

CERTAIN CURVES ASSOCIATED WITH f -KENMOTSU MANIFOLDS

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ABSTRACT. The aim of the present paper is to study slant magnetic curves, magnetic curves and biharmonic curves on a 3-dimensional f -Kenmotsu manifold. Next we deal with curve for which tangent vector of the curve is parallel to ξ . Also we characterize ξ -vertical hypersurface in a 3-dimensional f -Kenmotsu manifold.

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1. INTRODUCTION

In the paper [6], Cabrerizo et al. have introduced a geometric approach to the study of magnetic fields on three dimensional Sasakian manifolds. In a 3-dimensional f -Kenmotsu manifold, a curve γ is said to be magnetic curve if $\nabla_{\dot{\gamma}}\dot{\gamma} = \phi\dot{\gamma}$, where ∇ is the Levi-Civita connection. A Frenet curve is called a slant curve

if it makes a constant angle with the Reeb vector field ξ ([7]). A magnetic curve is the trajectory of magnetic fields. Geodesics on a smooth manifold are curves which do not experience any kind of forces where the magnetic curves experience due to magnetic fields. The magnetic curves become geodesics if the magnetic field disappears. Thus a magnetic curve is a generalization of a geodesic.

Examples of known classical geometric variational problem provided by an elastic curve. A plane curve is said to be an elastic curve if it is a critical point of elastic energy, or equivalently a critical point of the total squared curvature ([13]). In ([10]), Inoguchi et al. have studied another geometric variational problem in Riemannian 2-manifolds of constant curvature. They have investigated biminial curves in 2-dimensional space forms. The Euler-Lagrange equation studied in the paper ([10]) is derived from the theory of biharmonic maps in Riemannian geometry. For more details we refer to the readers ([5], [10], [15]) and references therein.

Next we recall the following fundamental result of Loubeau and Montaldo ([14]):

Lemma 1.1. *A unit speed curve $\gamma(s)$ in a Riemannian 2-manifold of Gaussian curvature K is biminimal if and only if its signed curvature $k(s)$ satisfies*

$$(1) \quad k'' - k^3 + kK = 0.$$

Legendre curves on contact manifolds have been studied by Baikoussis and Blair in the paper ([2]). The classical characterization of these curves is the Bertrand-Lancret-de Saint Venant Theorem ([4]). In the paper [9], the author has introduced the concept of ξ -vertical and ξ -horizontal. Following it we shall call the hypersurface of a three dimensional f -Kenmotsu manifold as ξ -vertical if the tangent vector fields of the hypersurface is orthogonal to ξ . Also Mondal et al. have studied some curves in α -Sasakian and trans-Sasakian manifolds respectively in ([16], [17]). It should be mentioned that Sarkar et al. have studied slant and magnetic curves in Kenmotsu manifolds ([22], [23]). Here we investigate the biharmonic magnetic curves on three dimensional f -Kenmotsu manifolds. Motivated by the above studies in the present paper we are interested in studying biminimal curves in ξ -vertical hypersurface of three-dimensional f -Kenmotsu manifolds with constant curvature.

The paper is organized as follows:

After preliminaries in Section 2, we consider slant magnetic curves, magnetic curves and biharmonic curves on 3-dimensional f -Kenmotsu manifolds in section

3. In the next section we deal with ξ -vertical hypersurface in a 3-dimensional f -Kenmotsu manifold.

2. PRELIMINARIES

An odd dimensional smooth manifold M^{2n+1} ($n \geq 1$) is said to admit an almost contact structure, sometimes called a (ϕ, ξ, η) -structure, if it admits a tensor field ϕ of type $(1, 1)$, a vector field ξ and a 1-form η satisfying ([3])

$$(2) \quad \phi^2 = -I + \eta \otimes \xi, \quad \eta(\xi) = 1, \quad \phi\xi = 0, \quad \eta \circ \phi = 0.$$

