

AXIALLY SYMMETRIC BULK VISCOUS COSMOLOGICAL MODEL IN $F(R,T)$ THEORY OF GRAVITY

G.R. AVCHAR¹, D.R. GOLECHHA² AND S.D. TADE³

^{1,2}DEPARTMENT OF MATHEMATICS, SHRI SHIVAJI EDUCATION SOCIETY,
AMRAVATI'S SCIENCE COLLEGE, CONGRESS NAGAR, NAGPUR 440012,
MAHARASHTRA, INDIA

³DEPARTMENT OF MATHEMATICS, JAWAHARLAL NEHRU ARTS, COMMERCE AND
SCIENCE COLLEGE, KACHIMET, NAGPUR 440023, MAHARASHTRA, INDIA
E-MAILS: ¹G.AVCHAR@REDIFFMAIL.COM, ²RICHAGOLECHHA@GMAIL.COM AND
³DRSTADE25@GMAIL.COM

(Received: 26 May 2020, Accepted: 1 February 2021)

ABSTRACT. In this paper, axially symmetric space-time in presence of perfect fluid with viscosity has been considered and studied within the framework of $F(R,T)$ theory of gravity proposed by (Harko et al. [14]). The solutions of the field equations are evaluated by assuming the proportionality of expansion (θ) to the shear scalar (σ). The physical and kinematical aspects of the model have also been discussed.

AMS Classification: 54H20.

Keywords: Axially symmetric metric; $F(R,T)$ theory of gravity; Bulk viscosity.

1. INTRODUCTION

Recent astronomical observations such as high red shift supernovae (SN Ia) experiments [4, 25, 30], cosmic microwave background (CMB) radiation [44, 45], large scale structure [47] confirm the idea that the current universe is not only expanding but also accelerating. These observations changed the concept of matter-energy

composition of the universe.

In view of this, it is now believed that the energy composition of the universe has 4% ordinary matter, 20% dark matter and 76% dark energy amongst which dark energy (DE) is one of the main components which is responsible for this cosmic acceleration of the universe. The negative pressure (i.e. positive energy density) has sufficient power to work against gravity and it may provide the accelerated expansion of the universe. Though dark energy is said to be main cause for cosmic acceleration issue, its nature is still changing. In order to understand the exotic nature of the dark energy and the consequent the late time cosmic acceleration of the universe, in recent years two different approaches in GR have gain more importance. GR can be modified in two different ways: either the matter source can be modified or the underlying geometry can be modified. The later approach has attracted more and more attention. In particular, Nojiri S. and Odintsov S. [22] formulated $F(R)$ theory of gravity and [8, 12, 21, 23, 24] are some of the author who discussed various aspects of $F(R)$ theory of gravity. [7, 31, 37, 38] developed the scalar-tensor theory of gravitation. Very recently, Harko T. [14] have proposed a modified $F(R, T)$ theory of gravity in where the gravitational lagarangian L_m is given by an arbitrary function of the Ricci scalar R and of the trace T of the stress energy tensor. There are so many authors who have widely investigated various cosmological models in $F(R, T)$ theory of gravity. Some of them are [33, 34, 35, 36, 39, 41, 46]. Adhav K. S. [1] studied LRS Bianchi type-I cosmological model in $F(R, T)$ theory of gravity. Dynamics of Non-Adiabatic charged spherical gravitational collapse in $F(R, T)$ gravity has been discussed by Ahmed R. and Abbas G. [2]. Chaubey R. and Shukla A. K. [10] have investigated a new class of Bianchi cosmological models in $F(R, T)$ gravity. Reddy D. R. K., Naidu R. L. and Satyanarayana B. [29] derived the Kaluza-Klein cosmological model in $F(R, T)$ gravity. The anisotropic cosmological models in $F(R, T)$ gravity with $\Lambda(T)$ have been investigated by Shukla A. K. and Chaubey R. [11]. Katore S. D., Sancheti M. M. and Hatkar S. P. [15] explained the magnetized anisotropic dark energy cosmological models in scale covariant theory of gravitation. In the presence of massive scalar field, Bianchi type-V dark energy cosmological model in general relativity, have been thoroughly explained by Naidu R. L., Aditya Y. and Reddy D. R. K. [20]. Singh V. and Singh C. [43] have studied the modified $F(R, T)$ theory and scalar

field cosmology. Ladke L. S., Hiwarkar R. A. and Jaiswal V. K. [16] have extensively investigated Cosmological model with Λ decaying in $F(R, T)$ gravity.

