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DYNAMICAL STUDY OF A VECTOR HOST EPIDEMIC MODEL WITH NON-MONOTONE INCIDENCE

SEEMA RAUT[1], SUJATHA JANARDHAN[2] AND GANESH KHEKARE[3]

[1]DEPARTMENT OF APPLIED MATHEMATICS,
G. H. RAISONI. INSTITUTE OF ENGINEERING & TECHNOLOGY, NAGPUR, MAHARASHTRA, INDIA

[2]DEPARTMENT OF MATHEMATICS,
ST. FRANCIS DE SALES COLLEGE, NAGPUR,MAHARASHTRA, INDIA

[3]DEPARTMENT OF INFORMATION TECHNOLOGY,
G.H. RAISONI COLLEGE OF ENGINEERING, NAGPUR, MAHARASHTRA, INDIA

E-MAILS: [1]SEEMA.SSK83@YAHOO.IN, [2]SUJATA_JANA@YAHOO.COM AND [3]KHEKARE.123@GMAIL.COM

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Abstract. Epidemics of vector borne disease studied in this paper when disease incidence rate is non- monotone. Non monotonic behaviors of infected hosts as well as infected vectors are studied for influence of increase in disease on susceptible host population and effect of repellents and insecticides on vectors, respectively. Suitable Liapunov functions are constructed to discuss the global asymptotic stabilities at the equilibrium points. Results are verified by conducting numerical simulation. Mechanism of disease spread by vectors and hosts during the epidemic can be better understood with the help of this model.

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1. Introduction

Infectious pathogens such as bacteria, viruses or parasites can be carried from human to human or from animals to humans by vectors. Infectious diseases caused by vectors like mosquitoes,ticks and fleas, etc. are called as vector borne diseases which is the leading cause of deaths yearly[4]. Malaria, Dengue, Chikungunya fever, West Nile fever, and Zika virus fever, etc. are some examples of vector borne diseases [1, 13, 14]. Such diseases badly affect the ecology as well as global economy in many aspects [7]. Once the vector is infectious, it can spread the infection throughout its life span. So vector control is the one of the method to control the disease caused by vectors. Behavioral change of humans plays a vital role in reducing such diseases during outbreaks. Vector borne diseases can also be averted by better community mobilization and taking precautionary measures. Outbreak and recurrence of vector borne diseases inspired to prepare mathematical model on it. When the disease spread rapidly, people try to prevent it through isolation of susceptible, quarantine of infectious, social distancing and lock down in community, etc. [15]. So in this case even if the numbers of infected cases are large in number, the secondary infections slow down due to behavioral changes of hosts. At the same time control of vector population by spraying insecticides in atmosphere, applying vector repellent creams on skin, household insecticide, etc. help to slow down the infections [8]. Since vector borne epidemics are of high priority in the world at present, we propose a model on epidemics of a disease which is vector borne and we used non-monotone incidence rate for vectors as well as humans. Incidence rate for humans includes the parameter which measures the changes in susceptible host population taken for their protection from disease and incidence rate for vectors includes the parameter which measures effect of vector control.

The incidence rate for hosts $f(V) = \frac{k_1\lambda_1 S(t)V(t)}{(1+a_1V^2(t))}$ increases when count of V decreases and $f(V)$ decreases when count of V increases. Similarly, the incidence rate for vectors $f(I) = \frac{k_2\lambda_2 M(t)I(t)}{1+a_2I(t)}$ increases when count of I decreases and $f(I)$ decreases when count of I increases. Thus, the incidence rates used in this model referred as non-monotone. It interprets that when count of infection increases beyond some limit; people become more cautious and aware about the disease. They follow the protective measures strictly in that case and it results in decrease of infection.

2. MODEL FORMULATION

The model formulated below on vector borne disease is given by system (2.1) of differential equations in which non-monotone incidence rates for vector as well as host is used.

$$\begin{aligned}
\frac{dS(t)}{dt} &= \alpha_1 - \frac{k_1\lambda_1 S(t)V(t)}{1+a_1V^2(t)} - \beta_1 S(t)\\
\frac{dI(t)}{dt} &= \frac{k_1\lambda_1 S(t)V(t)}{1+a_1V^2(t)} - \gamma I(t) - \beta_1 I(t)\\
\frac{dR(t)}{dt} &= \gamma I(t) - \beta_1 R(t)\\
\frac{dM(t)}{dt} &= \alpha_2 - \frac{k_2\lambda_2 M(t)I(t)}{1+a_2I(t)} - \beta_2 M(t)\\
\frac{dV(t)}{dt} &= \frac{k_2\lambda_2 M(t)I(t)}{1+a_2I(t)} - \beta_2 V(t)
\end{aligned} \tag{2.1}$$

where,

$S(t)$ denotes the count of susceptible individuals in hosts

$I(t)$ denotes the count of infected individuals in hosts

$R(t)$ denotes the count of recovered individuals in hosts

such that $N_1(t) = S(t) + I(t) + R(t)$

$M(t)$ denotes the count of susceptible vectors

$V(t)$ denotes the count infective vectors

such that $N_2(t) = M(t) + V(t)$

α_1 is the recruitment rate of host population

λ_1 is the transmission rate from vector to host

β_1 is natural death rate of host population

γ is per capital recovery rate of host

α_2 is the recruitment rate of vector population

β_2 is natural death rate of vector population and

λ_2 is the transmission rate from host to vector

a_1 is the parameter which measures the changes in susceptible host population taken for their protection

a_2 is the parameter which measures effect of vector control.