The first and one of the remaining three relations in (2) imply the other two relations in (2). An almost contact structure is said to be normal if the induced almost complex structure J on $M^n \times \mathbb{R}$ defined by

$$(3) \quad J(X, f \frac{d}{dt}) = (\phi X - f\xi, \eta(X) \frac{d}{dt})$$

is integrable, where X is tangent to M , t is the coordinate of $\mathbb{R}$ and f is a smooth function on $M^n \times \mathbb{R}$. Let g be a compatible Riemannian metric with (ϕ, ξ, η) , structure, that is,

$$(4) \quad g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y)$$

or equivalently,

$$g(X, \phi Y) = -g(\phi X, Y)$$

and

$$g(X, \xi) = \eta(X),$$

for all vector fields X, Y tangent to M . Then M becomes an almost contact metric manifold equipped with an almost contact metric structure (ϕ, ξ, η, g) .

An almost contact metric structure becomes a contact metric structure if

$$g(X, \phi Y) = d\eta(X, Y),$$

for all X, Y tangent to M . The 1-form η is then a contact form and ξ is its characteristic vector field. For further use, we recall the following definitions ([3], [8], [21]).

Olszak and Rosca [18] studied normal locally conformal almost cosymplectic manifold. They gave a geometric interpretation of f -Kenmotsu manifolds and studied some curvature properties.

We say that (M, ϕ, ξ, η, g) is an f -Kenmotsu manifold if the covariant differentiation of ϕ satisfies [19]:

$$(5) \quad (\nabla_X \phi)(Y) = f(g(\phi X, Y)\xi - \eta(Y)\phi X),$$

where $f \in C^\infty(M)$ such that $df \wedge \eta = 0$. In this connection we can observe the followings:

- If $f = \alpha = \text{constant} \neq 0$, then the manifold is a α -Kenmotsu manifold [11].
- 1-Kenmotsu manifold is a Kenmotsu manifold ([12], [20]).
- If $f = 0$, then the manifold is cosymplectic [11].

An f -Kenmotsu manifold is said to be regular if $f^2 + f' \neq 0$, where $f' = \xi f$, where f' denotes derivative of f . In an f -Kenmotsu manifold from (2) it follows that

$$(6) \quad \nabla_X \xi = f(X - \eta(X)\xi).$$

The condition $df \wedge \eta = 0$ holds if dimension of M is greater or equal to 5. In general this does not hold if dimension of M equals to 3 [18]. In a 3-dimensional f -Kenmotsu manifold, we have ([18], [20], [24])

$$(7) \quad \begin{aligned} R(X, Y)Z &= \left(\frac{r}{2} + 2f^2 + 2f'\right)\{g(Y, Z)X - g(X, Z)Y\} \\ &\quad - \left(\frac{r}{2} + 3f^2 + 3f'\right)[\eta(X)\{g(Y, Z)\xi - g(\xi, Z)Y\} \\ &\quad + \eta(Y)\{g(\xi, Z)X - g(X, Z)\xi\}]. \end{aligned}$$

$$(8) \quad S(X, Y) = \left(\frac{r}{2} + 2f^2 + 2f'\right)g(Y, Z)X - \left(\frac{r}{2} + 3f^2 + 3f'\right)\eta(X)\eta(Y),$$

where r is a scalar curvature of M and $f' = \xi f$.

3. SLANT MAGNETIC CURVES IN 3-DIMENSIONAL f -KENMOTSU MANIFOLDS

Let γ be a slant curve in 3-dimensional f -Kenmotsu manifold. Then

$$(9) \quad g(\dot{\gamma}, \xi) = \cos \theta,$$

where θ is a constant angel between tangent vector and ξ . Differentiating (9) with respect to $\dot{\gamma}$, we have

$$(10) \quad g(\nabla_{\dot{\gamma}}\dot{\gamma}, \xi) + g(\dot{\gamma}, \nabla_{\dot{\gamma}}\xi) = 0.$$

In view of (6), we get

$$(11) \quad g(\nabla_{\dot{\gamma}}\dot{\gamma}, \xi) + g(\dot{\gamma}, f(\dot{\gamma} - \eta(\dot{\gamma})\xi)) = 0.$$

$$(12) \quad g(\nabla_{\dot{\gamma}}\dot{\gamma}, \xi) + f\{1 - (\eta(\dot{\gamma}))^2\} = 0.$$

