

BULK VISCOUS BIANCHI TYPE-III COSMOLOGICAL MODEL WITH MODIFIED TAKABAYASI STRING IN THE PRESENCE OF VARYING Λ

SHOMA SEN¹, G. S. KHADEKAR² AND SHILPA SAMDURKAR³

¹DEPT. OF APPLIED MATHEMATICS, PRIYADARSHINI INSTITUTE OF ENGINEERING
AND TECHNOLOGY, HINGNA ROAD, NAGPUR-440019, MAHARASHTRA, INDIA

²DEPT. OF MATHEMATICS, RASHTRASANT TUKADOJI MAHARAJ NAGPUR
UNIVERSITY, MAHATMA JYOTIBA PHULE EDUCATIONAL CAMPUS, AMRAVATI
ROAD, NAGPUR-440033, MAHARASHTRA, INDIA

³DEPT. OF MATHEMATICS, VIDYA VIKAS ARTS COMMERCE AND SCIENCE COLLEGE,
SAMUDRAPUR-442305, DIST. WARDHA, MAHARASHTRA, INDIA

EMAILS: ¹SHOMASEN0808@GMAIL.COM, ²GKHADEKAR@YAHOO.COM AND

³RPAHADE2008@REDIFFMAIL.COM

(Received: 5 June 2020, Accepted: 12 January 2021)

ABSTRACT. In the present study, the solutions are obtained for Bianchi type III cosmological models with modified Takabayasi string of the form $\rho = (1 + \omega)\lambda + \Lambda$ in the presence of bulk viscous fluid. To obtain the realistic model we assume the relation between metric potential i. e $B = C^n$. We studied the role of Λ term in the evolution of universe. Here, we investigated two different cases for the cosmological model i. e case i) $\omega \rightarrow \infty$ and case ii) $\omega \rightarrow 0$. In both the cases, we found the physical significance of the cosmological term Λ in quadratic form i. e $\Lambda = \Lambda_0 + \Lambda_1 H + \Lambda_2 H^2$. All the physical and geometrical aspects of the model for both the cases are discussed for the corresponding solutions. It is observed that the resultant models follows the present day observations and literature favour the accelerating model of the universe. Also, we have plotted the graphs for detailed study of each case.

AMS Classification: 54H20.

Keywords: Bianchi type III spacetime, Takabayasi string; Bulk viscosity; Cosmological term Λ .

1. INTRODUCTION

At the advent of formation of universe string theory evolved wherein cosmic strings arise after the big bang explosion during phase transitions as the temperature goes down below critical temperature [37, 12, 35]. Stachel [30] has considered massless strings. Letelier [14] studied massive strings formed by a geometric string with particles attached along its extension that form the cloud are the generalization of Takabayasi realistic model of string [32] called as p strings [13]. Letelier [14] mentioned that the examples of equation of state for strings as follows:

a) Geometric string is $\rho = \lambda$ ($\rho_p = 0$),

where ρ_p is the cloud rest energy density.

b) Takabayasi strings is $\rho = (1 + \omega) \lambda$,

where ω is a constant such that $\omega > 0$.

c) Barotropic equation is $\rho = \rho(\lambda)$ ($\rho_p = \rho - \lambda$).

d) The modified EoS for cloud of string is $\rho = (1 + \omega) \lambda + \Lambda(t)$

where $\Lambda(t)$ is cosmological constant.

Banerjee et al. [2] have investigated some cosmological solutions of massive strings for Bianchi type I spacetime. Rahman et al. [24] have discussed the case of geometric string, Takabayasi string and barotropic string in their research work.

Bianchi Type-III cosmological models with gravitational constant G and the cosmological constant Λ have been discussed by [28]. Yadav et al. [36] have studied some Bianchi Type III string cosmological models with bulk Viscosity. Bali and Pradhan [1] have studied Bianchi Type III string cosmological models with time dependent bulk viscosity.

Tiwari and Sharma [33] have studied non existence of shear in Bianchi Type-III string cosmological models with bulk viscosity and time-dependent Λ term. Singh [29] studied anisotropic Bianchi V universe with magnetic field and bulk viscous fluid by taking different string models like geometric (Nambu string), Takabayasi string (p -string) and Reddy string and discussed the physical and geometrical aspects of

each string model. Inhomogeneous Bianchi type-VI α String Cosmological Model for Stiff Perfect Fluid Distribution in General Relativity have been studied by [27].

Nojiri and Odinstov [20] studied inhomogeneous equation of state of the universe: Phantom era, future singularity and crossing the phantom barrier. Cosmological scenarios with a time-varying Λ were proposed by several researchers. A number of models with different decay laws for the variation of cosmological term were investigated during last two decades [8, 23, 7, 15, 16].

The present universe is subject to an acceleration, which can be explained in terms of an ideal fluid (dark energy) weakly interacting with usual matter and which has an uncommon EOS. The origin of this dark energy is really dark: the proposed explanations vary from the modifications of gravity to the introduction of new fields with really strange properties. In the study of celebrated dark energy problem (for a review, see [9]) the ideal fluid with specific (sometimes, strange) EOS remains to be the simplest possibility for the description of the current cosmic acceleration. Various examples of ideal fluid EOS may be considered for this purpose: constant EOS with negative pressure, imperfect EOS [6], general EOS [19, 31], inhomogeneous EOS [20, 21, 5, 3, 25] and so on for extensive review, see [9]. Moreover, general relativity with ideal fluid of any type may be rewritten in the equivalent form [5] as modified gravity [17]. Of course, the introduction of ideal fluid with complicated EOS is somehow phenomenological approach because no explanation of the origin of such dark fluid follows.

