

TOWARDS A NOVEL FAMILY OF ALGORITHMS IN PURSUIT NAVIGATION

HASSAN ATTARCHI¹, BEHROZ BIDABAD¹, SEYED MEHDI KAZEMI
TORBAGHAN² AND MOHAMMAD REZA KHOSRAVI³

¹DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE,
AMIRKABIR UNIVERSITY OF TECHNOLOGY, TEHRAN, IRAN.

²DEPARTMENT OF MATHEMATICS, UNIVERSITY OF BOJNORD,
BOJNORD, IRAN.

³MANAGEMENT AND ACCOUNTING, ALLAMEH TABATABA'I UNIVERSITY,
TEHRAN, IRAN.

E-MAILS: HASSAN.ATTARCHI@AUT.AC.IR, BIDABAD@AUT.AC.IR,
MEHDIKAZEMI@AUT.AC.IR, MOREKHOSRAVI@GMAIL.COM

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ABSTRACT. The problem of tracking and navigation towards a moving target is always one of the main topics in robotics. In this paper, following our recent work in the plane, a novel on-line strategy is used to guide a pursuer in space. This strategy is called hybrid pursuit navigation and it is a combination of two well-known pursuit algorithms the pure-pursuit and the pure-rendezvous navigation. To show the reliability of this new method, it is proved that the pursuer reaches the target under the hybrid pursuit navigation. Moreover, by simulating different scenarios, the advantage of this algorithm is demonstrated in comparison with two well-known algorithms.

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1. INTRODUCTION

One of the topics in navigation is the act of guiding a pursuer to reach a moving point. This act has applications in structured footballer robots, rescue robots, military equipments, etc. The related literature on this topic have various ranges and the most of them consider the target as a stationary point [2, 5, 6, 8]. One of the merits of this work is considering a moving goal. In this case, the target's path is unknown to the pursuer and off-line algorithms do not work properly. Therefore, the algorithm presented here is an on-line one.

In the case of a moving target, there exist mainly two classes of algorithms: Two-point navigation law and Three-point navigation law [9]. In the three-point navigation law, the algorithm uses a reference point for guiding the pursuer. The algorithm conducts the pursuer in a manner that it remains on the line of sight (a virtual line that connect the reference point to the target). In some literature this navigation is known as the Line-of-Sight algorithm [2, 5, 6, 8]. On the other hand in the two-point navigation law, there is no reference point and algorithms are based on two points: pursuer and target. There are two well-known pursuit algorithms for the two-point navigation law: pure-pursuit and pure-rendezvous navigation.

In three-point navigation law, because of commuting data between pursuer and reference point, the amount of noise and error in data increase. This shows the preference of using two-point navigation law in contrast with three-point navigation law. According to this, one of the superiorities of our method (hyper-pursuit navigation) is being of two-point navigation law.

In addition, most algorithms for pursuing a moving target are considered in 2-dimensional space $\mathbb{R}^2$ such as soccer or rescue robots [1, 10]. Our algorithm in this work is considered on the 3-dimensional real space $\mathbb{R}^3$. This is one of the differences between the present work and our previous paper [1]. The other differences of these works are about the method and the number of algorithms which are introduced. Here, we present a family of pursuing algorithms, but in the previous work we just introduced a single algorithm in 2-dimensional space by a completely different method and approach.

In this paper, our new family of pursuing algorithms is based on the Pure Pursuit (or PP) and Pure Rendezvous (or PR) navigations [3, 7]. This family of algorithms is called Hybrid Pursuit navigation (HP-navigation) and its kinematic property in

3-dimensional space $\mathbb{R}^3$ is studied here. Our goal for providing these algorithms is improving the accuracy and decreasing the trajectories length of two well-known algorithms PP and PR-navigation. In this aim, in Section 2, the formulation of the problem is presented and notations are fixed. In Section 3, the kinematic of the two well-known algorithms (PP and PR navigations) are briefly reviewed. In Section 4, the kinematic of Hybrid Pursuit navigation is presented as a combination of PP and PR navigation and the following proposition is proved:

Proposition 1. *Under the control strategy given by (5), and the assumptions made previously on pursuing problems, the pursuer reaches successfully its moving target. This proposition ensures that in the Hybrid Pursuit navigation algorithm, the target will be reached by pursuer. In section 5, two real examples are modeled and the trajectories of three algorithms PP, PR and HP-navigation are compared. These two examples are considered for different scenarios, in one the target passes through a predictable path (i.e. the direction of target's path does not change frequently or changes slightly) while in the other one the path of target is assumed to be unpredictable (i.e. the direction of target's path changes frequently).*

2. PROBLEM FORMULATION AND KINEMATICS MODELS

We shall refer to the target as T and to the pursuer as P and their velocity vectors as $v_T(t)$ and $v_P(t)$, respectively. In the Cartesian coordinates the target's path and the pursuer's path are denoted by $(x_T(t), y_T(t), z_T(t))$ and $(x_P(t), y_P(t), z_P(t))$, respectively. Here, our phase space in the control system is $\mathbb{R}^3$ and $(x_P(t), y_P(t), z_P(t))$ is our state variable [4]. The time when these two paths intercept is called the hitting moment and denoted by t_h . That is,

$$(x_T(t_h), y_T(t_h), z_T(t_h)) = (x_P(t_h), y_P(t_h), z_P(t_h)).$$

Moreover, the following conditions are typically assumed in pursuing problems and also in this paper:

- (1) The pursuer moves faster than target (here, we suppose that $|v_P(t)| = 1$ and $|v_T(t)| < 1$ for all t . $|\cdot|$ is the standard Euclidean norm on $\mathbb{R}^3$.)
- (2) The target's path is smooth.
- (3) The pursuer is able to measure the position and velocity of target.
- (4) The initial conditions of the pursuer agree with the navigation law of hybrid pursuit algorithm.

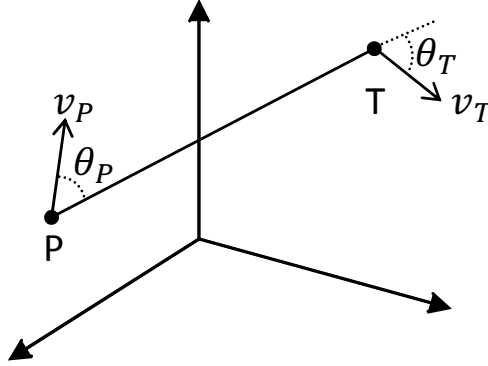


FIGURE 1. Geometry of the pursuing problem.

In Figure 1, the ray started at the pursuer P and directed to the target T is called the *line of sight* and it is denoted by $\overrightarrow{PT}(t)$. The angel between $v_T(t)$ and $\overrightarrow{PT}(t)$ is denoted by $\theta_T(t)$. Similarly, the angle between $v_P(t)$ and $\overrightarrow{PT}(t)$ is defined and denoted by $\theta_P(t)$. We denote in the sequel the length of the line of sight vector $\overrightarrow{PT}(t)$ by the real valued function $r(t)$, (i.e. $r(t) = |\overrightarrow{PT}(t)|$). If Δt , the variation of time, is sufficiently small, then the velocity of pursuer $v_P(t)$ and the velocity of target $v_T(t)$ can be considered (or at least approximately) as constants. It is well-known that, in order P meets T along the line of sight $\overrightarrow{PT}(t)$, $\Delta r(t)$ which is equal to the difference of displacement vectors of P and T along the line of sight should be negative. This displacement (i.e. $\Delta r(t)$) can be determined by projection of $v_T(t)$ and $v_P(t)$ on $\overrightarrow{PT}(t)$ as follows,

$$\begin{aligned} \Delta r(t) &= (|Proj_{\overrightarrow{PT}(t)} v_T(t)| - |Proj_{\overrightarrow{PT}(t)} v_P(t)|) \Delta t \\ &= (|v_T(t)| \cos(\theta_T(t)) - |v_P(t)| \cos(\theta_P(t))) \Delta t. \end{aligned}$$

Dividing both sides by Δt , as Δt approaches zero, we have:

$$(1) \quad \dot{r}(t) = |v_T(t)| \cos(\theta_T(t)) - |v_P(t)| \cos(\theta_P(t)).$$

3. PURE PURSUIT AND PURE RENDEZVOUS NAVIGATION

In Pure Pursuit navigation, the velocity vector field of pursuer $v_p(t)$ is always directed towards the target while in Pure Rendezvous navigation, $v_p(t)$ is directed towards some points in front of the target which are achieved by prediction of the path of target. This prediction forces the line of sight $\overrightarrow{PT}(t)$ to have constant direction during the pursuing process. In Figure 2, line of sights for two above navigations are presented.