k_1 and k_2 are proportionality constants

From system (2.1), we can easily obtain

$$\frac{d}{dt}N_1(t) = \alpha_1 - \beta_1 N_1(t)$$

$$\frac{d}{dt}N_2(t) = \alpha_2 - \beta_2 N_2(t)$$

The population sizes of vector and host are asymptotically steady in total and are given as follows:

$$\lim_{t\to\infty} N_1(t) = \alpha_1/\beta_1,$$

$$\lim_{t\to\infty} N_2(t) = \alpha_2/\beta_2$$

So we can consider with no loss of generality that

$$N_1(t) = \alpha_1/\beta_1,\ N_2(t) = \alpha_2/\beta_2 \text{ for all } t \geq 0.$$

So system (2.1) is qualitatively equivalent to the following system (2.2)

$$\frac{dS(t)}{dt} = \alpha_1 - \frac{k_1\lambda_1 S(t)V(t)}{1 + a_1 V^2(t)} - \beta_1 S(t)$$

$$\frac{dI(t)}{dt} = \frac{k_1\lambda_1 S(t)V(t)}{1 + a_1 V^2(t)} - \gamma I(t) - \beta_1 I(t) \tag{2.2}$$

$$\frac{dV(t)}{dt} = \frac{k_2\lambda_2}{1 + a_2 I(t)}\left(\frac{\alpha_2}{\beta_2} - V(t)\right) I(t) - \beta_2 V(t)$$

Values of recovered hosts and infected vectors can be determined from $R = \frac{\alpha_1}{\beta_1} - S - I$ and $M = \frac{\alpha_2}{\beta_2} - V$, respectively.

For biological reasons we require non-negative solutions. So the system (2.2) is studied in following closed set

$$\Gamma = \left\{(S, I, V) \in \mathbb{R}^3_+/0 \le S + I \le \alpha_1/\beta_1, 0 \le V \le \alpha_2/\beta_2, S \ge 0, I \ge 0\right\}$$

$E_0(\alpha_1/\beta_1, 0, 0)$ is disease free equilibrium for system (2.2).
Endemic equilibrium for system (2.2) obtained by

$$\alpha_1 - \frac{k_1\lambda_1 S(t)V(t)}{1 + a_1V^2(t)} - \beta_1 S(t) = 0$$

$$\frac{k_1\lambda_1 S(t)V(t)}{1 + a_1V^2(t)} - \gamma I(t) - \beta_1 I(t) = 0$$

$$\frac{k_2\lambda_2}{1 + a_2I(t)}\left(\frac{\alpha_2}{\beta_2} - V(t)\right) I(t) - \beta_2 V(t) = 0$$

This gives

$$a_1\beta_1\beta_2^2(\gamma + \beta_1)V^2 + [k_1k_2\alpha_1\lambda_1\lambda_2 + k_1\lambda_1\beta_2(\gamma + \beta_1) + a_2k_1\lambda_1\alpha_1\beta_2]V+$$

$$\beta_1\beta_2(\gamma + \beta_1)[1 - R_0] = 0.$$

Subsequently basic reproduction number is

$$R_0 = \frac{k_1k_2\alpha_1\alpha_2\lambda_1\lambda_2}{\beta_1\beta_2^2(\gamma + \beta_1)} \tag{2.3}$$

$E^*(S^*, I^*, V^*)$ is endemic equilibrium which is given by:

$$S^* = \frac{\alpha_1(1 + a_1 {V^*}^2)}{k_1\lambda_1 V + \beta_1[1 + a_1 {V^*}^2]} \tag{2.4}$$

$$I^* = \frac{k_1\alpha_1\lambda_1 V^*}{(\gamma + \beta_1)[k_1\lambda_1 V + \beta_1[1 + a_1 {V^*}^2]]} \tag{2.5}$$

$$V^* = \frac{-\Delta + \sqrt{\Delta^2 - 4a_1\beta_1^2\beta_2^2(\gamma + \beta_1)^2(1 - R_0)}}{2a_1\beta_1\beta_2(\gamma + \beta_1)} \tag{2.6}$$

where $\Delta = k_1k_2\alpha_1\lambda_1\lambda_2 + k_1\lambda_1\beta_2(\gamma + \beta_1) + a_2k_1\lambda_1\alpha_1\beta_2$

3. MODEL RESULTS

The properties of disease-free and the endemic equilibriums of system (2.2) are studied in this section and the stability conditions for these equilibriums are derived.

Theorem 3.1: When R_0 is less than one, E_0 is locally asymptotically stable and unstable for R_0 greater than one.