Brevik et al. [4] studied a model where there is an ideal fluid with an inhomogeneous equation of state of the form $p = \omega(t)\rho + \Lambda(t)$, in which the parameters $\omega(t)$ and $\Lambda(t)$ depend linearly on time. One of these proposals deals with dark fluids, which possess an inhomogeneous EOS that may depend on time [20, 22, 5].

In 2008, Jamila and Rashid [10] have investigated the model of dark energy interacting with dark matter by choosing inhomogeneous equation of state for dark energy and a nonlinear interaction term for the underlying interaction. The equations of state have dependencies either on the energy densities, the redshift, the Hubble parameter or the bulk viscosity. By considering these possibilities the authors have derived the effective equations of state for the dark energy and studied each case in detail. Recently, Tripathi [34] have investigated Inhomogeneous Bianchi Type I Cosmological Model for Stiff Perfect Fluid Distribution. Also, FRW viscous

fluid cosmological model with time-dependent inhomogeneous equation of state have been studied by [11].

With this motivation in this paper, we have studied the modified Takabayasi string of the form $\rho = (1 + \omega)\lambda + \Lambda$ with limiting case of ω in the presence of bulk viscous fluid ξ in the framework of Bianchi III spacetime. The outline of the work is as follows: The brief idea of the introduction is presented in section 1. In section 2, we describe metric and field equations. Section 3 summarizes the results of the paper, i.e Case i) $\omega \rightarrow \infty$ and Case ii) $\omega \rightarrow 0$. Our findings are described in section (4). The paper ends with some concluding remarks in the last section.

2. METRIC AND FIELD EQUATIONS

We consider the Bianchi type-III spacetime of the form

$$(1) \quad ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{2\alpha x} dy^2 + C^2 dz^2.$$

Here, we assume the energy momentum tensor with bulk stress in the form,

$$(2) \quad T_{ij} = \rho u_i u_j - \lambda x_i x_j - \xi \theta (u_i u_j + g_{ij}),$$

where, ρ is the energy density, ξ is the coefficient of bulk viscosity and λ is the string tension. u_i is the four velocity vector of the fluid satisfying the relation $u_i u^i = -1$ and the string is assumed in the z -direction i. e. $-x_3 x^3 = -1$.

Here we consider modified EoS for Takabayasi string of the following form,

$$(3) \quad \rho = (1 + \omega)\lambda + \Lambda(t).$$

where we take the cosmological term Λ in quadratic form i. e.

$$(4) \quad \Lambda = \Lambda_0 + \Lambda_1 H + \Lambda_2 H^2.$$

The Einsteins field equations are given by

$$(5) \quad R_{ij} - \frac{1}{2} R g_{ij} = -T_{ij},$$

For the metric (1) and energy momentum tensor (2) in comoving system of co-ordinates the field equation (5) yields,

$$(6) \quad \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{\alpha^2}{A^2} = \rho,$$

$$(7) \quad \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = \xi \theta,$$

$$(8) \quad \frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = \xi\theta,$$

$$(9) \quad \frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\alpha^2}{A^2} = \lambda + \xi\theta,$$

$$(10) \quad \frac{\dot{A}}{A} - \frac{\dot{B}}{B} = 0.$$

Integrating Eq. (10), we get

$$(11) \quad A = mB,$$

where m is integrating constant.

We assume that the expansion θ in the model is proportional to shear scalar σ which leads to the condition,

$$(12) \quad B = C^n.$$

Einstein field equations (6) to (9) can be written as,

$$(13) \quad 2n\frac{\ddot{C}}{C} + n(3n-2)\frac{\dot{C}^2}{C^2} - \frac{\alpha^2}{m^2C^{2n}} = \lambda + \xi\theta.$$

$$(14) \quad n(n+2)\frac{\dot{C}^2}{C^2} - \frac{\alpha^2}{m^2C^{2n}} = \rho,$$

$$(15) \quad (n+1)\frac{\ddot{C}}{C} + n^2\frac{\dot{C}^2}{C^2} = \xi\theta,$$

For the metric (1), we define the spatial volume V , generalized Hubble parameter H , expansion factor θ , shear scalar σ^2 , deceleration parameter q , anisotropy parameter A_h as,

$$(16) \quad V = R^3 = C^{2n+1},$$

$$(17) \quad H = \frac{1}{3}(H_1 + H_2 + H_3) = \frac{(2n+1)}{3}\frac{\dot{C}}{C},$$

$$(18) \quad \theta = u_{;i}^i = (2n+1)\frac{\dot{C}}{C},$$

$$(19) \quad \sigma^2 = \frac{1}{2}\left(\sum_{i=1}^3 H_i^2 - \frac{\theta^2}{3}\right) = \frac{(n-1)^2}{3}\frac{\dot{C}^2}{C^2},$$

$$(20) \quad q = -\frac{R\ddot{R}}{\dot{R}^2} = -\frac{3}{(2n+1)} \frac{\ddot{C}/C}{\dot{C}^2/C^2} + \frac{3}{(2n+1)} - 1,$$

$$(21) \quad A_h = \frac{1}{3} \sum \left(\frac{\Delta H_i}{H} \right)^2 = 2 \frac{(n-1)^2}{(2n+1)^2}.$$