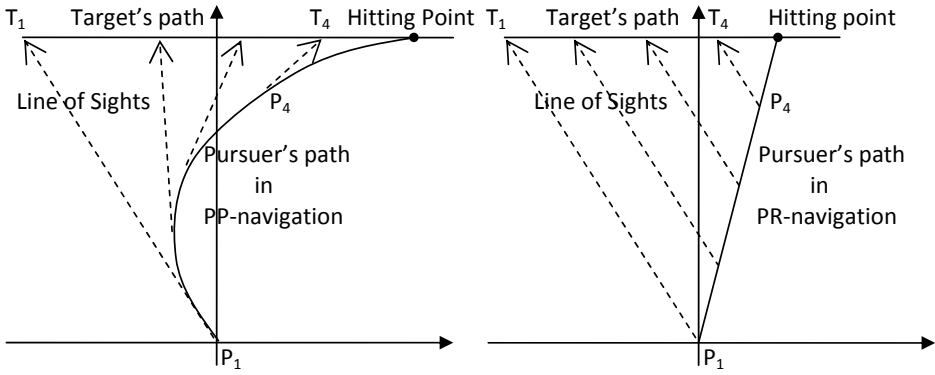


FIGURE 2. Line of sights for PP and PR-navigation.

3.1. Pure Pursuit navigation. The fundamental rule in Pure Pursuit navigation is that the line of sight $\overrightarrow{PT}(t)$ and the velocity of pursuer $v_p(t)$ are always in the same direction. That means $\theta_p(t) = 0$, and by replacing this equation in Eq. (1), we obtain:

$$(2) \quad \dot{r}(t) = |v_T(t)| \cos(\theta_T(t)) - |v_p(t)|.$$

Therefore, by means of the inequality $1 = |v_p(t)| > |v_T(t)|$, which is always supposed to be true in pursuit problems, we conclude from Eq. (2) that $\dot{r}(t)$ is negative. Indeed in the Pure Pursuit navigation, the fact $\dot{r}(t) < 0$ asserts that the pursuer will reach the target along the line of sight. The corresponding differential equation which gives the trajectories of this method is given by the following equation, cf. [3]

$$(3) \quad v_p(t) = \frac{\overrightarrow{PT}(t)}{r(t)}.$$

3.2. Pure Rendezvous navigation. The fundamental rule in Pure Rendezvous navigation is that the direction of line of sight $\overrightarrow{PT}(t)$ is constant. It is equivalent to the following equation:

$$(4) \quad v_P(t) = v_T(t) + C(t) \frac{\overrightarrow{PT}(t)}{r(t)},$$

where,

$$C(t) := -\frac{v_T(t) \cdot \overrightarrow{PT}(t)}{r(t)} + \sqrt{\left(\frac{v_T(t) \cdot \overrightarrow{PT}(t)}{r(t)} \right)^2 - |v_T(t)|^2 + 1} > 0,$$

provided $|v_P(t)| = 1$ be satisfied. From Eq. (4), it is easy to see that:

$$\begin{aligned} v_P(t) \cdot \overrightarrow{PT}(t) &> v_T(t) \cdot \overrightarrow{PT}(t), \\ |v_P(t)| \cos(\theta_P(t)) &> |v_T(t)| \cos(\theta_T(t)). \end{aligned}$$

Inserting the last equation in (1), we obtain:

$$\dot{r}(t) < 0.$$

This fact shows that the pursuer meets the target in this algorithm. Moreover, the trajectories of this method are achieved by Eq. (4).