Proof: Jacobian matrix at E_0 becomes

$$J(E_0) = \begin{bmatrix} -\beta_1 & 0 & -\frac{k_1\lambda_1\alpha_1}{\beta_1} \\ 0 & -(\gamma+\beta_1) & \frac{k_1\lambda_1\alpha_1}{\beta_1} \\ 0 & \frac{k_2\lambda_2\alpha_2}{\beta_2} & -\beta_2 \end{bmatrix} \tag{3.1}$$

and the characteristic equation gives:

$$(\beta_1+\ell)[\ell^2+(\gamma+\beta_1+\beta_2)\ell+(1-R_0)(\gamma+\beta_1)\beta_2]=0$$

All eigen values of above characteristic equation contain negative real part for $R_0 < 1$ using Routh-Hurwitz condition [5] and E_0 is locally asymptotically stable. One eigen value is positive and two are negative for $R_0 > 1$ and in this case disease free equilibrium i.e. E_0 is unstable.

Theorem 3.2: E_0 is globally asymptotically stable for R_0 less than one and it is unstable for R_0 greater than one.

Proof: Consider function

$$L_1 = \frac{k_1\lambda_1\alpha_1}{\beta_1\beta_2}V + I$$

Then,

$$\frac{dL_1}{dt} = \frac{k_1\lambda_1\alpha_1}{\beta_1\beta_2}\left[\frac{k_2\lambda_2 I}{1+a_2I}\left(\frac{\alpha_2}{\beta_2}-V\right)-\beta_2V\right]+\frac{k_1\lambda_1SV}{1+a_1V^2}-(\gamma+\beta_1)I$$

$$= -(\gamma+\beta_1)\left[1-\frac{k_1k_2\alpha_1\alpha_2\lambda_1\lambda_2}{\beta_1\beta_2^2(\gamma+\beta_1)(1+a_2I)}\right]I-\frac{k_1k_2\alpha_1\alpha_2\lambda_1\lambda_2}{\beta_1\beta_2(1+a_2I)}VI-$$

$$\frac{k_1\alpha_1\lambda_1}{\beta_1}V+\frac{k_1\lambda_1SV}{1+a_1V^2}$$

$$\leq -(\gamma+\beta_1)[1-R_0]I - \frac{k_1k_2\alpha_1\lambda_1\lambda_2}{\beta_1\beta_2(1+a_2I)}VI - \frac{k_1\alpha_1\lambda_1}{\beta_1}V + \frac{k_1\lambda_1SV}{1+a_1V^2}$$

$$\leq -(\gamma+\beta_1)[1-R_0]I - \frac{k_1k_2\alpha_1\lambda_1\lambda_2}{\beta_1\beta_2(1+a_2I)}VI - \frac{k_1\alpha_1\lambda_1}{\beta_1}V + k_1\lambda_1SV$$

$$\leq -(\gamma+\beta_1)[1-R_0]I - \frac{k_1k_2\alpha_1\lambda_1\lambda_2}{\beta_1\beta_2(1+a_2I)}VI \quad \text{as } S \leq \frac{\alpha_1}{\beta_1}$$

We get $\frac{dL_1}{dt} \leq 0$ when $R_0 \leq 1$ for all $t \geq 0$ and hence the function L_1 is a Liapunov function. The equality $\dfrac{dL_1}{dt} = 0$ holds at $E_0(\alpha_1/\beta_1, 0, 0)$. Thus, in the closed set Γ, $\{E_0\}$ is the largest invariant. Hence, according to LaSalle's invariance principle [10,11], global stability of E_0 is proved.

The Jacobian matrix at the endemic equilibrium becomes,

$$(3.2) \qquad J(E^*) = \begin{bmatrix} -\frac{k_1\lambda_1V^*}{1+a_1V^{*2}} - \beta_1 & 0 & -\frac{k_1\lambda_1S^*(1-a_1V^{*2})}{(1+a_1V^{*2})^2} \\ \frac{k_1\lambda_1V^*}{1+a_1V^{*2}} & -\gamma-\beta_1 & \frac{k_1\lambda_1S^*(1-a_1V^{*2})}{(1+a_1V^{*2})^2} \\ 0 & \left(\frac{\alpha_2}{\beta_2} - V^*\right)\frac{k_2\lambda_2}{(1+a_2I^*)^2} & -\frac{k_2\lambda_2I^*}{1+a_2I^*} - \beta_2 \end{bmatrix}$$

$J^{[2]}(E^*)$ i.e. second additive compound matrix is obtained by

$$J^{[2]}(E^*) =$$

(3.3)

$$\begin{bmatrix} -\frac{k_1\lambda_1V^*}{1+a_1V^{*2}} - 2\beta_1 - \gamma & \frac{k_1\lambda_1S^*(1-a_1V^{*2})}{(1+a_1V^{*2})^2} & \frac{k_1\lambda_1S^*(1-a_1V^{*2})}{(1+a_1V^{*2})^2} \\ \left(\frac{\alpha_2}{\beta_2} - V^*\right)\frac{k_2\lambda_2}{(1+a_2I^*)^2} & -\frac{k_1\lambda_1V^*}{1+a_1V^{*2}} - \beta_1 - \frac{k_2\lambda_2I^*}{1+a_2I^*} - \beta_2 & 0 \\ 0 & \frac{k_1\lambda_1V^*}{1+\alpha_1V^{*2}} & -\frac{k_2\lambda_2I^*}{1+a_2I^*} - \gamma - \beta_1 - \beta_2 \end{bmatrix}$$

Theorem 3.3: Local asymptotic stability of endemic equilibrium E^* of the system (2.2) attains for R_0 greater than one.