Moreover, as the viscosity plays an important role in cosmology, therefore in the recent years there have been an immense interest to investigate the cosmological model with viscous fluid. On the evolution of cosmological models, the effect of viscosity has been studied by Misner C. W. [18]. On the nature of initial singularity, its effect has been studied by Murphy G. L. [19]. In order to evaluate rate of cosmological entropy production, Weinberg S. [49] derived general formula for shear and bulk viscosity and used it. Some other authors like [3, 9, 13, 17, 27, 28, 32, 40, 42, 48, 50] also studied several different cosmological models in the presence of viscous fluid.

Motivated by the above investigations in this paper we have constructed axially symmetric bulk viscous cosmological model in the framework of $F(R, T)$ theory of gravity. The paper is organized as follows: in section 2, Metric and field equations have been considered. In section 3, we obtained the corresponding field equations and in section 4, some physical and kinematical parameters are analyzed with the help of graphical representation. The last section 5 contains some conclusions.

2. METRIC AND FIELD EQUATIONS

We consider the axially symmetric metric given by,

$$(1) \quad ds^2 = dt^2 - A^2(t)[d\chi^2 + f^2(\chi)d\phi^2] - B^2(t)dz^2,$$

where A and B are functions of cosmic time t alone while f is a function of the co-ordinate χ .

The field equations in $F(R, T)$ theory of gravity for the function $F(R, T) = R + 2f(T)$ [14] are

$$(2) \quad R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij} + 2f'(T)T_{ij} + [2pf'(T) + f(T)]g_{ij}.$$

where $f(T)$ is an arbitrary function of trace of the stress energy tensor of matter,

and overhead prime indicates differentiation with respect to argument.

The energy momentum tensor for perfect fluid with viscous fluid is given by,

$$(3) \quad T_i^j = (p' + \rho)u_i u^j - p' \delta_i^j + \eta g^{j\beta} [u_{i;\beta} + u_{\beta;i} - u_i u^\alpha u_{\beta;\alpha} - u_\beta u^\alpha u_{i;\alpha}],$$

$$(4) \quad \text{where} \quad p' = p - \left(\xi - \frac{2}{3}\eta\right)u^i_{;i},$$

and u^i is four velocity vector satisfying the condition

$$(5) \quad g_{ij}u^i u^j = 1.$$

Here ρ is energy density, p is the pressure, η and ξ are the coefficients of shear and bulk viscosity respectively.

where,

$$(6) \quad \eta > 0, \xi > 0$$

We have taken them as constant.

Using co-moving co-ordinate and a particular choice of function given by,

$$(7) \quad f(T) = \mu T,$$

where μ is constant.

With the help of equation (3) to (7), the field equations (2) for the metric (1) yield the set of independent equations as:

$$(8) \quad \frac{A''}{A} + \frac{A'B'}{AB} + \frac{B''}{B} = (3\mu + 8\pi)p - \rho\mu - [(5\mu + 8\pi)\xi + (\frac{14}{3}\mu + \frac{32}{3}\pi)\eta]\frac{A'}{A} - 2\mu\eta\frac{B'}{B}.$$

$$(9) \quad \frac{2A''}{A} + \frac{A'^2}{A^2} - \frac{f''}{fA^2} = (3\mu + 8\pi)p - \rho\mu - [(5\mu + 8\pi)\xi + (\frac{8}{3}\mu + \frac{32}{3}\pi)\eta]\frac{B'}{B} - 4\mu\eta\frac{A'}{A}.$$

$$(10) \quad \frac{A'^2}{A^2} + \frac{2A'B'}{AB} - \frac{f''}{fA^2} = -(3\mu + 8\pi)\rho + p\mu - (\frac{4A'}{A} + \frac{2B'}{B})\mu\eta.$$

Now, the functional dependence of the metric imply

$$(11) \quad \frac{f''}{f} = k^2 \quad \text{and} \quad k^2 = \text{constant}.$$

Now, if $k = 0$, then $f(\chi) = k_1\chi + k_2, 0 < \chi < \infty$, where k_1 and k_2 are constant of integration.