4. KINEMATICS OF HYBRID PURSUIT ALGORITHMS

In this section we introduce a novel family of pursuit algorithms in navigation problems which are combination of both PP and PR navigation. Here, both algorithms presented in the subsections 3.1 and 3.2 are combined to introduce a family of algorithms which benefit advantages of these two algorithms. We refer to these new algorithms as Hybrid Pursuit navigation or simply HP-navigation. In the HP-navigation, the velocity vector of pursuer is a linear combination of the target velocity $v_T(t)$ and the line of sight $\overrightarrow{PT}(t)$ with non-negative coefficients.

The control system of HP navigation is given by:

$$v_P(t) = (\dot{x}_P(t), \dot{y}_P(t), \dot{z}_P(t)) = f(t, (x_P(t), y_P(t), z_P(t)), \vec{u}(t))$$

where $(x_P(t), y_P(t), z_P(t))$ is our state variable and $\vec{u}(t)$ is our control variable. The control variable $\vec{u}(t)$ is characterized by the position of target (i.e. $(x_T(t), y_T(t), z_T(t))$) and the velocity of target (i.e. $v_T(t) = (\dot{x}_T(t), \dot{y}_T(t), \dot{z}_T(t))$).

In HP-navigation, the velocity of pursuer $v_P(t)$ at the time t is determined to be in the plane spanned by $\overrightarrow{PT}(t)$ and $v_T(t)$. Therefore, the control system which

present the velocity of pursuer $v_P(t)$ in these algorithms can be written in the following form:

$$(5) \quad \begin{cases} v_P(t) := A(t)v_T(t) + B(t)\overrightarrow{PT}(t), & \forall t \geq 0, \\ 0 \leq A(t) \leq 1, \quad 0 < B(t) \leq \frac{1}{r(t)}, & \forall t \geq 0, \end{cases}$$

where coefficients $A(t)$ and $B(t)$ are two smooth non-negative functions which force $v_P(t)$ to be unitary vector. Obviously, if $A(t) = 0$ for all t , then by means of Eq. (5) and $|v_P(t)| = 1$ we obtain $B(t) = \frac{1}{r(t)}$. Hence, Eq. (3) yields that the Hybrid Pursuit algorithm coincides with Pure Pursuit algorithm when $A(t) \equiv 0$. Moreover, if $A(t) = 1$ for all t , by means of Eq. (4) the Hybrid Pursuit algorithm will coincide with Pure Rendezvous algorithm. Recalling the inequality $|v_T(t)| < |v_P(t)| = 1$, which is supposed to be true for all pursuing problems, one can show that the pursuer will reach the target in Hybrid Pursuit algorithms.

Proposition 4.1. *Under the control strategy given by (5), and the assumptions made previously on pursuing problems, the pursuer reaches successfully its moving target.*

Proof. To prove this theorem, we have to show that the length $r(t) = |\overrightarrow{PT}(t)|$ is a decreasing function of t or equivalently $\dot{r}(t)$ is negative in Eq. (1). By definition, we have

$$|v_T(t)| \cos(\theta_T(t)) = \frac{v_T(t) \cdot \overrightarrow{PT}(t)}{r(t)},$$

and

$$\cos(\theta_P(t)) = \frac{v_P(t) \cdot \overrightarrow{PT}(t)}{r(t)}.$$

Plugging Eq. (5) in the last equation we get

$$(6) \quad \cos(\theta_P(t)) = \frac{(A(t)v_T(t) + B(t)\overrightarrow{PT}(t)) \cdot \overrightarrow{PT}(t)}{r(t)} = \frac{A(t)v_T(t) \cdot \overrightarrow{PT}(t)}{r(t)} + B(t)r(t).$$

Moreover, using Eq. (5) and the fact $|v_P(t)| = 1$, we obtain

$$r(t)B(t) = -A(t)|v_T(t)| \cos(\theta_T(t)) + \sqrt{1 - (\sin(\theta_T(t))A(t)|v_T(t)|)^2},$$

where $\cos(\theta_P(t)) = \sqrt{1 - (\sin(\theta_T(t))A(t)|v_T(t)|)^2}$.