Proof: Equation (3.2) gives,

$$trJ(E^*) = -\frac{k_1\lambda_1V^*}{1+a_1V^{*2}} - \frac{k_2\lambda_2I^*}{1+a_2I^*} - 2\beta_1 - \gamma - \beta_2 < 0$$

and

$$detJ(E^*) =$$

$$\left(-\frac{k_1\lambda_1 V^*}{1+a_1{V^*}^2}-\beta_1\right)$$
$$\left[(-\gamma-\beta_1)\left(-\frac{k_2\lambda_2 I^*}{1+a_2I^*}-\beta_2\right)-\frac{k_1\lambda_1 S^*(1-a_1{V^*}^2)}{(1+a_1{V^*}^2)^2}\left(\frac{\alpha_2}{\beta_2}-V^*\right)\frac{k_2\lambda_2}{(1+a_2I^*)^2}\right]$$
$$-\frac{k_1\lambda_1 S^*(1-a_1{V^*}^2)}{(1+a_1{V^*}^2)^2}\frac{k_1\lambda_1 V^*}{(1+a_1{V^*}^2)}\left(\frac{\alpha_2}{\beta_2}-V^*\right)\frac{k_2\lambda_2}{(1+a_2I^*)^2}$$
$$=-\left(\frac{k_1\lambda_1 V^*}{(1+a_1{V^*}^2)}+\beta_1\right)(\gamma+\beta_1)\left(\frac{k_2\lambda_2 I^*}{1+a_2I^*}+\beta_2\right)$$
$$-\frac{k_1\lambda_1\beta_1 S^*(a_1{V^*}^2-1)}{(1+a_1{V^*}^2)^2}\left(\frac{\alpha_2}{\beta_2}-V^*\right)\frac{k_2\lambda_2}{(1+a_2I^*)^2}$$

$\therefore\ detJ(E^*)<0,\qquad$ provided ${V^*}^2>1/a_1$

From equation (3.3), we have

$$detJ^{[2]}(E^*)=$$
$$\left(-\frac{k_1\lambda_1 V^*}{1+a_1{V^*}^2}-2\beta_1-\gamma\right)\left(-\frac{k_1\lambda_1 V^*}{1+a_1{V^*}^2}-\beta_1-\frac{k_2\lambda_2 I^*}{1+a_2I^*}-\beta_2\right)$$
$$\left(-\frac{k_2\lambda_2 I^*}{1+a_1I^*}-\gamma-\beta_1-\beta_2\right)$$
$$-\frac{k_1\lambda_1 S^*(1-a_1{V^*}^2)}{(1+a_1{V^*}^2)^2}\left(\frac{\alpha_2}{\beta_2}-V^*\right)\frac{k_2\lambda_2}{(1+a_2I^*)^2}\left(-\frac{k_2\lambda_2 I^*}{1+a_2I^*}-\gamma-\beta_1-\beta_2\right)$$
$$+\frac{k_1\lambda_1 S^*(1-a_1{V^*}^2)}{(1+a_1{V^*}^2)^2}\left(\frac{\alpha_2}{\beta_2}-V^*\right)\frac{k_2\lambda_2}{(1+a_2{I^*}^2)}\frac{k_1\lambda_1 V^*}{(1+a_1{V^*}^2)}$$
$$=-\left(\frac{k_1\lambda_1 V^*}{1+a_1{V^*}^2}+2\beta_1+\gamma\right)\left(\frac{k_1\lambda_1 V^*}{(1+a_1{V^*}^2)}\frac{k_2\lambda_2 I^*}{(1+a_2I^*)}+\beta_2\right)$$
$$\left(\frac{k_2\lambda_2 I^*}{1+a_2I^*}+\gamma+\beta_1+\beta_2\right)$$
$$-\frac{k_1\lambda_1 S^*(a_1{V^*}^2-1)}{(1+a_1{V^*}^2)^2}\left(\frac{\alpha_2}{\beta_2}-V^*\right)\frac{k_2\lambda_2}{(1+a_2{I^*}^2)}\left(\frac{k_2\lambda_2 I^*}{1+a_2I^*}+\gamma+\beta_1+\beta_2\right)$$
$$-\frac{k_1\lambda_1 S^*(a_1{V^*}^2-1)}{(1+a_1{V^*}^2)^2}\left(\frac{\alpha_2}{\beta_2}-V^*\right)\frac{k_2\lambda_2}{(1+a_2I^*)^2}-\frac{k_1\lambda_1 V^*}{(1+a_1{V^*}^2)}$$

$<0,\qquad$ as $\dfrac{\alpha_2}{\beta_2}-V^*>0,$

provided ${V^*}^2>1/a_1$.

We obtained negative values of tr $J(E^*)$, det $J(E^*)$, and det $J^{[2]}(E^*)$, so $J(E^*)$ has all of the eigen values with negative real part [2]. Hence the theorem follows.