If the curve is magnetic curve. Then we have

$$(13) \quad g(\phi\dot{\gamma}, \xi) + f(1 - (\eta(\dot{\gamma}))^2) = 0.$$

As $\phi\xi = 0$ and $f \neq 0$, therefore

$$(14) \quad (\eta(\dot{\gamma}))^2 = 1.$$

$$(15) \quad \eta(\dot{\gamma}) = \pm 1.$$

Since g is positive definite, we obtain

$$(16) \quad \eta(\dot{\gamma}) = g(\dot{\gamma}, \xi) \geq 0,$$

and hence

$$(17) \quad \eta(\dot{\gamma}) = 1 = \cos 2n\pi.$$

Therefore tangent vector is parallel to ξ .

Then we can state the following:

Theorem 3.1. *For a slant magnetic curve γ in a 3-dimensional f -Kenmotsu manifold, the tangent vector is parallel to ξ .*

Let γ be a magnetic curve on 3-dimensional f -Kenmotsu manifolds.
Then

$$(18) \quad \nabla_{\dot{\gamma}}\dot{\gamma} = \phi\dot{\gamma}.$$

Let $\sigma = \eta(\dot{\gamma})$.

In view of $\dot{\gamma}$, $\frac{\phi\dot{\gamma}}{\sqrt{1-\sigma^2}}$ and $\frac{\xi-\sigma\dot{\gamma}}{\sqrt{1-\sigma^2}}$ are orthonormal vectors along γ .

Again

$$(19) \quad \nabla_{\dot{\gamma}} E_1 = \nabla_{\dot{\gamma}} \dot{\gamma} = \sqrt{1-\sigma^2} \cdot \frac{\phi\dot{\gamma}}{\sqrt{1-\sigma^2}} + 0 \cdot \frac{\xi-\sigma\dot{\gamma}}{\sqrt{1-\sigma^2}}.$$

Then

$$\kappa = \sqrt{(\sqrt{1-\sigma^2})^2 + 0^2}.$$

Therefore

$$(20) \quad \kappa = \sqrt{1-\sigma^2}.$$

Now

$$(21) \quad E_2 = \frac{1}{\kappa} \nabla_{\dot{\gamma}} E_1 = \frac{1}{\sqrt{1-\sigma^2}} \nabla_{\dot{\gamma}} \dot{\gamma} = \frac{1}{\sqrt{1-\sigma^2}} \phi\dot{\gamma}.$$

Differentiating along $\dot{\gamma}$, we obtain

$$(22) \quad \begin{aligned} \nabla_{\dot{\gamma}} E_2 &= \nabla_{\dot{\gamma}} \left(\frac{1}{\sqrt{1-\sigma^2}} \phi\dot{\gamma} \right) = \nabla_{\dot{\gamma}} \left(\frac{1}{\sqrt{1-\sigma^2}} \right) \phi\dot{\gamma} + \frac{1}{\sqrt{1-\sigma^2}} \nabla_{\dot{\gamma}} (\phi\dot{\gamma}). \\ &= \frac{\sigma\dot{\sigma}}{(1-\sigma^2)^{\frac{3}{2}}} \phi\dot{\gamma} + \frac{1}{\sqrt{1-\sigma^2}} \nabla_{\dot{\gamma}} (\phi\dot{\gamma}). \end{aligned}$$

$$(23) \quad \begin{aligned} \nabla_{\dot{\gamma}} (\phi\dot{\gamma}) &= (\nabla_{\dot{\gamma}} \phi) \dot{\gamma} + \phi (\nabla_{\dot{\gamma}} \dot{\gamma}) \\ &= f(g(\phi\dot{\gamma}, \dot{\gamma}) - \eta(\dot{\gamma})\phi\dot{\gamma}) + \phi^2 \dot{\gamma} \\ &= -f\eta(\dot{\gamma})\phi\dot{\gamma} - \dot{\gamma} + \eta(\dot{\gamma})\xi \\ &= \eta(\dot{\gamma})\xi - f\eta(\dot{\gamma})\phi\dot{\gamma} - \dot{\gamma}. \end{aligned}$$