(Since, $\sin(\theta_T(t))A(t)|v_T(t)|$ is the length of normal component of $v_T(t)$ to $\overrightarrow{PT}(t)$ or equivalently $\sin(\theta_P(t)) = \sin(\theta_T(t))A(t)|v_T(t)|$.)

Therefore, by means of Eqs. (1) and (6) and $|v_P(t)| = 1$, we have

$$\dot{r}(t) = |v_T(t)| \cos(\theta_T(t)) - \cos(\theta_P(t)) = (1 - A(t)) \frac{v_T(t) \cdot \vec{PT}(t)}{r(t)} - B(t)r(t).$$

From which we obtain

$$\dot{r}(t) = (1 - A(t)) \frac{v_T(t) \cdot \vec{PT}(t)}{r(t)} + A(t)|v_T(t)| \cos(\theta_T(t)) - \sqrt{1 - (\sin(\theta_T(t))A(t)|v_T(t)|)^2}.$$

Finally using the inequality

$$||v_T(t)| \cos(\theta_T(t))| < |\cos(\theta_T(t))| < \sqrt{1 - (\sin(\theta_T(t))A(t)|v_T(t)|)^2},$$

we obtain

$$\dot{r}(t) = |v_T(t)| \cos(\theta_T(t)) - \sqrt{1 - (\sin(\theta_T(t))A(t)|v_T(t)|)^2} < 0.$$

This shows that the $r(t)$ is a decreasing function and hence the proof of theorem is complete. $\square$

5. SIMULATING TWO DIFFERENT SCENARIOS OF HYBRID PURSUIT ALGORITHMS

In order to pursue an unknown target in real world, the trajectory of the target should be observed regularly. The path of the target is unpredictable if its direction changes frequently. In this section, two examples of Hybrid Pursuit algorithms are given and their trajectories are compared with those of the two other well-known algorithms.

In these examples, we fix the coefficient $A(t)$ to be approximately 1 at beginning moments and then decreases smoothly to approach zero in the hitting moment. It means that the Hybrid Pursuit algorithm, in these examples, starts with prediction of the target's path by help of $v_T(t)$ and by getting closer to the target this method change smoothly to Pure Pursuit algorithm for more accuracy.

Now, we are in a position to present the coefficients $A(t)$ and $B(t)$. As explained, we put

$$(7) \quad A(t) := \frac{r(t)}{r(t) + |v_T(t)|}.$$

Next, by using the assumption $|v_P(t)| = 1$ in Eq. (5), the function $B(t)$ becomes:

$$(8) \quad B(t) := -\frac{\vec{PT}(t) \cdot v_T(t)}{r(t) + |v_T(t)|} + \sqrt{\left(\frac{\vec{PT}(t) \cdot v_T(t)}{r(t) + |v_T(t)|}\right)^2 - \left(\frac{r(t)|v_T(t)|}{r(t) + |v_T(t)|}\right)^2} + 1.$$

Now, we state our examples here. In Example 1, the target travels a predictable path (i.e. the direction of $v_T(t)$ does not change frequently or changes slightly) while in Example 2 the path of the target is assumed to be unpredictable (i.e. the direction of $v_T(t)$ changes frequently).

Example 1: Assume that there is a target moving along the parametric curve $(\frac{t}{2} - 5, \frac{t}{2} - 5, 5)$. Therefore, the velocity vector $v_T(t)$ and the line of sight $\overrightarrow{PT}(t)$ are given by

$$v_T(t) = (\frac{1}{2}, \frac{1}{2}, 0), \quad \overrightarrow{PT}(t) = (\frac{t}{2} - 5 - x(t), \frac{t}{2} - 5 - y(t), 5 - z(t)),$$

respectively, where $(x(t), y(t), z(t))$ is the parametric trajectory of the pursuer. To find the trajectory of pursuer in the Hybrid Pursuit algorithm given by Eqs. (7) and (8), we have to solve Eq. (5). It yields the following system of first order differential equations

$$(9) \quad \begin{cases} \dot{x}(t) = \frac{1}{2}A(t) + \frac{B(t)}{r(t)}(\frac{t}{2} - 5 - x(t)), \\ \dot{y}(t) = \frac{1}{2}A(t) + \frac{B(t)}{r(t)}(\frac{t}{2} - 5 - y(t)), \\ \dot{z}(t) = \frac{B(t)}{r(t)}(5 - z(t)). \end{cases}$$