Theorem 3.4: The system (2.2) is uniformly persistent when R_0 greater than one, that means, there exists $\in > 0$ (not dependent on initial conditions) such that $\lim\inf_{t\to\infty} S(t) > 0$, $\lim\inf_{t\to\infty} I(t) > 0$ and $\lim\inf_{t\to\infty} V(t) > 0$.

Proof: First to prove following results:

(i) Omega-limit point on the boundary of Γ is E_0 only.

(ii) Any orbit in interior of Γ can not have E_0 as its omega-limit point when R_0 is greater than 1.

(i) Boundary of Γ is transversal to vector field, excluding in the S-axis. Also the vector field is invariant for system (2.2).

On S-axis $\frac{dS}{dt}$ becomes

$$\frac{dS}{dt} = \alpha_1 - \beta_1 S$$

which gives $S \to \alpha_1/\beta_1$ as $t \to \infty$. Hence for system (2.2) omega-limit point on the boundary of Γ is E_0 only.

(ii) We define the function L_2 in Γ such that

$$L_2(t) = \frac{\beta_1\beta_2(1+R_0)}{2k_1\lambda_1\alpha_1} I(t) + V(t)$$

$$\frac{dL_2(t)}{dt} = \frac{\beta_1\beta_2(1+R_0)}{2k_1\lambda_1\alpha_1}\left(\frac{k_1\lambda_1 S(t)V(t)}{1+a_1V^2(t)} - (\gamma+\beta_1)I(t)\right) +$$

$$\frac{k_2\lambda_2}{1+a_2 I}\left(\frac{\alpha_2}{\beta_2} - V(t)\right) I(t) - \beta_2 V(t)$$

$$= \frac{\beta_1\beta_2(1+R_0)S(t)V(t)}{2\alpha_1(1+a_1V^2(t))} - \beta_2 V(t) + \frac{k_2\lambda_2}{1+a_2 I}\left(\frac{\alpha_2}{\beta_2} - V(t)\right) I(t) -$$

$$\frac{(1+R_0)}{2R_0}\frac{k_2\lambda_2\alpha_2}{\beta_2} I(t)$$

$$\geq \frac{\beta_1\beta_2(1+R_0)S(t)V(t)}{2\alpha_1(1+a_1V^2(t))} - \beta_2 V(t) + k_2\lambda_2\left(\frac{\alpha_2}{\beta_2} - V(t) - \frac{1}{2}\left(\frac{1}{R_0}+1\right)\frac{\alpha_2}{\beta_2}\right) I(t)$$

$$\geq \frac{\beta_1\beta_2(1+R_0)}{2\alpha_1}\left[S(t) - \frac{2\alpha_1}{\beta_1(1+R_0)}\right]V(t)+$$

$$k_2\lambda_2\left(\frac{\alpha_2}{\beta_2} - V(t) - \frac{1}{2}\left(\frac{1}{R_0}+1\right)\frac{\alpha_2}{\beta_2}\right)I(t)$$

Since $R_0 > 1$, then $\frac{1}{2}\left(\frac{1}{R_0}+1\right)$ and $\frac{2}{(1+R_0)}$ are less than one, therefore, E_0 has a neighborhood N such that in $(S, I, V) \in N \cap Int\Gamma$, the expression $\left[S(t) - \frac{2\alpha_1}{\beta_1(1+R_0)}\right]$ becomes positive.

In this neighborhood $N(E_0)$, we have that $\dfrac{dL_2(t)}{dt} > 0$ in $N(E_0) - \{E_0\}$. Note that level sets of L_2 are the plane

$$\frac{\beta_1\beta_2(1+R_0)}{2k_1\lambda_1\alpha_1}I(t) + V(t) = C$$

As C increases, it moves away from S-axis. So, all solutions of system (2.2) move away from E_0 as L_2 goes on increasing along the orbits initiating in $N \cap Int\Gamma$.

Theorem 3.5: Global asymptotic stability of E^* attains when R_0 greater than one.

Proof: The system (2.2) is uniformly persistent when R_0 greater than one and local asymptotic stability of E^* is attained. To prove the proposition, we prove that the system (2.2) has the property of stability of periodic orbits [6].

Let system (2.2) has periodic solution $P(t) = (S(t), I(t), V(t))$. To prove the stability of periodic orbits, it is sufficient to prove asymptotic stability of following linear non-autonomous system.

Consider the linear non-autonomous system

$$U'(t) = (J^{[2]}P(t))U(t) \tag{3.4}$$

To prove periodic orbits are stable, we prove the asymptotic stability of system (3.4).