Also

$$(24) \quad \sigma = g(\dot{\gamma}, \xi)$$

Differentiating along $\dot{\gamma}$, we have

$$(25) \quad \begin{aligned} \dot{\sigma} &= \nabla_{\dot{\gamma}} g(\dot{\gamma}, \xi) = \dot{\gamma}(g(\dot{\gamma}, \xi)) \\ &= g(\nabla_{\dot{\gamma}} \dot{\gamma}, \xi) + g(\dot{\gamma}, \nabla_{\dot{\gamma}} \xi) \\ &= g(\phi\dot{\gamma}, \xi) + g(\dot{\gamma}, f(\dot{\gamma} - \eta(\dot{\gamma})\xi)) \\ &= f(1 - (\eta(\dot{\gamma}))^2). \end{aligned}$$

Then from (22) and (23) and (25), we get

$$\begin{aligned}
 \nabla_{\dot{\gamma}} E_2 &= \frac{f\sigma}{(1-\sigma^2)^{\frac{3}{2}}}(1-(\eta(\dot{\gamma}))^2)\phi\dot{\gamma} + \frac{1}{(1-\sigma^2)^{\frac{1}{2}}}(\eta(\dot{\gamma})\xi - f\eta(\dot{\gamma})\phi\dot{\gamma} - \dot{\gamma}) \\
 &= \frac{f\sigma}{\sqrt{1-\sigma^2}}\phi\dot{\gamma} + \frac{\sigma}{\sqrt{1-\sigma^2}}\xi - \frac{f\sigma}{\sqrt{1-\sigma^2}}\phi\dot{\gamma} \\
 &\quad - \frac{1}{\sqrt{1-\sigma^2}}\dot{\gamma} \\
 (26) \quad &= \sigma \frac{\xi - \sigma\dot{\gamma}}{\sqrt{1-\sigma^2}} - \sqrt{1-\sigma^2}\dot{\gamma}.
 \end{aligned}$$

Again

$$\begin{aligned}
 \tau E_3 &= \nabla_{\dot{\gamma}} E_2 + \kappa E_1 \\
 (27) \quad &= \sigma \frac{\xi - \sigma\dot{\gamma}}{\sqrt{1-\sigma^2}} + 0.\dot{\gamma}.
 \end{aligned}$$

Therefore

$$(28) \quad \tau = \sigma.$$

Hence

$$(29) \quad \kappa = \sqrt{1-\sigma^2},$$

and

$$(30) \quad \tau = \sigma.$$

Thus we can state the following:

Theorem 3.2. *Let M be a 3-dimensional f -Kenmotsu manifold. Then for the magnetic curve γ it's curvature and torsion are given by $\kappa = \sqrt{1-\sigma^2}$, $\tau = \sigma$.*

Definition 3.3. [23] *A unit speed curve γ on a smooth manifold is called biharmonic with respect to the Levi-Civita connection ∇ if it satisfies*

$$(31) \quad \nabla_T^3 T + R(\nabla_T T, T)T = 0,$$

where $T = \dot{\gamma}$.

Again suppose a magnetic curve γ on a 3-dimensional f -Kenmotsu manifold. Then we have