3. SOLUTION OF THE FIELD EQUATIONS

From Eq. (13) and Eq. (15), we get the following equation,

$$(22) \quad \lambda = (n-1) \frac{\ddot{C}}{C} + 2n(n-1) \frac{\dot{C}^2}{C^2} - \frac{\alpha^2}{m^2 C^{2n}}.$$

By using modified equation of state from Eq. (3), Eq. (14) becomes,

$$(23) \quad \lambda = \frac{1}{(1+\omega)} \left[n(n+2) \frac{\dot{C}^2}{C^2} - \frac{\alpha^2}{m^2 C^{2n}} - \Lambda \right].$$

Now, equating Eq. (22) and Eq. (23), we get,

$$(24) \quad \begin{aligned} & \frac{\ddot{C}}{C} + \left[2n - \frac{n(n+2)}{(n-1)(1+\omega)} \right] \frac{\dot{C}^2}{C^2} \\ & - \frac{\alpha^2 \omega}{m^2 (n-1)(1+\omega)} \frac{1}{C^{2n}} + \frac{\Lambda}{(n-1)(1+\omega)} = 0. \end{aligned}$$

With the help of Eq. (4), the above equation reduces to,

$$(25) \quad \begin{aligned} & \frac{\ddot{C}}{C} + \left[2n - \frac{n(n+2)}{(n-1)(\omega+1)} + \frac{\Lambda_2 (2n+1)^2}{9(n-1)(\omega+1)} \right] \frac{\dot{C}^2}{C^2} \\ & + \frac{\Lambda_1 (2n+1)}{3(n-1)(\omega+1)} \frac{\dot{C}}{C} - \frac{\alpha^2 \omega}{m^2 (n-1)(\omega+1)} \frac{1}{C^{2n}} + \frac{\Lambda_0}{(n-1)(\omega+1)} = 0. \end{aligned}$$

The exact solution of the above differential equation is not possible. Hence, here we discuss the following two limiting cases depending upon ω as follows:

Case (i): $\omega \rightarrow \infty$ and **Case (ii):** $\omega \rightarrow 0$.

3.1. **Case (i):** $\omega \rightarrow \infty$. As $\omega \rightarrow \infty$, the differential equation (25) can be written as,

$$(26) \quad \frac{\ddot{C}}{C} + 2n \frac{\dot{C}^2}{C^2} - \frac{A_0}{C^{2n}} = 0,$$

where

$$(27) \quad A_0 = \frac{\alpha^2}{m^2 (n-1)}.$$

Using the substitution $C = e^\beta$ in the differential equation Eq. (26), we get

$$(28) \quad \ddot{\beta} + (2n+1) \dot{\beta}^2 = A_0 e^{-2n\beta},$$

Again, we put $\dot{\beta}^2 = p$ in Eq. (28), reduces to,

$$(29) \quad \frac{dp}{d\beta} + 2(2n+1)p = 2A_0 e^{-2n\beta},$$

After integrating, we get,

$$(30) \quad \int \frac{d\beta}{\sqrt{\frac{A_0 e^{-2n\beta}}{(n+1)} + c_0 e^{-2(2n+1)\beta}}} = (t - t_0),$$

where c_0 and t_0 are integrating constant.

We are unable to find out the solution of the above differential equation. Therefore, we take the particular case i.e $c_0 = 0$. Hence, the differential Eq. (30), becomes

$$(31) \quad \int e^{n\beta} d\beta = \sqrt{\frac{A_0}{(n+1)}} (t - t_0),$$

After integrating Eq. (31), we get,

$$(32) \quad \beta = \log C = \frac{1}{n} \log \left[n \sqrt{\frac{A_0}{(n+1)}} (t - t_0) \right],$$

Substituting the value of A_0 from Eq. (27) in Eq. (32), we get,

$$(33) \quad C = \left[\frac{\alpha n}{m \sqrt{(n^2 - 1)}} (t - t_0) \right]^{\frac{1}{n}}.$$

3.1.1. *Some physical and geometrical aspects.* The expression for energy density ρ , string tension λ , cosmological term Λ and bulk viscosity coefficient ξ is given by,

$$(34) \quad \rho = \frac{(2n+1)}{n^2} \frac{1}{(t-t_0)^2}.$$

$$(35) \quad \lambda = 0.$$

$$(36) \quad \Lambda = \Lambda_0 + \frac{\Lambda_1 (2n+1)}{3n} \frac{1}{(t-t_0)} + \frac{\Lambda_2 (2n+1)^2}{9n^2} \frac{1}{(t-t_0)^2},$$

$$(37) \quad \xi = \frac{1}{n(2n+1)} \frac{1}{(t-t_0)},$$

Similarly, the expression for spatial volume V , Hubble parameter H , expansion scalar θ , shear scalar σ^2 as follows:

$$(38) \quad V = \left[\frac{\alpha n (t-t_0)}{m \sqrt{(n^2-1)}} \right]^{\frac{2n+1}{n}},$$

$$(39) \quad H = \frac{(2n+1)}{3n} \frac{1}{(t-t_0)},$$

$$(40) \quad \theta = \frac{(2n+1)}{n} \frac{1}{(t-t_0)},$$

$$(41) \quad \sigma^2 = \frac{(n-1)^2}{3n^2} \frac{1}{(t-t_0)^2}.$$

$$(42) \quad q = \frac{(n-1)}{(2n+1)},$$

$$(43) \quad A_h = \frac{2(n-1)^2}{(2n+1)^2}.$$

3.2. Case (ii): $\omega \rightarrow 0$. In this case, Eq. (3) reduces to $\rho = \lambda + \Lambda(t)$ which is a modified geometric string.

Hence Eq. (25) reduces to,

$$(44) \quad \frac{\ddot{C}}{C} - Z_1 \frac{\dot{C}^2}{C^2} + Z_2 \frac{\dot{C}}{C} + Z_3 = 0,$$

where $Z_1 = \left[n(4-n) - \frac{\Lambda_2(2n+1)^2}{9} \right] \frac{1}{(n-1)}$, $Z_2 = \frac{\Lambda_1(2n+1)}{3(n-1)}$, $Z_3 = \frac{\Lambda_0}{(n-1)}$.

Solving Eq. (44), we get

$$(45) \quad C = k_2 \exp \left[\frac{Z_2 t}{2(Z_1 - 1)} \right] \left[\sec \left(Z_4 \left(k_1 + \frac{t}{2} \right) \right) \right]^{\frac{1}{Z_1 - 1}}$$

where k_1 and k_2 are constants of integration

and $Z_4 = \sqrt{4Z_3 - 4Z_1Z_3 - Z_2^2}$.

3.2.1. Some physical and geometrical aspects. The expressions for energy density ρ , string tension λ , cosmological term Λ , coefficient of viscosity ξ , the spatial volume V , Hubble parameter H , expansion scalar θ , shear scalar σ^2 , deceleration parameter q and anisotropy parameter A_h as follows,

$$(46) \quad \rho = \frac{n(n+2)}{4(Z_1 - 1)^2} \left[Z_4 \tan \left[Z_4 \left(k_1 + \frac{t}{2} \right) \right] + Z_2 \right]^2$$

$$- \frac{\alpha^2}{m^2} \frac{1}{\left[k_2 \exp \left[\frac{Z_2 t}{2(Z_1 - 1)} \right] \left[\sec \left(Z_4 \left(k_1 + \frac{t}{2} \right) \right) \right]^{\frac{1}{Z_1 - 1}} \right]^{2n}},$$

$$\lambda = \frac{Z_4^2(n-1)}{4(Z_1 - 1)} \sec^2 \left[Z_4 \left(k_1 + \frac{t}{2} \right) \right]$$

$$+ \frac{(n-1)(2n+1)}{4(Z_1 - 1)^2} \left[Z_4 \tan \left[Z_4 \left(k_1 + \frac{t}{2} \right) \right] + Z_2 \right]^2$$

$$(47) \quad - \frac{\alpha^2}{m^2} \frac{1}{\left[k_2 \exp \left[\frac{Z_2 t}{2(Z_1 - 1)} \right] \left[\sec \left(Z_4 \left(k_1 + \frac{t}{2} \right) \right) \right]^{\frac{1}{Z_1 - 1}} \right]^{2n}},$$

$$\Lambda = \Lambda_0 + \frac{\Lambda_1(2n+1)}{6(Z_1 - 1)} \left[Z_4 \tan \left[Z_4 \left(k_1 + \frac{t}{2} \right) \right] + Z_2 \right]$$

$$(48) \quad + \frac{\Lambda_2(2n+1)^2}{36(Z_1 - 1)^2} \left[Z_4 \tan \left[Z_4 \left(k_1 + \frac{t}{2} \right) \right] + Z_2 \right]^2,$$

$$\xi = \frac{(n+1)Z_4^2}{2(2n+1)} \frac{\sec^2 \left[Z_4 \left(k_1 + \frac{t}{2} \right) \right]}{\left[Z_4 \tan \left[Z_4 \left(k_1 + \frac{t}{2} \right) \right] + Z_2 \right]}$$

$$(49) \quad + \frac{(n^2 + n + 1)}{2(2n + 1)(Z_1 - 1)} \left[Z_4 \tan \left[Z_4 \left(k_1 + \frac{t}{2} \right) \right] + Z_2 \right],$$

$$(50) \quad V = \left[k_2 \exp \left[\frac{Z_2 t}{2(Z_1 - 1)} \right] \left[\sec \left(Z_4 \left(k_1 + \frac{t}{2} \right) \right) \right]^{\frac{1}{Z_1 - 1}} \right]^{2n+1},$$

$$(51) \quad H = \frac{(2n + 1)}{6(Z_1 - 1)} \left[Z_4 \tan \left[Z_4 \left(k_1 + \frac{t}{2} \right) \right] + Z_2 \right],$$

$$(52) \quad \theta = \frac{(2n + 1)}{2(Z_1 - 1)} \left[Z_4 \tan \left[Z_4 \left(k_1 + \frac{t}{2} \right) \right] + Z_2 \right],$$