Now,

$$J^{[2]}(S, I, V) =$$

$$\begin{bmatrix} -\frac{k_1\lambda_1 V}{1+a_1V^2} - 2\beta_1 - \gamma & -\frac{k_1\lambda_1 S(1-a_1V^2)}{(1+a_1V^2)^2} & \frac{k_1\lambda_1 S(1-a_1V^2)}{(1+a_1V^2)^2} \\ \left(\frac{\alpha_2}{\beta_2} - V\right)\frac{k_2\lambda_2}{(1+a_2I)^2} & -\frac{k_1\lambda_1 V}{1+a_1V^2} - \beta_1 - \frac{k_2\lambda_2 I}{1+a_2I} - \beta_2 & 0 \\ 0 & \frac{k_1\lambda_1 V}{1+a_1V^2} & -\frac{k_2\lambda_2 I}{1+a_2I} - \gamma - \beta_1 - \beta_2 \end{bmatrix}$$

For the values of $P(t)$, equation (3.4) becomes,

$$U_1'(t) = -\left(\frac{k_1\lambda_1 V}{1+a_1V^2} + 2\beta_1 + \gamma\right)U_1(t) + \frac{k_1\lambda_1 S(1-a_1V^2)}{(1+a_1V^2)^2}U_2(t) + \frac{k_1\lambda_1 S(1-a_1V^2)}{(1+a_1V^2)^2}U_3(t)$$

$$(3.5) \quad U_2'(t) = \left(\frac{\alpha_2}{\beta_2} - V\right)\frac{k_2\lambda_2}{(1+a_2I)^2}U_1(t) - \left(\frac{k_1\lambda_1 V}{1+a_1V^2} + \beta_1 + \frac{k_2\lambda_2 I}{1+a_2I} + \beta_2\right)U_2(t)$$

$$U_3'(t) = \frac{k_1\lambda_1 V}{1+a_1V^2}U_2(t) - \left(\frac{k_2\lambda_2 I}{1+a_2I} + \gamma + \beta_1 + \beta_2\right)U_3(t)$$

Consider the function

$$L_3\left(U(t), U_2(t), U_3(t), S(t), I(t), V(t)\right) = \left\|U_1(t), \frac{I(t)}{V(t)}U_2(t), \frac{I(t)}{V(t)}U_3(t)\right\|$$

Where the norm $\|\cdot\|$ is defined in $\mathbb{R}^3$ as

$$\|U_1(t), U_2(t), U_3(t)\| = \sup\left\{|U_1|, |U_2 + U_3|\right\}$$

So, we get that the orbit of $P(t)$ remains at a positive distance from the boundary of Γ. Thus constant $c > 0$ exists such that

$$L_3\left(U_1(t), U_2(t), U_3(t), S(t), I(t), V(t)\right) \geq c\left\|U_1(t), U_2(t), U_3(t)\right\| \tag{3.6}$$

Let a solution of system (3.5) be $(U_1(t), U_2(t), U_3(t))$ and

$$L_3(t) = \sup\left\{|U_1(t)|, \frac{I(t)}{V(t)}|U_2(t) + U_3(t)|\right\} \tag{3.7}$$

Thus, we get inequalities as follows

$$D_+|U_1(t)| \leq -\left(\frac{k_1\lambda_1 V}{1+a_1V^2} + 2\beta_1 + \gamma\right)|U_1(t)| + \frac{k_1\lambda_1 S(1-a_1V^2)}{(1+a_1V^2)^2}(|U_2(t) + U_3(t)|)$$

$$\leq -\left(\frac{k_1\lambda_1 V}{1+a_1V^2}+2\beta_1+\gamma\right)|U_1(t)|+\frac{k_1\lambda_1 S(1-a_1V^2)}{(1+a_1V^2)^2}\frac{V}{I}\left(\frac{I}{V}|U_2(t)+U_3(t)|\right)$$

$$(3.8)\quad D_+|U_2(t)|\leq\left(\frac{\alpha_2}{\beta_2}-V\right)\frac{k_2\lambda_2}{(1+a_2I)^2}|U_1(t)|-$$

$$\left(\frac{k_1\lambda_1 V}{1+a_1V^2}+\beta_1+\frac{k_2\lambda_2 I}{1+a_2I}+\beta_2\right)|U_2(t)|$$

$$D_+|U_3(t)|\leq\frac{k_1\lambda_1 V}{1+a_1V^2}|U_2(t)|-\left(\frac{k_2\lambda_2 I}{1+a_2I}+\gamma+\beta_1+\beta_2\right)|U_3(t)|$$

We have

$$D_+(|U_2(t)+U_3(t)|)$$

$$\leq\left(\frac{\alpha_2}{\beta_2}-V\right)\frac{k_2\lambda_2}{(1+a_2I)^2}|U_1(t)|-\left(\beta_1+\frac{k_2\lambda_2 I}{1+a_2I}+\beta_2\right)|U_2(t)|-$$

$$\left(\frac{k_2\lambda_2 I}{1+a_2I}+\gamma+\beta_1+\beta_2\right)|U_3(t)|$$

$$\leq\left(\frac{\alpha_2}{\beta_2}-V\right)\frac{k_2\lambda_2}{(1+a_2I)^2}|U_1(t)|-\left(\beta_1+\frac{k_2\lambda_2 I}{1+a_2I}+\beta_2\right)|U_2(t)+U_3(t)|$$

Now, we get

$$D_+\left(\frac{I}{V}|U_2(t)+U_3(t)|\right)=\left(\frac{I'}{I}-\frac{V'}{V}\right)\frac{I}{V}|U_2(t)+U_3(t)|+\frac{I}{V}D_+|U_2(t)+U_3(t)|$$

$$\leq\left(\frac{I'}{I}-\frac{V'}{V}\right)\frac{I}{V}|U_2(t)+U_3(t)|+\left(\frac{\alpha_2}{\beta_2}-V\right)\frac{k_2\lambda_2}{(1+a_2I)^2}\frac{I}{V}|U_1(t)|-$$

$$\frac{I}{V}\left(\beta_1+\frac{k_2\lambda_2 I}{1+a_2I}+\beta_2\right)|U_2(t)+U_3(t)|$$