$$(32) \quad \nabla_{\dot{\gamma}} \dot{\gamma} = \phi\dot{\gamma}.$$

Differentiating the foregoing equation along $\dot{\gamma}$, we obtain

$$\begin{aligned}
\nabla_{\dot{\gamma}}^2 \dot{\gamma} &= \nabla_{\dot{\gamma}}(\phi \dot{\gamma}) \\
&= (\nabla_{\dot{\gamma}} \phi) \dot{\gamma} + \phi (\nabla_{\dot{\gamma}} \dot{\gamma}) \\
&= (\nabla_{\dot{\gamma}} \phi) \dot{\gamma} + \phi^2 \dot{\gamma} \\
&= (\nabla_{\dot{\gamma}} \phi) \dot{\gamma} - \dot{\gamma} + \eta(\dot{\gamma}) \xi \\
&= f(g(\phi \dot{\gamma}, \dot{\gamma}) \xi - \eta(\dot{\gamma}) \phi \dot{\gamma}) - \dot{\gamma} + \eta(\dot{\gamma}) \xi \\
&= -f\eta(\dot{\gamma}) \phi \dot{\gamma} - \dot{\gamma} + \eta(\dot{\gamma}) \xi \\
(33) \quad &= \eta(\dot{\gamma})(\xi - f\phi \dot{\gamma}) - \dot{\gamma}.
\end{aligned}$$

Differentiating the above equation (33) along $\dot{\gamma}$, we have

$$\begin{aligned}
\nabla_{\dot{\gamma}}^3 \dot{\gamma} &= \nabla_{\dot{\gamma}}(\nabla_{\dot{\gamma}}^2 \dot{\gamma}) \\
&= \nabla_{\dot{\gamma}}(\eta(\dot{\gamma})(\xi - f\phi \dot{\gamma})) - \nabla_{\dot{\gamma}} \dot{\gamma} \\
&= \nabla_{\dot{\gamma}} g(\dot{\gamma}, \xi)(\xi - f\phi \dot{\gamma}) + \eta(\dot{\gamma})(\nabla_{\dot{\gamma}} \xi \\
&\quad - \nabla_{\dot{\gamma}}(f\phi \dot{\gamma})) - \phi \dot{\gamma} \\
&= (g(\nabla_{\dot{\gamma}} \dot{\gamma}, \xi) + g(\dot{\gamma}, \nabla_{\dot{\gamma}} \xi))(\xi - f\phi \dot{\gamma}) \\
&\quad + \eta(\dot{\gamma})(f\dot{\gamma} - f\eta(\dot{\gamma})\xi - (\dot{\gamma}f)\phi \dot{\gamma} - f\nabla_{\dot{\gamma}}(\phi \dot{\gamma})) - \phi \dot{\gamma} \\
&= (g(\phi \dot{\gamma}, \xi) + g(\dot{\gamma}, f(\dot{\gamma} - \eta(\dot{\gamma})\xi)))(\xi - f\phi \dot{\gamma}) \\
&\quad + \eta(\dot{\gamma})(f\dot{\gamma} - f\eta(\dot{\gamma})\xi - (\dot{\gamma}f)\phi \dot{\gamma} - f\nabla_{\dot{\gamma}}^2 \dot{\gamma}) - \phi \dot{\gamma} \\
&= (0 + f - f(\eta(\dot{\gamma}))^2)(\xi - f\phi \dot{\gamma}) + \eta(\dot{\gamma})(f\dot{\gamma} - f\eta(\dot{\gamma})\xi \\
&\quad - (\dot{\gamma}f)\phi \dot{\gamma} - f\eta(\dot{\gamma})(\xi - f\phi \dot{\gamma}) + f\dot{\gamma}) - \phi \dot{\gamma} \\
&= f\xi - 3f(\eta(\dot{\gamma}))^2\xi + 2f^2(\eta(\dot{\gamma}))^2\phi \dot{\gamma} - \eta(\dot{\gamma})(\dot{\gamma}f)\phi \dot{\gamma} \\
(34) \quad &\quad + 2\eta(\dot{\gamma})f\dot{\gamma} - \phi \dot{\gamma} - f^2\phi \dot{\gamma}.
\end{aligned}$$