Without any loss of generality, we take $k_1 = 1$ and $k_2 = 0$, so that we have $f(\chi) = \chi$ resulting in flat model of the universe.

Hence, field equations (8)-(10) reduces to,

$$(12) \quad \frac{A''}{A} + \frac{A'B'}{AB} + \frac{B''}{B} = (3\mu + 8\pi)p - \rho\mu - [(5\mu + 8\pi)\xi + (\frac{14}{3}\mu + \frac{32}{3}\pi)\eta]\frac{A'}{A} - 2\mu\eta\frac{B'}{B}.$$

$$(13) \quad \frac{2A''}{A} + \frac{A'^2}{A^2} = (3\mu + 8\pi)p - \rho\mu - [(5\mu + 8\pi)\xi + (\frac{8}{3}\mu + \frac{32}{3}\pi)\eta]\frac{B'}{B} - 4\mu\eta\frac{A'}{A}.$$

$$(14) \quad \frac{A'^2}{A^2} + \frac{2A'B'}{AB} = -(3\mu + 8\pi)\rho + p\mu - (\frac{4A'}{A} + \frac{2B'}{B})\mu\eta.$$

3. SOLUTION OF THE FIELD EQUATIONS

The field equations, we obtained, is a system of three independent equations with four unknowns A , B , p and ρ . Therefore to obtain the determinate solution of the system, we need one more physical assumption amongst the unknown parameters. We assume the condition that the expansion scalar (θ) is proportional to the component of shear tensor (σ) which also represents anisotropy of the universe. This lead to a polynomial relation between the metric coefficients

$$(15) \quad A = B^m,$$

where $m \neq 0$ is proportionality constant.

Here, we take help of special law of variation proposed by [5] for the Hubble parameter that yield constant value of deceleration parameter. We consider constant deceleration parameter model defined by

$$(16) \quad q = -\frac{RR''}{R'^2} = \text{constant},$$

where R is scale factor given by

$$(17) \quad R = (A^2 B)^{\frac{1}{3}}.$$

Solving equation (16), we obtain

$$(18) \quad R = (at + b)^{\frac{1}{1+q}}.$$

where $(a \neq 0)$ and b are constant of integration, and the equation implies condition for expansion of the universe i.e. $1 + q > 0$.

With the help of (17) and (18), we get

$$(19) \quad A = (at + b)^{\frac{3(\beta-1)}{2\alpha\beta}}.$$

and

$$(20) \quad B = (at + b)^{\frac{3}{\alpha\beta}}.$$

where $\alpha = (q+1)$ and $\beta = (2m+1)$ are constants and a, b are integrating constants.

Without any loss of generality, we take $a = 1$ and $b = 0$, then equation (1) becomes

$$(21) \quad ds^2 = dt^2 - t^{\frac{3(\beta-1)}{\alpha\beta}} [d\chi^2 + f^2(\chi)d\phi^2] - t^{\frac{6}{\alpha\beta}} dz^2.$$

Also, the field equations (12)-(14) reduces to,

$$(22) \quad (3\mu + 8\pi)p - \rho\mu = \frac{6\mu\eta}{t\alpha\beta} + [(5\mu + 8\pi)\xi + (\frac{14}{3}\mu + \frac{32}{3}\pi)\eta] \frac{3(\beta-1)}{2t\alpha\beta} + \frac{3(3\beta^2 - 2\alpha\beta^2 - 2\alpha\beta + 9)}{4t^2\alpha^2\beta^2}.$$

$$(23) \quad (3\mu + 8\pi)p - \rho\mu = \frac{(6\mu\eta)(\beta - 1)}{t\alpha\beta} + [(5\mu + 8\pi)\xi + (\frac{8}{3}\mu + \frac{32}{3}\pi)\eta]\frac{3}{t\alpha\beta} + \frac{3(9\beta^2 - 4\alpha\beta^2 + 4\alpha\beta - 18\beta + 9)}{4t^2\alpha^2\beta^2}.$$

$$(24) \quad (3\mu + 8\pi)\rho - p\mu = -\frac{9(\beta^2 - 6\beta + 5)}{4t^2\alpha^2\beta^2} - \frac{6\mu\eta}{t\alpha}.$$