Here, the coefficients $A(t)$ and $B(t)$ are determined by $v_T(t)$, $\overrightarrow{PT}(t)$ and $r(t) =$

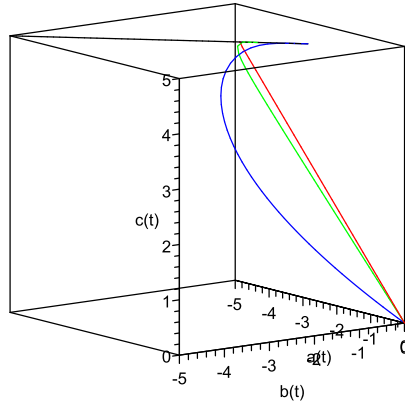


FIGURE 3. Trajectories of pursuer in PP, PR and CP navigations in Example 1.

$|\overrightarrow{PT}(t)|$ in Eqs. (7) and (8). Therefore, we are able to solve Eq. (9) with the initial condition $(x(0), y(0), z(0)) = (0, 0, 0)$. Similarly, trajectories of Pure Pursuit and Pure Rendezvous navigation are achieved by solving the system of first order differential equations (3) and (4), respectively, with the same initial conditions.

These equations are solved with Maple software and their trajectories in PP, PR and HP-navigation are sketched in Figure 3. The wasting time for meeting the target in PP, PR and HP-navigation are 7.55 s, 5.83 s and 6.30 s, respectively.

In Examples 1 and 2, target's paths are denoted by black curves. The trajectories of pursuer in PP, PR and HP algorithms are denoted by Blue, Green and Red curves, respectively. This example shows the advantage of HP-navigation with respect to the PP-navigation, whenever the target's path is predictable.

Example 2: We assume that the path of target is a helix given by:

$$(2 \cos(\frac{\sqrt{2}}{4}t), 2 \sin(\frac{\sqrt{2}}{4}t), \frac{t}{2} + 5).$$

The trajectories of these three algorithms are sketched by maple in Figure 4. Here, we use same notation as the Example 1 for presenting the paths.

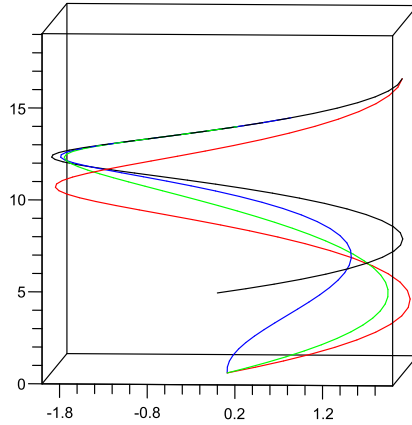


FIGURE 4. Trajectories of pursuer in PP, PR and CP navigations in example 2.

The wasting time for meeting the target in PP, PR and HP-navigations are 18.98 s, 22.15 s and 18.09 s, respectively. This example shows the prominence of

the HP-navigation in comparison with respect to the algorithms of PP and PR-navigation.

6. CONCLUSION AND AN OPEN PROBLEM

In this paper, we presented a family of algorithms for navigating a robot toward a moving goal. The target's path is not a priori known to the robot and we use a real-time strategy in this family of navigation. Another advantage of this method is that its model space is the 3-dimensional real space $\mathbb{R}^3$, unlike many other control laws which base on 2-dimensional model space. Moreover, it is proven that under the suggested navigation law, the robot reaches successfully the moving goal. Considering the times and trajectories of these three algorithms in these examples show the advantage of Hybrid Pursuit navigation. In fact, flexibility of the HP-navigation intensifies its influence. More precisely, the coefficients $A(t)$ and $B(t)$ in Eq. (5) can be chosen to increase the accuracy in optimal time.

Regarding the coefficients $A(t)$ and $B(t)$ in Eq. (5), one can propose the following interesting problem: "Characterize the coefficients $A(t)$ and $B(t)$ in Eq. (5) with respect to the target's path, to minimize the interception time."

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