It follows that

$$\begin{aligned}
 R(\nabla_{\dot{\gamma}}\dot{\gamma}, \dot{\gamma})\dot{\gamma} &= \left(\frac{r}{2} + 2f^2 + 2f'\right)(g(\dot{\gamma}, \dot{\gamma})\nabla_{\dot{\gamma}}\dot{\gamma} \\
 &\quad - g(\nabla_{\dot{\gamma}}\dot{\gamma}, \dot{\gamma})\dot{\gamma}) - \left(\frac{r}{2} + 3f^2 + 3f'\right)(\eta(\nabla_{\dot{\gamma}}\dot{\gamma})(g(\dot{\gamma}, \dot{\gamma})\xi \\
 &\quad - g(\xi, \dot{\gamma})\dot{\gamma}) + \eta(\dot{\gamma})(g(\xi, \dot{\gamma})\nabla_{\dot{\gamma}}\dot{\gamma} - g(\nabla_{\dot{\gamma}}\dot{\gamma}, \dot{\gamma})\xi)) \\
 &= \left(\frac{r}{2} + 2f^2 + 2f'\right)(\nabla_{\dot{\gamma}}\dot{\gamma} - g(\phi\dot{\gamma}, \dot{\gamma})\dot{\gamma}) \\
 &\quad - \left(\frac{r}{2} + 3f^2 + 3f'\right)(\eta(\phi\dot{\gamma})(\xi - \eta(\dot{\gamma})\dot{\gamma}) + \eta(\dot{\gamma})(\eta(\dot{\gamma})\phi\dot{\gamma} \\
 &\quad - g(\phi\dot{\gamma}, \dot{\gamma})\xi)) \\
 (35) \qquad &= \left(\frac{r}{2} + 2f^2 + 2f'\right)\phi\dot{\gamma} - \left(\frac{r}{2} + 3f^2 + 3f'\right)(\eta(\dot{\gamma}))^2\phi\dot{\gamma}.
 \end{aligned}$$

Now, if the curve is biharmonic. Then

$$(36) \qquad \nabla_{\dot{\gamma}}^3\dot{\gamma} + R(\nabla_{\dot{\gamma}}\dot{\gamma}, \dot{\gamma})\dot{\gamma} = 0.$$

This implies

$$\begin{aligned}
 &f\xi - 3f(\eta(\dot{\gamma}))^2\xi + 2f^2(\eta(\dot{\gamma}))^2\phi\dot{\gamma} - \eta(\dot{\gamma})(\dot{\gamma}f)\phi\dot{\gamma} + 2\eta(\dot{\gamma})f\dot{\gamma} - \phi\dot{\gamma} - f^2\phi\dot{\gamma} \\
 (37) \quad &+ \left(\frac{r}{2} + 2f^2 + 2f'\right)\phi\dot{\gamma} - \left(\frac{r}{2} + 3f^2 + 3f'\right)(\eta(\dot{\gamma}))^2\phi\dot{\gamma} = 0.
 \end{aligned}$$

Taking inner product with ξ in the equation (37), we have

$$(38) \qquad f - 3f(\eta(\dot{\gamma}))^2 + 2f(\eta(\dot{\gamma}))^2 = 0.$$

Since $f \neq 0$, we get

$$(39) \qquad (\eta(\dot{\gamma}))^2 = 1.$$

$$(40) \qquad \eta(\dot{\gamma}) = \pm 1.$$

As the metric g is positive definite

$$(41) \qquad \eta(\dot{\gamma}) = g(\dot{\gamma}, \xi) \geq 0$$

Therefore

$$(42) \qquad \eta(\dot{\gamma}) = 1 = \cos 2n\pi.$$

Therefore the curve is slant curve and tangent is parallel to ξ . Thus we can state the following:

Theorem 3.4. *A magnetic biharmonic curve in a 3-dimensional f -Kenmotsu manifold is a slant curve and its tangent vector is parallel to ξ .*

4. ξ -VERTICAL HYPERSURFACE IN A 3-DIMENSIONAL f -KENMOTSU MANIFOLD

A Riemannian manifold with constant sectional curvature λ is known as space of constant curvature and its curvature tensor is given

$$(43) \quad R(X, Y)Z = \lambda(g(Y, Z)X - g(X, Z)Y).$$

Let us consider a 3-dimensional f -Kenmotsu manifold. In view of (7) we have

$$(44) \quad \begin{aligned} R(X, Y)Z &= \left(\frac{r}{2} + 2f^2 + 2f'\right)(g(Y, Z)X - g(X, Z)Y) \\ &\quad - \left(\frac{r}{2} + 3f^2 + 3f'\right)(\eta(X)g(Y, Z)\xi - \eta(X)\eta(Z)Y \\ &\quad + \eta(Y)\eta(Z)X - \eta(Y)g(X, Z)\xi). \end{aligned}$$