$$(53) \quad \sigma^2 = \frac{(n - 1)^2}{12(Z_1 - 1)^2} \left[Z_4 \tan \left[Z_4 \left(k_1 + \frac{t}{2} \right) \right] + Z_2 \right]^2,$$

$$(54) \quad q = \frac{-3Z_4^2(Z_1 - 1)}{(2n + 1)} \frac{\sec^2 \left[Z_4 \left(k_1 + \frac{t}{2} \right) \right]}{\left[Z_4 \tan \left[Z_4 \left(k_1 + \frac{t}{2} \right) \right] + Z_2 \right]^2} - 1,$$

$$(55) \quad A_h = \frac{2(n - 1)^2}{(2n + 1)^2}.$$

4. OUR FINDINGS:

Here, we have obtained solutions of the Einstein field equations for two cases:

Case (i): $\omega \rightarrow \infty$

- From the graph (1), it is observed that as t increases, scale factors also increase while all the other parameters as given by fig (2), fig (3) and fig (4) i.e energy density ρ , cosmological term Λ and Hubble parameter H decreases as t increases.
- The energy density condition $\rho > 0$ is satisfied. Here, from Eq. (35), it is clear that $\lambda = 0$.
- It is observed that the cosmological term Λ is positive and decreases as t increases.

Case (ii): $\omega \rightarrow 0$, here we discuss the modified geometric string.

- From the fig (6), it is clear that energy density ρ is a decreasing function of time t for the range $1 < n < 4$. Also, from Eq. (46), $\rho \geq 0$ accordingly values of constants are chosen.
- As t increases, Hubble parameter H and expansion scalar θ also increases.

- It is observed that as $t \rightarrow 0$, λ becomes constant.
- It can be seen that the spatial volume is zero as $t = 0$ for suitable values of constants and it increases with the increase of t .
- Since $\lim_{t \rightarrow \infty} \frac{\sigma}{\theta} = \text{constant}$, the model does not approach isotropy at late time.
- For the suitable values of constants, we get $q < 0$ which favours accelerating models. Hence, we can say that the result is consistent with the current observations of SNe Ia and CMBR.
- The dynamics of the mean anisotropy parameter depends on the value of n .
- As $n \rightarrow 1$, $q \rightarrow 0$ and A_h vanishes.

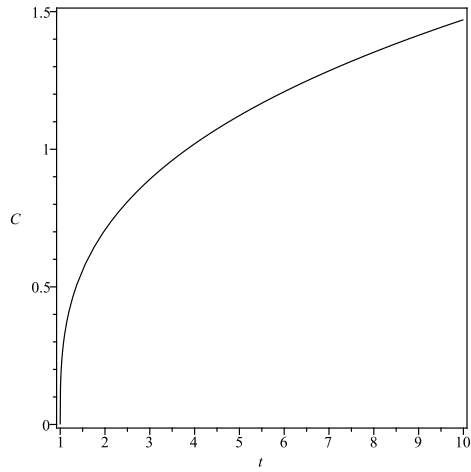


FIGURE 1. Case (i) $\omega \rightarrow \infty$: relation between C and time t ,
 $n = 3$, $\alpha = 1$, $m = 3$, $\Lambda_0 = 3$, $\Lambda_1 = 1$, $\Lambda_2 = 2$, $t_0 = 1$.

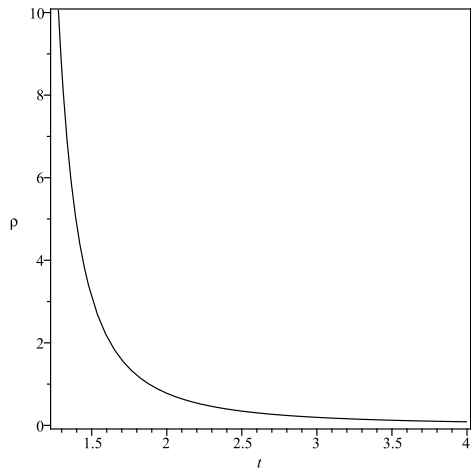


FIGURE 2. Case (i) $\omega \rightarrow \infty$: relation between ρ and time t ,
 $n = 3$, $\alpha = 1$, $m = 3$, $\Lambda_0 = 3$, $\Lambda_1 = 1$, $\Lambda_2 = 2$, $t_0 = 1$.

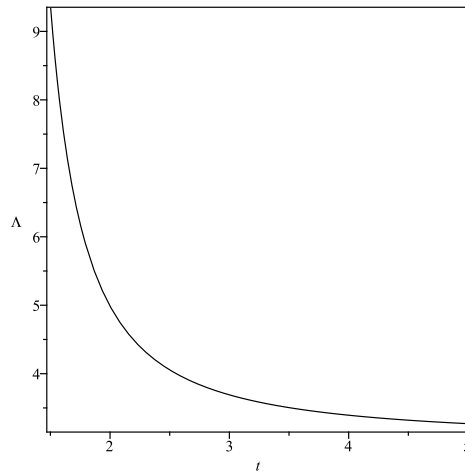


FIGURE 3. Case (i) $\omega \rightarrow \infty$: relation between Λ and time t ,
 $n = 3$, $\alpha = 1$, $m = 3$, $\Lambda_0 = 3$, $\Lambda_1 = 1$, $\Lambda_2 = 2$, $t_0 = 1$.