4. SOME PHYSICAL AND KINEMATICAL PROPERTIES OF THE MODEL

Equation (21) represents axially symmetric bulk viscous cosmological model in $F(R, T)$ theory of gravity which is physically significant for discussion of various stages of the universe. The physical and kinematical parameters which are important to discuss the physics of the cosmological model (21) are obtained with the help of equations (22)-(24) as:

Proper energy density is given by,

$$(25) \quad \rho = \frac{\mu}{(8\pi + 2\mu)(t\alpha\beta)(4\mu + 8\pi)} [6\mu\eta + \frac{3(\beta - 1)}{2} [(5\mu + 8\pi)\xi + (\frac{14\mu}{3} + \frac{32\pi}{3})\eta]] + \frac{1}{4(t^2\alpha^2\beta^2)(8\pi + 2\mu)} [\frac{3\mu(15\beta^2 - 2\alpha\beta^2 - 2\alpha\beta - 72\beta + 69)}{4\mu + 8\pi} - 9(\beta^2 - 6\beta + 5)] + \frac{6\mu\eta}{(t\alpha)(8\pi + 2\mu)} [\frac{\mu}{4\mu + 8\pi} - 1].$$

Figure-1 represents the graph of density (ρ) verses time (t). It is observed that density $\rho > 0$ throughout, and tends to zero as time t tends to ∞ which implies

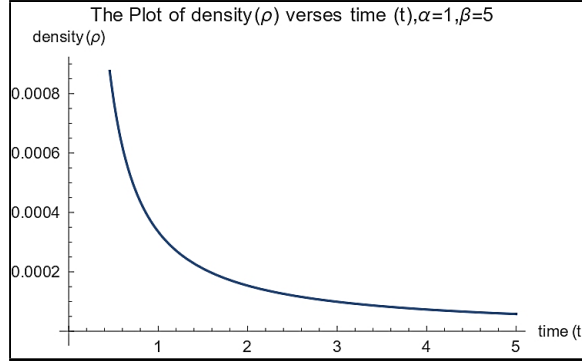


FIGURE 1. The plot of density (ρ) verses time (t), $\alpha = 1, \beta = 5$

accelerated expansion of the universe.

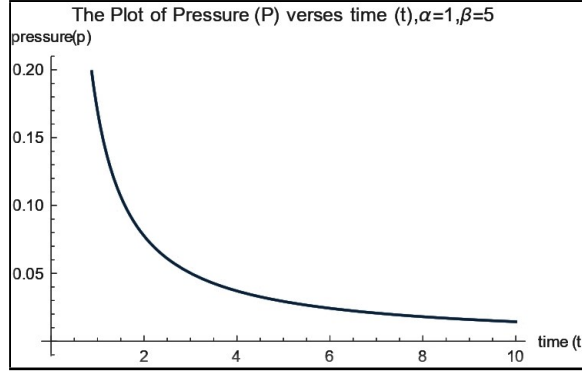
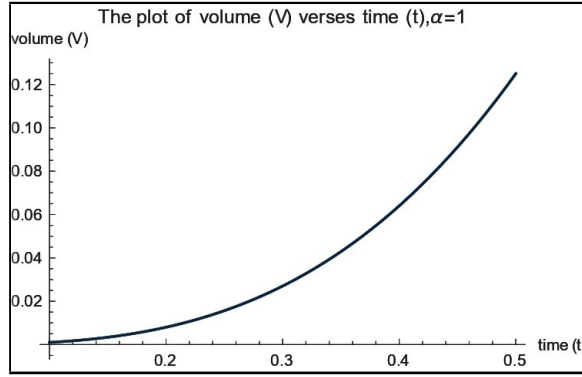
An Isotropic pressure is obtain as,