From (43) and (44), we obtain

$$(45) \quad \begin{aligned} \lambda(g(Y, Z)X - g(X, Z)Y) &= \left(\frac{r}{2} + 2f^2 + 2f'\right)(g(Y, Z)X - g(X, Z)Y) \\ &\quad - \left(\frac{r}{2} + 3f^2 + 3f'\right)(\eta(X)g(Y, Z)\xi \\ &\quad - \eta(X)\eta(Z)Y + \eta(Y)\eta(Z)X \\ &\quad - \eta(Y)g(X, Z)\xi). \end{aligned}$$

It follows that

$$(46) \quad \begin{aligned} &\left(\lambda - \frac{r}{2} - 2f^2 - 2f'\right)(g(Y, Z)X - g(X, Z)Y) \\ &= -\left(\frac{r}{2} + 3f^2 + 3f'\right)(\eta(X)g(Y, Z)\xi - \eta(X)\eta(Z)Y \\ &\quad + \eta(Y)\eta(Z)X - \eta(Y)g(X, Z)\xi). \end{aligned}$$

Taking $Y = \xi$ in the foregoing equation, we get

$$(47) \quad \begin{aligned} &\left(\lambda - \frac{r}{2} - 2f^2 - 2f'\right)(\eta(Z)X - g(X, Z)\xi) \\ &= -\left(\frac{r}{2} + 3f^2 + 3f'\right)(\eta(X)\eta(Z)\xi \\ &\quad - \eta(X)\eta(Z)\xi + \eta(Z)X - g(Y, Z)\xi). \end{aligned}$$

From which it follows that

$$(48) \quad \lambda = -(f^2 + f').$$

Then from (43) we obtain

$$(49) \quad R(X, Y)Z = -(f^2 + f')(g(Y, Z)X - g(X, Z)Y).$$

Taking inner product with W , we have

$$\begin{aligned} g(R(X, Y)Z, W) &= -(f^2 + f')g(g(Y, Z)X - g(X, Z)Y, W) \\ (50) \quad &= -(f^2 + f')(g(g(Y, Z)X, W) - g(g(X, Z)Y, W)). \end{aligned}$$

Let $\{e_i : 1 \leq i \leq 3\}$ be an orthonormal basis of the tangent space at each point of the manifold. Then taking $Y = Z = e_i$ and summing over i , we obtain

$$(51) \quad g(R(X, e_i)e_i, W) = -(f^2 + f')(g(e_i, e_i)g(X, W) - g(X, e_i)g(e_i, W)).$$

$$(52) \quad S(X, W) = -(f^2 + f')(3g(X, W) - g(X, W)).$$

$$(53) \quad S(X, W) = -2(f^2 + f')g(X, W).$$

Contracting X, W in the foregoing equation, we obtain

$$(54) \quad r = -2(f^2 + f')3.$$

It follows that

$$(55) \quad r = -6(f^2 + f').$$

Let S_1 be the Ricci tensor and r_1 be the scalar curvature of the ξ -vertical hyper surface of the manifold. Then we have

$$(56) \quad S_1(X, W) = -(f^2 + f')g(X, W),$$

where X, W are orthogonal to ξ .

Again contracting X, W in (56), we obtain

$$(57) \quad r_1 = -2(f^2 + f').$$

Then from (55) and (57) we have

$$(58) \quad r_1 = \frac{1}{3}r.$$

Theorem 4.1. *The scalar curvature of the ξ -vertical hypersurface of a 3-dimensional f -Kenmotsu manifold having space of constant curvature is $r_1 = \frac{1}{3}r$, where r is scalar curvature of the manifold.*

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