & 5) = 6	0.1667	3	0.3333	cph.	cph.
5			2.5	0.4		

where A = Customer for service in Sr. No. , B = Inter-arrival time between two successive customers in hours (a) , C = Arrival rate of customers (1/a) , D = Service time in hours (b), E = Service rate (1/b) , F = α Mean value of (1/a) , G = β Mean value of (1/b)

(2) Server stands idle, (from equation (6))

$$\begin{aligned}
 P_0 &= 1 \div \left[\sum_{x=0}^R \frac{R!}{(R-x)!} \left(\frac{\alpha}{\beta} \right)^x \right] \\
 &= 1 \div \left[\sum_{x=0}^5 \frac{5!}{(5-x)!} (0.5233)^x \right] \\
 &= 0.0318
 \end{aligned}$$

(3) Expected waiting time of a customer in the queue, (from equation (10))

$$\begin{aligned}
 T_q &= \frac{(R-1)!(R-1)^2 P_0}{\beta(R+1)} \int_{x=1}^{R-1} \frac{x}{(R-x)!} \left(\frac{\alpha}{\beta} \right)^x \\
 &= \frac{4! \times 4^2 \times 0.0318}{0.3505 \times 6} \int_{x=1}^4 \frac{x}{(5-x)!} (0.5233)^x \\
 &= 3.6471 \text{ hrs.}
 \end{aligned}$$

(4) Expected waiting time of a customer in the system = (Expected waiting time in the queue) + (service time)

$$\begin{aligned}
 T_s &= T_q + \frac{1}{\beta} \\
 &= 3.6471 + \frac{1}{0.3505} = 6.5002 \text{ hrs.}
 \end{aligned}$$

- (5) Expected number of customers in the queue, (Using Little's formula [4])

$$N_q = \alpha' T_q$$

where

$$\alpha' = \alpha \sum_{x=0}^R (R-x) P_x$$

therefore

$$N_q = 0.3388 \times 3.6471 = 1.2356 \text{ customers}$$

- (6) Expected number of customers in the system, (Using Little's formula [4])

$$N_s = \alpha' T_s$$

where

$$\alpha' = \alpha \sum_{x=0}^R (R-x) P_x$$

therefore

$$N_s = 0.3388 \times 6.5002 = 2.2023 \text{ customers}$$

- (7) Expected waiting time of a customer in the queue for busy system,

$$\begin{aligned} T_b &= \frac{T_q}{(\text{Prob. of system being engaged})} = \frac{T_q}{1 - P_0} \\ &= \frac{3.6471}{1 - 0.0318} = 3.7669 \text{ hrs.} \end{aligned}$$

- (8) Expected number of customers served per busy period,

$$\begin{aligned} N_b &= \frac{N_s}{(\text{Prob. of system being engaged})} = \frac{N_s}{1 - P_0} \\ &= \frac{2.2023}{1 - 0.0318} = 2.2746 \text{ customers} \end{aligned}$$

TABLE 2. Calculated results, Existing results and % Decrease in the value of Queuing Parameters of the Single Server Finite Source Queuing Model (Time measured in hrs).

Sr. No.	Queuing Parameters	Single Server		
		CalculatedResults	Existing Results [2, 3, 8]	% Decrease
1	T_q	3.6471	6.4284	43.27
2	T_s	6.5002	9.2815	29.97
3	N_q	1.2356	2.1815	43.36
4	N_s	2.2023	3.1497	30.08
5	T_b	3.7669	6.6395	43.27
6	N_b	2.2746	3.2532	30.08

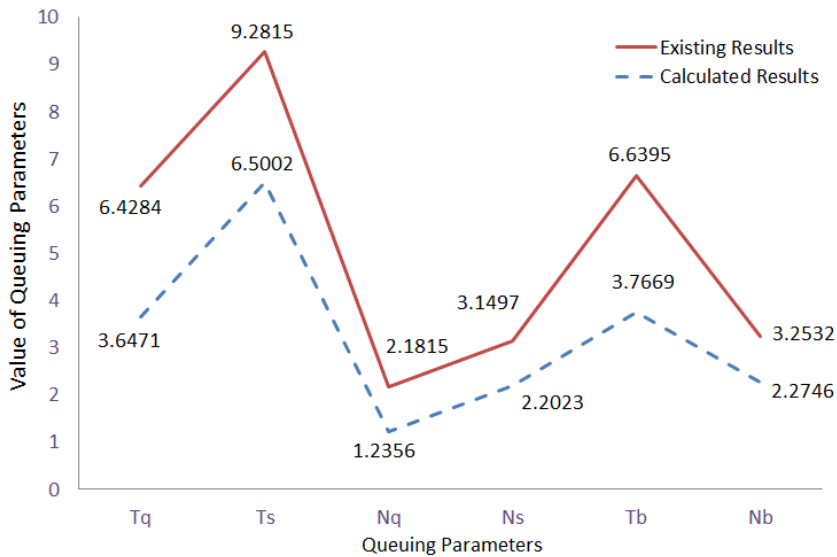


FIGURE 1. Graph showing calculated results and existing results of queuing parameters of single server limited source queuing model.

9. CONCLUSION

Use of Binomial Distribution to the Limited Source Queuing Model gives the time interval of particular number of customers arrived and the time interval required

serving the particular number of customers. The application of Binomial Distribution to the limited source Queuing Model shows the reduction in the value of the various parameters calculated by derived formulae as compared to the results given by existing formulae. It indicates that there increases the satisfaction level of customers which improve the result analysis.

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