$$(26) \quad p = \frac{1}{4\mu + 8\pi} \left(\frac{\mu}{8\pi + 2\mu} + 1 \right) \left[\frac{1}{(t\alpha\beta)} \left[6\mu\eta + \frac{3(\beta - 1)}{2} [(5\mu + 8\pi)\xi + (\frac{14\mu}{3} + \frac{32\pi}{3})\eta] \right] \right. \\ \left. + \frac{6\mu\eta}{t\alpha} + \frac{3(15\beta^2 - 2\alpha\beta^2 - 2\alpha\beta - 72\beta + 69)}{4t^2\alpha^2\beta^2} \right] + \left[\frac{-9(\beta^2 - 6\beta + 5)}{4t^2\alpha^2\beta^2} - \frac{6\mu\eta}{(t\alpha)} \right] \frac{1}{(8\pi + 2\mu)}.$$

Figure-2 shows that pressure decreases exponentially with increase in time t , and finally tends to 0 as t tends to ∞ which depicts expansion of the universe. Also, at an initial epoch, it gets diverges, and finally vanishes at infinity.

Spatial Volume V is given by,

$$(27) \quad V = A^2 B = t^{\frac{3}{\alpha}}.$$

FIGURE 2. The plot of pressure (p) verses time (t), $\alpha = 1, \beta = 5$ FIGURE 3. The plot of volume (V) verses time (t), $\alpha = 1, \beta = 5$

From figure-3, we observed that the volume increases exponentially with increase in time, and attains infinity at $t = \infty$, which indicates that universe represented by our model is expanding, and showing late time acceleration which resembles the present scenario of modern cosmology.

Expansion scalar is given by,

$$(28) \quad \theta = \frac{3}{t\alpha}.$$

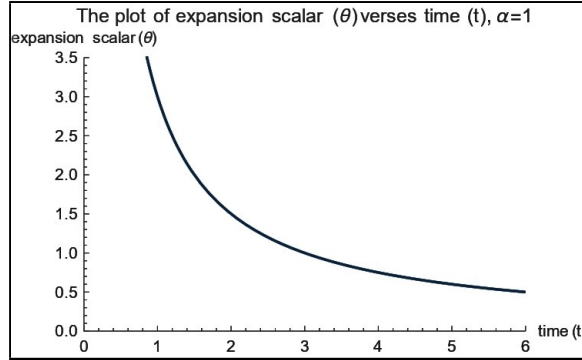


FIGURE 4. The plot of expansion scalar (θ) versus time (t) , $\alpha = 1$

From figure-4, we observe that as time t increases, expansion scalar decreases, and finally vanishes at t tends to infinity. Also, the expansion scalar shows epoch at an initial phase but later on it decreases exponentially, and converges to zero. It shows that universe has nonzero or infinite rate of expansion [15] at an initial phase which resembles with big bang model of universe. This rate of expansion get slows down with time.

Shear Scalar is obtain as,

$$(29) \quad \sigma^2 = \frac{3(m-1)^2}{t^2 \alpha^2 \beta^2}.$$

In figure-5, we can see that at an initial epoch shear scalar diverges. It can also be depict that as time t increases, shear scalar decreases, and finally vanishes at $t \rightarrow \infty$. Hence, at large time anisotropic universe will turn completely to isotropic [15, 26].

The Mean Anisotropic Parameter is given by,

$$(30) \quad A_m = \frac{2(m-1)^2}{\beta^2}.$$

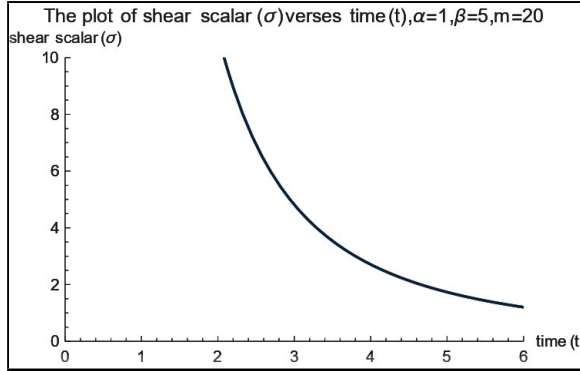


FIGURE 5. The plot of shear scalar (σ) versus time (t), $\alpha = 1, \beta = 5, m = 20$

and the ratio of anisotropy is obtain as,

$$(31) \quad \frac{\sigma^2}{\theta^2} = \frac{(m-1)^2}{3(2m+1)^2} \neq 0, m \neq 1.$$