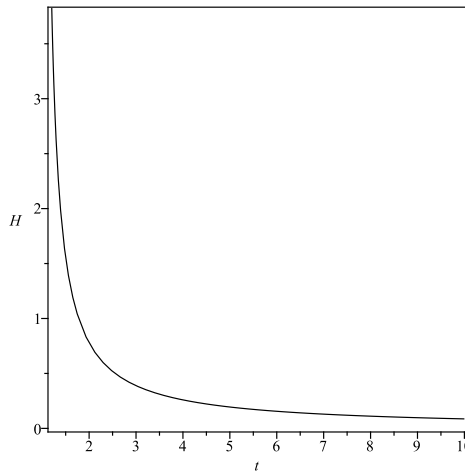


FIGURE 4. Case (i) $\omega \rightarrow \infty$: relation between H and time t ,
 $n = 3$, $\alpha = 1$, $m = 3$, $\Lambda_0 = 3$, $\Lambda_1 = 1$, $\Lambda_2 = 2$, $t_0 = 1$.

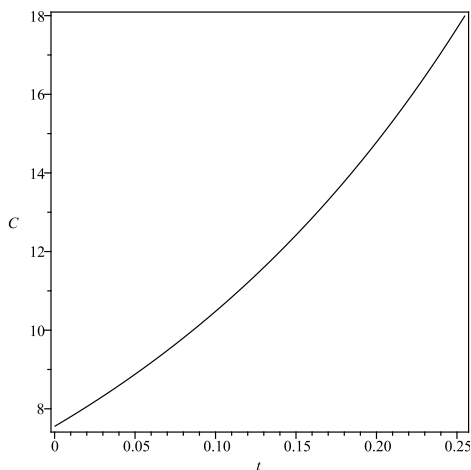


FIGURE 5. Case (ii) $\omega \rightarrow 0$, relation between C and time t for $n = 3$, $\Lambda_0 = -3$, $\Lambda_1 = 1$, $\Lambda_2 = 0.05$, $\alpha = 1$, $m = 3$, $k_1 = 1$, $k_2 = 2$, $Z_1 = 1.36$, $Z_2 = 1.16$, $Z_3 = -1.5$, $Z_4 = 0.9024$.

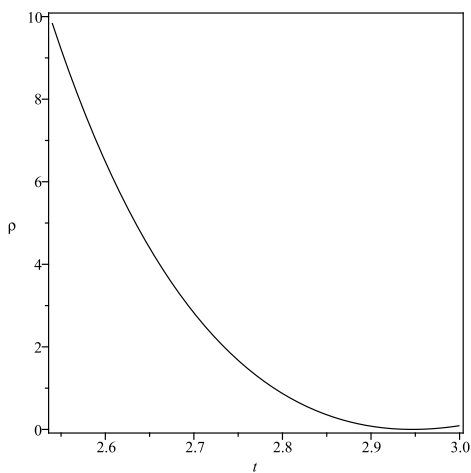


FIGURE 6. Case (ii) $\omega \rightarrow 0$, relation between ρ and time t for $n = 3$, $\Lambda_0 = -3$, $\Lambda_1 = 1$, $\Lambda_2 = 0.05$, $\alpha = 1$, $m = 3$, $k_1 = 1$, $k_2 = 2$, $Z_1 = 1.36$, $Z_2 = 1.16$, $Z_3 = -1.5$, $Z_4 = 0.9024$.

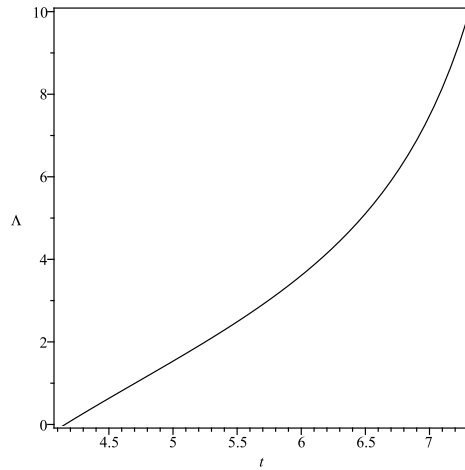


FIGURE 7. Case (ii) $\omega \rightarrow 0$, relation between Λ and time t for $n=3$, $\Lambda_0 = -3$, $\Lambda_1 = 1$, $\Lambda_2 = 0.05$, $\alpha = 1$, $m = 3$, $k_1 = 1$, $k_2 = 2$, $Z_1 = 1.36$, $Z_2 = 1.16$, $Z_3 = -1.5$, $Z_4 = 0.9024$.