(3.9)

$$\leq\left(\frac{\alpha_2}{\beta_2}-V\right)\frac{k_2\lambda_2}{(1+a_2I)^2}\frac{I}{V}|U_1(t)|+\left[\frac{I`}{I}-\frac{V`}{V}-\beta_1-\frac{k_2\lambda_2 I}{1+a_2I}-\beta_2\right]\frac{I}{V}|U_2(t)+U_3(t)|$$

We get

$$(3.10)\qquad D_+L_3(t)\leq\sup\{g_1(t),g_2(t)\}\,U(t)$$

where

$$g_1(t) = -\left(\frac{k_1\lambda_1 V}{1+a_1V^2} + 2\beta_1 + \gamma\right) + \frac{k_1\lambda_1 S(1-a_1V^2)}{(1+a_1V^2)^2}\frac{V}{I}$$

$$g_2(t) = \frac{k_2\lambda_2}{(1+a_2I)^2}\left(\frac{\alpha_2}{\beta_2} - V\right)\frac{I}{V} + \frac{I'}{I} - \frac{V'}{V} - \beta_1 - \frac{k_2\lambda_2 I}{1+a_2I} - \beta_2$$

We have

$$g_1(t) \leq -\left(\frac{k_1\lambda_1 V}{1+a_1V^2} + 2\beta_1 + \gamma\right) + \frac{k_1\lambda_1 S}{(1+a_1V^2)^2}\frac{V}{I}$$

$$= -\frac{k_1\lambda_1 V}{1+a_1V^2} - \beta_1 - \beta_1 - \gamma + \frac{k_1\lambda_1 S}{(1+a_1V^2)^2}\frac{V}{I}$$

$$= -\frac{k_1\lambda_1 V}{1+a_1V^2} - \beta_1 + \frac{I'}{I}$$

$$g_1(t) \leq -\beta_1 + \frac{I'}{I} \tag{3.11}$$

$$g_2(t) = \frac{V'}{V} + \frac{I'}{I} - \frac{V'}{V} - \beta_1 - \frac{k_2\lambda_2 I}{1+a_2I}$$

$$g_2(t) = \frac{I'}{I} - \beta_1 - \frac{k_2\lambda_2 I}{1+a_2I}$$

$$g_2(t) \leq -\beta_1 + \frac{I'}{I} \tag{3.12}$$

Hence

$$\sup\{g_1(t), g_2(t)\} \leq -\beta_1 + \frac{I'}{I}$$

Using Gronwall's inequality and (3.10), we get

$$L_3(t) \leq L_3(0)I(t)e^{-\beta_1 t} < L_3(0)\frac{\alpha_1}{\beta_1}e^{-\beta_1 t}$$

So it implies $L_3(t)$ tends to zero as t tends to infinity. From (3.6), we have

$$(U_1(t), U_2(t), U_3(t)) \to 0 \ as \ t \to \infty$$

This proves the asymptotic stability of linear system (3.5) and eventually periodic solution is asymptotically orbitally stable. Hence, we proved global asymptotic stability of endemic equilibrium E^*.

4. NUMERICAL RESULTS

Validity of theoretical results of proposed model is checked through simulation using MATLAB. Disease free equilibrium continues when R_0 less than one, i.e. disease dies out which is shown in fig. 4.1. Clearly from fig. 4.2, disease persists in the population for R_0 greater than one. We observe from fig. 4.3 that number of infective individual decreases as the host population starts taking precautions. Similarly, fig. 4.4 shows that number of infective individual decreases when the vector population is in control.

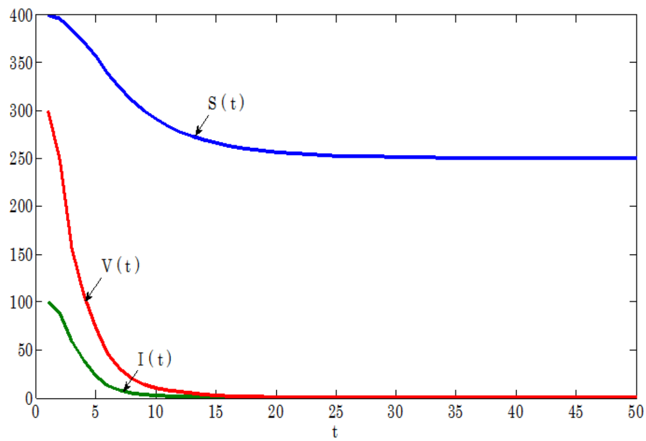

Fig. (a) Here $S(0) = 400, I(0) = 100, V(0) = 300, k_1 = 0.02, k_2 = 0.3, \alpha_1 = 10, \alpha_2 = 110, \lambda_1 = 0.006, \lambda_2 = 0.009, \beta_1 = 0.04, \beta_2 = 0.31, \gamma = 0.15, a_1 = 0.001, a_2 = 0.012, R_0 = 0.488$.