From the above equation, it can be observed that for $m = 1$, the ratio becomes zero, hence implies that the universe becomes isotropic. For $m = 0$, the ratio becomes constant, in this case also, universe becomes isotropic. For $m = -\frac{1}{2}$, the ratio becomes infinite. Thus, we can say that for $m \neq -\frac{1}{2}$, $\frac{\sigma^2}{\theta^2}$ becomes constant or zero, indicating that universe approaches isotropy.

5. CONCLUSION

From the recent cosmological observations, it has been noted that the modified theory of gravity especially $F(R, T)$, attracted much attention. Hence, we have studied axially symmetric bulk viscous cosmological model in framework of $F(R, T)$ theory of gravity. It is observed that universe is expanding and accelerating which resembles the present scenario of modern cosmology. The physical parameters, energy density and pressure tends to zero as time t tends to infinity, and while both get diverge at an initial epoch. However, the volume increases exponentially with increase in the time showing the late time acceleration of the universe. The kinematical parameters, expansion scalar and shear both converges to zero. It can

be concluded that our model is anisotropic. From the ratio of anisotropy, it is to be noted that for $m \neq -\frac{1}{2}$ the universe represented by our model becomes isotropic.

REFERENCES

- [1] Adhav K. S., LRS Bianchi Type-I Cosmological Model in $F(R, T)$ Theory of Gravity, Astrophysics and Space Science, Vol. 339, Number 2, 365 (2012).
- [2] Ahmed R., and Abbas G., Dynamics of Non-Adiabatic Charged Spherical Gravitational Collapse in $F(R, T)$ Gravity, Chinese Journal of Physics, Vol. 65, 177 (2020).
- [3] Bali R. and Yadav M. K., Bianchi Type-IX Viscous Fluid Cosmological Model in General Relativity, Pramana - Journal of Physics, Vol. 64, Number 2, 187 (2005).
- [4] Bennett C. L. et al., First Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Preliminary Maps and Basic Results, The Astrophysical Journal Supplement Series, Vol. 148, 1 (2003).
- [5] Berman M. S., A Special Law of Variation for Hubbles Parameter, Il Nuovo Cimento B Series 11, Vol. 74, Number 2, 182 (1983).
- [6] Bhattacharya S. et al., Uniform Anisotropic Cosmological Model with String Source, Astrophysics and Space Science, Vol. 202, Number 1, 69 (1993).
- [7] Brans C. and Dicke R. H., Machs Principle and a Relativistic Theory of Gravitation, Physical Review, Vol. 124, Number 3, 925 (1961).
- [8] Carroll S. M. et al., Is Cosmic Speed-up Due to New Gravitational Physics?, Physical Review D, Vol. 70, Number 4, 043528 (2004).
- [9] Charjan S. S. and Dhongle P. R., Bianchi Type I Bulk Viscous Fluid String Dust Magnetized Cosmological Model with Λ -Term in Bimetric Theory of Gravitation, International Journal of Scientific Research, Vol. 6, Number 3, 35 (2019).
- [10] Chaubey R. and Shukla A. K., A New Class of Bianchi Cosmological Models in $F(R, T)$ Gravity, Astrophysics and Space Science, Vol. 343, Number 1, 415 (2013).
- [11] Chaubey R. and Shukla A. K., The Anisotropic Cosmological Models in $F(R, T)$ Gravity with $\Lambda(T)$, Pramana - Journal of Physics, Vol. 88, Number 65, 1 (2017).
- [12] Copeland E. J. et al., Dynamics of Dark Energy, International Journal of Modern Physics D, Vol. 15, Number 11, 1753 (2006).
- [13] Debnath P. S., Bulk Viscous Cosmological Model in $F(R, T)$ Theory of Gravity, International Journal of Geometric Methods in Modern Physics, Vol. 16, Number 1, 1950005 (2019).
- [14] Harko T. et al., $F(R, T)$ Gravity, Physics Review Journal D, arXiv: 1104.2669v2, (2011).
- [15] Katore S. D., Sancheti M. M. and Hatkar S. P., Magnetized Anisotropic Dark Energy Cosmological Models in Scale Covariant Theory of Gravitation, International Journal of Modern Physics D, Vol. 23, Number 7, 1450065 (2014).
- [16] Ladke L. S., Hiwarkar R. A. and Jaiswal V. K., Cosmological Model with Decaying Λ in $F(R, T)$ Theory of Gravity, International Journal of Science and Research, Vol. 4, Number 12, 2245 (2015).