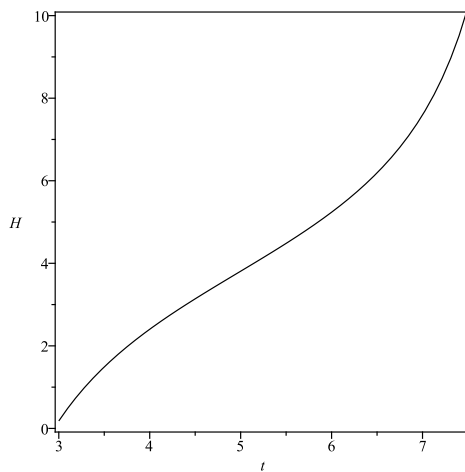


FIGURE 8. Case (ii) $\omega \rightarrow 0$, relation between H and time t for $n = 3$, $\Lambda_0 = -3$, $\Lambda_1 = 1$, $\Lambda_2 = 0.05$, $\alpha = 1$, $m = 3$, $k_1 = 1$, $k_2 = 2$, $Z_1 = 1.36$, $Z_2 = 1.16$, $Z_3 = -1.5$, $Z_4 = 0.9024$.

5. CONCLUSION:

In this paper, we have investigated the modified EoS of Takabayasi string of the form $\rho = (1 + \omega)\lambda + \Lambda(t)$ in the presence of bulk viscous fluid and obtained the solution of the field equation in the framework of Bianchi III model. We have analysed the two interesting cases depending on ω i. e the limiting cases $\omega \rightarrow \infty$ and $\omega \rightarrow 0$. It is noted that for $\omega \rightarrow \infty$, we solved the Eq.(26) for the scale factor C and obtained the various physical parameters. Similarly, for $\omega \rightarrow 0$, we solved the D. E (44) for the value of C and obtained the exact solution and discussed all the physical parameters in detail by plotting the graphs.

Case (i): $\omega \rightarrow \infty$

From the graph (1), it is observed that as t increases, scale factors also increase while all the other parameters as given by fig (2), fig (3) and fig (4) i.e energy density ρ , cosmological term Λ and Hubble parameter H decreases as t increases. The energy density condition $\rho > 0$ is satisfied. Here, from Eq. (35), it is clear that $\lambda = 0$. It is observed that the cosmological term Λ is positive and decreases as t increases.

Case (ii): $\omega \rightarrow 0$, here we discuss the modified geometric string.

From the fig (6), it is clear that energy density ρ is a decreasing function of time t for the range $1 < n < 4$. Also, from Eq. (46), $\rho \geq 0$ accordingly values of constants are chosen. As t increases, Hubble parameter H and expansion scalar θ also increases. It is observed that as $t \rightarrow 0$, λ becomes constant. It can be seen that the spatial volume is zero as $t = 0$ for suitable values of constants and it increases with the increase of t . Since $\lim_{t \rightarrow \infty} \frac{\sigma}{\theta} = \text{constant}$, the model does not approach isotropy at late time. For the suitable values of constants, we get $q < 0$ which favours accelerating models. Hence, we can say that the result is consistent with the current observations of SNe Ia and CMBR. The dynamics of the mean anisotropy parameter depends on the value of n . As $n \rightarrow 1$, $q \rightarrow 0$ and A_h vanishes.

The model is accelerating for $n > 1$. Hence, the anisotropy in the models is maintained throughout in both the models. Also, the model with negative deceleration parameter is compatible with recent Supernovae Ia observations that the universe is undergoing a late time acceleration. Therefore, the model describes a continuously expanding, shearing and non-rotating universe. The present day observations and literature favour accelerating model of the universe.

REFERENCES

- [1] Bali, R., and Pradhan, A., Bianchi Type-III String Cosmological Models with Time Dependent Bulk Viscosity, *Chin. Phys. Lett.*, Vol. 24, Number 2, 585 (2007).
- [2] Banerjee, A., Sanyal, A. K. and Chakraborty, S., String cosmology in Bianchi I space-time, *Pramana J. Phys.*, Vol. 34, 1 (1990).
- [3] Brevik, I., Gorbunova, O., Dark energy and viscous cosmology, *Gen. Rel. Grav.*, Vol. 37, 2039 (2005).
- [4] Brevik, I., Gorbunova, O. G. and Timoshkin, A. V., Dark Energy Fluid with Time-Dependent, Inhomogeneous Equation of State, *Eur. Phys. J.*, Vol. C51, 179 (2007).
- [5] Capozziello, S., Cardone, V., Elizalde, E., Nojiri, S. and Odintsov, S. D., Observational constraints on dark energy with generalized equations of state, *Phys. Rev. D.*, Vol. 73, 043512 (2006).
- [6] Cardone, V., Tortora, C., Troisi, A. and Capozziello, S., Beyond the perfect fluid hypothesis for the dark energy equation of state, *Phys. Rev. D.*, Vol. 73, 043508 (2006).
- [7] Carvalho, J. C., Lima, J. A. S. and Waga, I., Cosmological consequences of a time-dependent Λ term *Phys. Rev. D*, Vol. 46, 2404 (1992).
- [8] Chen, W. and Wu, Y. S., Implications of a cosmological constant varying as R^{-2} , *Phys. Rev. D*, Vol. 41, 695 (1990).
- [9] Copeland, E., Sami, M. and Tsujikawa, S., Dynamics of dark energy, *Int. Mod. Phys.*, Vol D15, 1753 (2006).
- [10] Jamila, M., Rashid, Interacting dark energy with inhomogeneous equation of state, *M. A., Eur. Phys. J. C* Vol. 56, 429 (2008).
- [11] Khadekar, G. S., Raut, D., FRW viscous fluid cosmological model with time-dependent inhomogeneous equation of state, *Int. J. Geom. Methods Mod. Phys.* Vol. 15, 1830001 (2018).
- [12] Kibble, T. W. B., Topology of cosmic domains and strings, *J. Phys. A: Mathematical and General*, Vol. 9, Number 8, 1387 (1976).
- [13] Letelier, P. S., Unified theory of direct interaction between particles, strings, and membranes, *J. Math. Phys.*, Vol. 19, 1898 (1978).
- [14] Letelier, P. S., String cosmologies, *Phys. Rev. D*, Vol. 28, 2414 (1983).
- [15] Lima, J. A. S. and Maia J. M. F., Deflationary cosmology with decaying vacuum energy density, *Phys. Rev. D.*, Vol 49, 5597 (1994).
- [16] Lima, J. A. S. and Trodden, M., Decaying vacuum energy and deflationary cosmology in open and closed universes, *Phys. Rev. D* Vol. 53, 4280 (1996).
- [17] Nojiri, S. and Odintsov, S. D., Quantum de Sitter cosmology and phantom matter, *Phys. Lett. B.*, Vol. 562, Issue 3-4, 147 (2003).
- [18] Nojiri, S. and Odintsov, S. D., Effective equation of state and energy conditions in phantom/tachyon inflationary cosmology perturbed by quantum effects, *Phys. Lett. B*, Vol. 571, Issue 1-2, 1(2003).