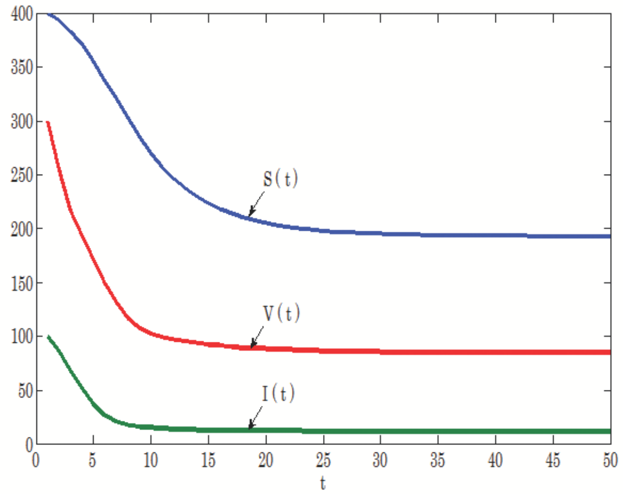

Fig. (b) Here $S(0) = 400, I(0) = 100, V(0) = 300, k_1 = 0.19, k_2 = 1.05, \alpha_1 = 10, \alpha_2 = 110, \lambda_1 = 0.006, \lambda_2 = 0.009, \beta_1 = 0.04, \beta_2 = 0.31, \gamma = 0.15, a_1 = 0.001, a_2 = 0.012, R_0 = 16.225$.

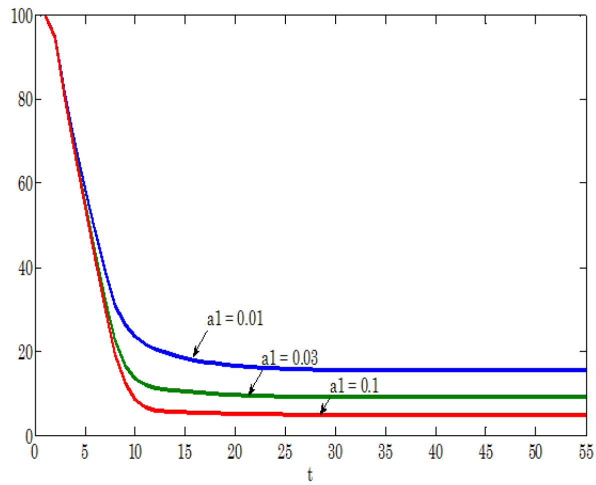

Fig. (c) Dependence of I^* on a_1

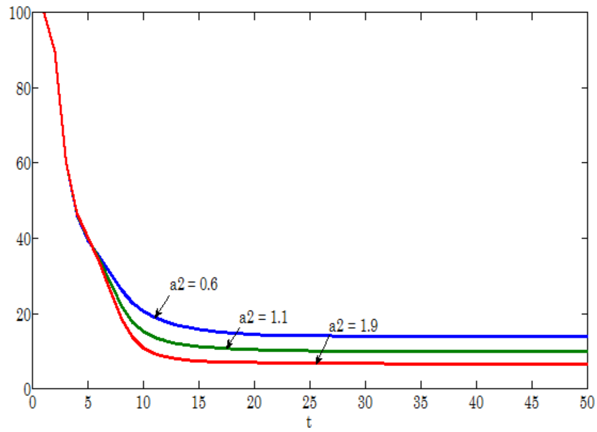

Fig. (d) Dependence of I^* on a_2

5. CONCLUDING REMARKS

We proposed vector-host disease transmission model with non-monotone incidence rates for vector and host as well. We used MATLAB and Simulink to validate results of model. We derived global asymptotic stability of disease free equilibrium when R_0 less than one and disease did not persist in the population. We proved endemic equilibrium becomes globally asymptotically stable for R_0 greater than one and disease became endemic. Expression (2.3) for R_0 shows that it is independent of a_1 and a_2 but numerical analysis shows that in endemic state decrease in number of infective hosts I^* takes place as parameters a_1 and a_2 increase. From the expression (2.3), it can be observed that if more effective preventive measures like quarantine and isolation are taken then it will help in reducing number of infective, rate of infection and also number of vectors. From steady state expression(2.6) of V^*, it is observed that V^* approaches zero as a_2 tends to infinity i.e. if proper hygiene is maintained and vector control measures such as indoor and outdoor spraying of insecticides, use of mosquito repellents, etc. are taken then lots of sufferings from diseases can be avoided. Also, from the steady state expression (2.5) of I^*, it is clear that I^* approaches zero as a_1 and a_2 tend to infinity.

The model can be used to interpret the effect of social awareness among the susceptible individuals. The vector borne diseases can be controlled if people use safe drinking water and improve sanitation. Simulation results show that preventive actions such as access to quality health care, early diagnosis of the disease, reduction of the breeding sources of vectors etc. will help in reducing disease transmission. We observe from the numerical simulation that basic reproduction number for some models are very low when compared to models given in $[2, 3, 5, 9, 10, 14]$ where the social, psychological factors were not included. The model helps the government to modify policies, to predict and control the growth of infectious diseases and will positively help in developing academic programs and literature on mathematical modeling of epidemiology.

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