- [17] Mishra B., Ray P. P. and Pacif S. K. J., Anisotropic Cosmological Models with Two Fluids, *Advances in High Energy Physics*, Vol. 2018, 6306848 (2018).
- [18] Misner C. W., Transport Processes in the Primordial Fireball, *Nature*, Vol. 214, 40 (1967).
- [19] Murphy G. L., Big-Bang Model Without Singularities, *Physical Review D*, Vol. 8, Number 12, 4231 (1973).
- [20] Naidu R. L., Aditya Y. and Reddy D. R. K., Bianchi Type-V Dark Energy Cosmological Model in General Relativity in the Presence of Massive Scalar Field, *Heliyon*, Vol. 5, Number 5, e01645 (2019).
- [21] Nojiri S. and Odintsov S., Introduction to Modified Gravity and Gravitational Alternative for Dark Energy, *International Journal of Geometric Methods in Modern Physics*, Vol. 4, Number 1, 115 (2007).
- [22] Nojiri S. and Odintsov S., Modified Gravity with Negative and Positive Powers of Curvature: Unification of Inflation and Cosmic Acceleration, *Physical Review D*, Vol. 68, Number 12, 123512 (2003a).
- [23] Nojiri S. and Odintsov S., Quantum de Sitter Cosmology and Phantom Matter, *Physics Letters B*, Vol. 562, Number 3-4, 147 (2003b).
- [24] Nojiri S. and Odintsov S., Gravity Assisted Dark Energy Dominance and Cosmic Acceleration, *Physics Letters B*, Vol. 599, Number 3-4, 137 (2004).
- [25] Perlmutter S. et al., Measurement of Ω and Λ from 42 High-Redshift Supernovae, *The Astrophysical Journal*, Vol. 517, Number 2, 565 (1999).
- [26] Ram S., Singh C. P. and Zeyauddin M., Anisotropic Bianchi-V Cosmological Models in Saez Ballester Theory of Gravitation, Vol. 23, Number 18, 2719 (2008)
- [27] Ram S., Singh S. K. and Verma M. K., Bulk Viscous BianchiI Cosmological Model in $F(R, T)$ Gravity Theory, *Physics and Astronomy International Journal*, Vol. 2, Number 4, 330 (2018).
- [28] Reddy D. R. K., Naidu R. L., Naidu K. D. and Prasad T. R., Kaluza-Klein Universe with Cosmic Strings and Bulk Viscosity in $F(R, T)$ Gravity, *Astrophysics and Space Science*, Vol. 346, Number 1, 261 (2013).
- [29] Reddy D. R. K., Naidu R. L. and Satyanarayana B., Kaluza-Klein Cosmological Model in $F(R, T)$ Gravity, *International Journal of Theoretical Physics*, Vol. 51, Number 10, 3222 (2012).
- [30] Riess A. G. et al., Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant To Appear in the *Astronomical Journal*, *The Astronomical Journal*, Vol. 116, Number 3, 1009 (1998).
- [31] Saez D. and Ballester V., A Simple Coupling with Cosmological Implications, *Physics Letters A*, Vol. 113, Number 9, 467 (1986).
- [32] Saha B., Bianchi Type I Universe with Viscous Fluid, *Modern Physics Letters A*, Vol. 20, Number 28, 2127 (2005).
- [33] Sahoo P. K., Mishra B. and Tripathy S. K., KaluzaKlein Cosmological Model in $F(R, T)$ Gravity with $\Lambda(T)$, *Indian Journal of Physics*, Vol. 90, Number 4, 485 (2015).
- [34] Sahoo P. K. and Sivakumar M., LRS Bianchi Type-I Cosmological Model in $F(R, T)$ Theory of Gravity with $\Lambda(T)$, *Astrophysics and Space Science*, Vol. 357, Number 1, 60 (2015).