- [19] Nojiri, S., Odintsov, S. D., Final state and thermodynamics of a dark energy universe, *Phys. Rev. D.*, Vol. 70, 103522 (2004).
- [20] Nojiri, S., Odintsov, S. D., Inhomogeneous equation of state of the universe: Phantom era, future singularity, and crossing the phantom barrier, *Phys. Rev. D.* Vol. 72, 023003 (2005).
- [21] Nojiri, S. and Odintsov, S. D., The oscillating dark energy: future singularity and coincidence problem *Phys. Lett. B*, Vol 637, Issue 3, 139 (2006).
- [22] Nojiri, S., Odintsov, S. D., Introduction to modified gravity and gravitational alternative for dark energy, *Int. J. Geom. Method. Mod. Phys.*, Vol. 4, No.1, 115 (2007).
- [23] Pavon, D., Jou, J., Bafaluy, D., *Class. Quant. Grav.*, Vol. 8, 357 (1991).
- [24] Rahman, F., Chakraborty, S., Hossain, M., Begum, N. and Das, S., String Cosmology in the Lyra Geometry, *Bulg. J. Phys.*, Vol. 28, No. 5-6, 217 (2001).
- [25] Ren, J., Meng, X. H., Cosmological model with viscosity media (dark fluid) described by an effective equation of state, *Phys. Lett. B.*, Vol. 633, Issue 1, 1 (2006).
- [26] Ren, J. and Meng, X. H., Modified equation of state, scalar field, and bulk viscosity in Friedmann universe, *Phys. Lett. B.*, Vol. 636, Issue 1, 5 (2006).
- [27] Sharma, A., Tyagi, A., Chhajed, D., Inhomogeneous Bianchi type-VI α String Cosmological Model for Stiff Perfect Fluid Distribution in General Relativity, *Prespacetime Journal*, Vol. 7, No 3, 615 (2016).
- [28] Singh, J. P., Tiwari, R. K. and Shukla, P., Bianchi Type-III Cosmological Models with Gravitational Constant G and the Cosmological Constant Λ , *Chin. Phys. Lett.*, Vol. 24, No. 12, 3325 (2007).
- [29] Singh C. P., Some exact solutions of magnetized viscous model in string cosmology, *Pramana Journal of Phys.*, Vol. 83, 63 (2014).
- [30] Stachel, J., Thickening the string. I. The string perfect dust, *Phys. Rev. D.* Vol. 21, 2171 (1980).
- [31] Stefancic, H., Expansion around the vacuum equation of state: Sudden future singularities and asymptotic behavior, *Phys. Rev. D*, Vol. 71, 084024 (2005).
- [32] Takabayasi T., in *Quantum Mechanics, Determinism, Casuality and Particles*, edited by M. Flato et al. (Riedel Dorrecht Holland), p. 179 (1976).
- [33] Tiwari, R., and Sharma S., Non Existence of Shear in Bianchi Type-III String Cosmological Models with Bulk Viscosity and Time-Dependent Λ Term, *Chin. Phys. Lett.*, Vol. 28, No. 2, 020401 (2011).
- [34] Tripathi, B. R., Tyagi, A., Parikh, S., Inhomogeneous Bianchi Type I Cosmological Model for Stiff Perfect Fluid Distribution, *Prespacetime Journal*, Vol. 7, Issue 12, 1666 (2016).
- [35] Vilenkin, A., Cosmic strings, *Phys. Rev. D.*, Vol. 24, 2082 (1981).
- [36] Yadav, M. K., Rai, A. and Pradhan, A., Some Bianchi Type III String Cosmological Models with Bulk Viscosity, *Int J Theor Phys.*, Vol. 46, 2677 (2007).
- [37] Zel'dovich, Ya. B., Cosmological consequences of spontaneous violation of discrete symmetry, *Sov. Phys. JETP.*, Vol. 40, 1 (1974).