- [35] Sahoo P., Taori B. and Mahanta K. L., Mixed Fluid Cosmological Model in $F(R, T)$ Gravity, Canadian Journal of Physics, Vol. 98, Number 11, (2020).
- [36] Sahoo P. K., Sahoo P. and Bishi B. K., Anisotropic Cosmological Models in $F(R, T)$ Gravity with Variable Deceleration Parameter, International Journal of Geometric Methods in Modern Physics, Vol. 14, Number 6, 1750097 (2017).
- [37] Sen D. K. and Dunn K. A., A ScalarTensor Theory of Gravitation in a Modified Riemannian Manifold, Journal of Mathematical Physics, Vol. 12, Number 4, 578 (1971).
- [38] Sen D. K., A Static Cosmological Model, Zeitschrift für Physik, Vol. 149, Number 3, 311 (1957).
- [39] Sharma U. K. and Pradhan A., Cosmology in Modified $F(R, T)$ -Gravity Theory in a Variant $\Lambda(T)$ Scenario-Revisited, International Journal of Geometric Methods in Modern Physics, Vol. 15, Number 1, 1850014 (2018).
- [40] Singh C. P. and Kumar P., Friedmann Model with Viscous Cosmology in Modified $F(R, T)$ Gravity Theory, European Physical Journal C, Vol. 74, Number 10, 3070 (2014).
- [41] Singh G. P., Bishi B. K. and Sahoo P. K., Cosmological Constant Λ in $F(R, T)$ Modified Gravity, International Journal of Geometric Methods in Modern Physics, Vol. 13, Number 5, 1650058 (2016).
- [42] Singh T. and Chaubey R., Bianchi Type-V Universe with a Viscous Fluid and Λ -Term, Pramana - Journal of Physics, Vol. 68, Number 5, 721 (2007).
- [43] Singh V. and Singh C. P., Modified $F(R, T)$ Gravity Theory and Scalar Field Cosmology, Astrophysics and Space Science, Vol. 356, Number 1, 153 (2015).
- [44] Spergel D. N. et al., ThreeYear Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Implications for Cosmology, The Astrophysical Journal Supplement Series, Vol. 170, Number 2, 377 (2007).
- [45] Spergel D. N. et al., First-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Determination of Cosmological Parameters, The Astrophysical Journal Supplement Series, Vol. 148, Number 1, 175 (2003).
- [46] Sontakke A. S. and Ghate H. R., Bianchi Type-IX Bulk Viscous String Cosmological Model in $F(R, T)$ Gravity with Special Form of Deceleration Parameter, International Journal of Theoretical and Mathematical Physics, Vol. 4, Number 6, 240 (2014).
- [47] Tegmark M. et al., Cosmological Parameters from SDSS and WMAP, Physical Review D, Vol. 69, Number 10, 103501 (2004).
- [48] Tiwari L. K. and Tiwari R. K., Bianchi Type-V Cosmological Models with Viscous Fluid & Varying Λ , Prespacetime Journal, Vol. 8, Number 14, 1509 (2017).
- [49] Weinberg S., Entropy Generation and the Survival of Protogalaxies in an Expanding Universe, The Astrophysical Journal, Vol. 168, 175 (1971).
- [50] Yadav A. K., Sharma L. K., Singh B. K. and Sahoo P. K., Existence of Bulk Viscous Universe in $F(R, T)$ Gravity and Confrontation with Observational Data, New Astronomy, Vol. 78, 101382 (2020).